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Influence of Disorder on the Fidelity Susceptibility in the BCS-BEC Crossover 6th APCWQIS, December 2012

Bilal Tanatar



December 6, 2012







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| Prologue | | | | |

- Observation of superconductivity in 1911.
- ✓ Theoretical Prediction of Bose-Einstein Condensate in 1925.
- ✓ Theory of Superconductivity (BCS) in 1957.
- ✓ Experimental Observation of Bose-Einstein Condensate in 1995.



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Image Source:

1. http://www.quantumconsciousness.org/penrose-hameroff/anesthesiahydrophobic.html

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| Cooling Techniques | | | | |



Sisyphus Cooling



- Six lasers applied opposite to each other from each direction.
- Light is red detuned to activate Doppler effect.
- Temperature attain $\lesssim 1 m K$.

 Opposite polarization of the laser beams create a potential crest and valley.

• Takes down temperature to $\lesssim 1 \mu K$.

Image Source:

2. http://www.physics.otago.ac.nz/research/jackdodd/resources/exp_aspects.html

3. http://cold-atoms.physics.lsa.umich.edu/projects/lattice/sis1.html

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 Cut the higher edge of the magnetic trap with rf spectra.
 Atoms with higher energy will leave the pot leaving cooler atoms inside.

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 Takes down temperature to nK level.

Sympathetic Cooling

- Evaporative cooling does not work well for fermions.
- Mix evaporative cooled bosons to fermions to cool the laser cooled fermions sympathetically.

Image Source:

4. http://www.physics.otago.ac.nz/research/jackdodd/resources/exp_aspects.htm

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| Cooling Techniques | | | | |



$$a = a_0 \Big[1 - \frac{\Delta B}{B - B_0} \Big]$$

 Image Source:

 5. http://cua.mit.edu/ketterle_group/experimental_setup/BEC_I/background.html

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- Leggett in 1980 showed that starting from a BCS ground state for a Fermi gas with attractive interaction one can reach composite boson limit¹ at T = 0 for sufficiently strong attraction.
- Nozieres and Schmitt-Rink in 1985 extended this to finite temperature².
- Evolution from weak coupling to strong coupling is smooth. So it is a crossover, not a phase transition.

^{1.} A. J. Leggett, J. Phys. 41, C7-19 (1980).

^{2.} P. Nozieres, S. Schmitt-Rink, J. Low. Temp. Phys. 59, 195 (1985). Image Source:

^{6.} http://jilawww.colorado.edu/research/highlights_archive/2006_spring/images/fermisea.jpg

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Phase diagram BCS-BEC Crossover



Image Source:

C. A. R. Sa de Melo, Physics Today,

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| Motivation | | | | |



Image Source:

7. Modern Quantum Mechanics, J. J. Sakurai





- Discovery of High Temperature Superconductors.
- High transition temperature and pseudogap.
- The order of $k_F \xi_{pair}$ is similar in the crossover and in HTSC.

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Image Source:

8. P.hD. thesis of Cindy Regal

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Beauty of BCS Wave function

 BCS ground state wave function can be extended to BEC limit under some constraint.

$$|\Psi
angle = \prod_{\mathbf{k}} \left[u_{\mathbf{k}} + v_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow}
ight] |0
angle$$

• Set
$$g_{\mathbf{k}} = v_{\mathbf{k}}/u_{\mathbf{k}}$$
,
 $|\Psi\rangle = \left(\prod_{\mathbf{k}'} u_{\mathbf{k}'}\right) \exp\left[\sum_{\mathbf{k}} g(\mathbf{k}) c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}^{\dagger}\right]|0\rangle$

- ▶ Define a new operator: $b^{\dagger} = \sum_{\mathbf{k}} g(\mathbf{k}) c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow}$
- ► $[b, b^{\dagger}] = \sum_{\mathbf{k}} |g(\mathbf{k})|^2 (1 n_{\mathbf{k}\uparrow} n_{-\mathbf{k}\downarrow}) \neq c$ number.
- Provided $\langle n_{\mathbf{k}\sigma} \rangle << 1 \Rightarrow [b, b^{\dagger}] = c$ number.
- $|\Psi\rangle = \exp(b^{\dagger})|0\rangle$ represents a Bosonic coherent state.

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The appropriate Hamiltonian:

$$H_{eff} = \int d\mathbf{r} \left\{ \sum_{\alpha} \Psi^{\dagger}(\mathbf{r}\alpha) H_{e}(\mathbf{r}) \Psi(\mathbf{r}\alpha) + V(\mathbf{r}) \Psi^{\dagger}(\mathbf{r}\alpha) \Psi(\mathbf{r}\alpha) \right\}$$
$$+ \int d\mathbf{r} \left\{ \Delta(\mathbf{r}) \Psi^{\dagger}(\mathbf{r}\uparrow) \Psi^{\dagger}(\mathbf{r}\downarrow) + \Delta^{*}(\mathbf{r}) \Psi(\mathbf{r}\downarrow) \Psi(\mathbf{r}\uparrow) \right\}$$

Perform unitary transformation³:

$$\Psi(\mathbf{r}\uparrow) = \sum_{n} \left(\gamma_{n\uparrow} u_{n}(\mathbf{r}) - \gamma_{n\downarrow}^{\dagger} v_{n}^{*}(\mathbf{r})\right)$$
$$\Psi(\mathbf{r}\downarrow) = \sum_{n} \left(\gamma_{n\downarrow} u_{n}(\mathbf{r}) + \gamma_{n\uparrow}^{\dagger} v_{n}^{*}(\mathbf{r})\right)$$

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3. P. D. de GennesSuperconductivity of Metals and Alloys, Addition-Wesley Publishing Company, Inc. 1989.

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Bogoliubov deGennes Equation

$$\epsilon u(\mathbf{r}) = [H_e + V(\mathbf{r})] u(\mathbf{r}) + \Delta(\mathbf{r})v(\mathbf{r})$$

$$\epsilon v(\mathbf{r}) = -[H_e^* + V(\mathbf{r})] v(\mathbf{r}) + \Delta^*(\mathbf{r})u(\mathbf{r})$$

• The order parameter is $\Delta(\mathbf{r}) = g \sum_{n} v_n^*(\mathbf{r}) u_n(\mathbf{r}) (1 - 2f_n)$.

Gap & Density Equation $\Delta(\mathbf{r}) = g \sum_{\epsilon_n > 0} u_n(\mathbf{r}) v_n^*(\mathbf{r})$ $n(\mathbf{r}) = 2 \sum_{\epsilon_n > 0} |v_n(\mathbf{r})|^2$

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Gap & Density Equation

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| The Theory of Crossover | | | | |

- Consider no external potential
- fermion-fermion interaction is mediated via short range contact potential.



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| The Theory of Crossover | | | | |

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| Anderson Localization (Al | _) | | | |

Definition

Beyond a critical amount of impurity, motion of the electron can come to a complete halt. The electron becomes trapped and the conductivity vanishes.

- Direct observation of AL for electron is very difficult.
 - Number of phenomena can mask single particle quantum effects genuinely induced by disorder.
 - Most of the evidences are indirect and stem from conductivity measurement.
- Cold atoms are good candidate for observation of AL:
 - Genuine quantum particles described as matter waves.
 - Single atom matter waves can be directly visualized by different imaging techniques in Bose-Einstein Condensate.

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In One Dimension





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In Three Dimension



Optical Speckle, ⁴⁰K



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| 3 Dis A E | order Anderson Localization Experiments | (AL) | | |
| ④ The (F | e <mark>Dirty Crossover</mark> Continuum Model (3D Fidelity Susceptibility (|) [FS] | | |
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| Continuum Model (3D) | | | | | | |
| What to Expect? | | | | | | |

- Disorder should not affect the BCS superfluid.
- Disorder should seriously affect the molecular BEC.

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What happens in the crossover?

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Image Source:

9. L. Han & C. A. R. Sa de Melo, New J. Phys. 13, 055012 (2011).

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| Continuum Model (3D) | | | | |
| Theory-I | | | | |

The Hamiltonian

$$H = \Psi_{\sigma}^{\dagger} \Big[(-\frac{\nabla^2}{2m} - \mu) + V(\mathbf{r}) \Big] \Psi_{\sigma} - g \Psi_{\uparrow}^{\dagger} \Psi_{\downarrow}^{\dagger} \Psi_{\downarrow} \Psi_{\uparrow}$$

The random potential originating from the scattering of fermions against impurity atoms:

$$V(\mathbf{r}) = \sum_{j} g_{d} \delta(\mathbf{r} - \mathbf{R}_{j}).$$

• Corresponding correlation function: $\langle V(-q)V(q)\rangle = \beta \delta_{i\omega_m,0}\kappa$, where $q = (\mathbf{q}, i\omega_m)$ and κ is the disorder strength.

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| Continuum Model (3D) | | | | |
| Theory-II | | | | |

$$-\frac{m}{4\pi a} = \sum_{k} \left[\frac{1}{2E_{k}} - \frac{1}{2\epsilon_{k}} \right]$$
$$n = \sum_{k} \left[1 - \frac{\xi_{k}}{E_{k}} \right] - \frac{\partial \Omega_{k}}{\partial \mu}$$

Disorder induced thermodynamic potential:

$$\Omega_B = \lim_{\beta \to 0} \frac{1}{2\beta} \sum_q \ln |M| = -\frac{\kappa}{2} \sum_{q,\omega_m=0} W^{\dagger} M^{-1} W$$

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| Continuum Model (3D) | | | | |
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| Continuum Model (3D) | | | | |

Analytical Extensions⁵:

$$\frac{1}{k_{F}a} = -\frac{2}{\pi} \left[\frac{2}{3I_2(x_0)}\right]^{1/3} I_1(x_0)$$
$$\frac{\Delta}{\epsilon_F} = \left[\frac{2}{3I_2(x_0)}\right]^{2/3}.$$

Disorder induced Density Equation^o

$$\begin{aligned} \frac{\Delta}{\epsilon_F} &= \left(\frac{2}{3h_1(x_0)}\right)^{2/3} + \frac{\eta}{\pi^2} l_3(x_0), \\ \frac{\Delta - \Delta(\eta = 0)}{\epsilon_F} &= \frac{\eta}{\pi^2} l_3(x_0), \ \eta = \kappa m^2/k_F \end{aligned}$$

5. M. Marini, F. Pistolesi, and G. C. Strinati, Eur. Phys. J. B 1, 151, (1998)

6. Personal communication with G. Orso is acknowledged

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| Continuum Model (3D) | | | | |

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What they say?

- Order parameter (Δ) remains unaffected by weak disorder in the BCS limit but follows the approximation to the hard core bosons in BEC limit i.e Δ − Δ₀ ∝ η/k_Fa.
- Chemical potential (μ) remains unaffected.

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| Continuum Model (3D) | | | | |

Condensate Fraction



$$n_c = \sum_k \left[\frac{\Delta(\eta)}{2E_k(\eta)}\right]^2$$

A. Khan, S. Basu, S. W. Kim, J. Phys. B 45, 135302 (2012).

- Nonmonotonicity of condensate fraction.
- BCS side follows mean field approximation i.e n_c ∝ Δ.
- BEC side follows correction due to disorder i.e

 $n_c - n_{c_0} \propto \eta \Delta.$

 Enhancement of condensate fraction around the crossover.

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What they say?

- Critical velocity (v_c) is nonmonotonic and the maxima is pinned in the vicinity of a → ∞.
- Sound velocity (v_s) get depleted in the BEC side might be due to additional random scattering rendered by impurity.

A. Khan, S. Basu, S. W. Kim, J. Phys. B 45, 135302 (2012).

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Fidelity in Quantum Phase Transition

- Fidelity is the measure of closeness of two quantum states, $F = |\langle \Psi | \Phi \rangle|$ (for normalized states).
- Quantum Phase Transition is a sudden change in the ground state of a many body system when a controlling parameter λ of the Hamiltonian crosses critical value λ_c.
- There should be an abrupt change in the fidelity
 F(λ + δλ, λ) = |⟨Ψ(λ + δλ)|Ψ(λ)⟩| in the vicinity of λ_c.
- A hasty drop of the ground-state fidelity at the critical point will then correspond to a divergence of the fidelity susceptibility,

$$\chi(\lambda) = rac{1}{\Omega} rac{\partial \langle \Psi(\lambda) |}{\partial \lambda} rac{\partial |\Psi(\lambda)
angle}{\partial \lambda}.$$

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| Fidelity Susceptibility (FS | 5) | | | |
| Fidelity in E | BCS-BEC Cross | over | | |

- Start with BCS ground state wave function⁷, $|\Psi(\lambda)\rangle = \prod_{\mathbf{k}} \left[u_{\mathbf{k}}(\lambda) + v_{\mathbf{k}}(\lambda)c^{\dagger}_{\mathbf{k}\uparrow}c^{\dagger}_{-\mathbf{k}\downarrow} \right] |0\rangle.$
- The fidelity-susceptibility is:

 $\chi(\lambda) = \int \frac{d\mathbf{k}}{(2\pi)^3} \left[\left(\frac{du_k}{d\lambda} \right)^2 + \left(\frac{dv_k}{d\lambda} \right)^2 \right].$

► The dependence of u_k and v_k on λ is determined by BCS gap and density equation:

$$\Delta_k = -\int \frac{d\mathbf{k}'}{(2\pi)^3} V_{\lambda}(\mathbf{k},\mathbf{k}') \frac{\Delta_{k'}}{2E_{k'}}, \ n = \int \frac{d\mathbf{k}}{(2\pi)^3} 2v_k^2.$$

The resulting fidelity susceptibility will be:

$$\chi(\lambda) = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{4E_k^4} \Big[\Delta_k \frac{d\mu}{d\lambda} + \xi_k \frac{d\Delta_k}{d\lambda} \Big]^2$$

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7. A. Khan and P. Pieri, Phys. Rev. A 80, 012303 (2009).





Dirty Fermi Gas

A.Khan, S. Basu and BT, submitted in Phys. Lett. A.

Notations

• χ is in dimensions of k_F^{-3} in both the plots.

•
$$\lambda = (k_F a)^{-1}$$

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| Fidelity Susceptibility (FS |) | | | |



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| Fidelity Susceptibility (FS |) | | | |



Statistical Analysis

Definition

Skewness

$$S = rac{\langle (x - \langle x
angle)^3
angle}{\langle (x - \langle x
angle)^2
angle^{3/2}}.$$

Kurtosis

$$\kappa = rac{\langle (x - \langle x
angle)^4
angle}{\langle (x - \langle x
angle)^2
angle^2} - 3.$$

•
$$x = (k_F a)^{-1}$$
.

- Both S and κ monotonically moves towards zero.
- $\eta_c = 10 \sim 13$ obtained from the linear fit of the data.

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| Fidelity Susceptibility (FS) | | | | |

Density of States (DOS)

$$N(\omega) = \sum_{k} u_k^2 \delta(\omega - E_k) + v_k^2 \delta(\omega + E_k)$$





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| Fidelity Susceptibility (FS |) | | | |

Observations

- N(\u03c6) is low in the BCS and BEC regions but the singular pile up is high at the unitarity.
- Distinct reduction of spectral gap at the unitarity.
- S and κ data predict for a phase transition at a moderate to high disorder value.
- From the behavior of DOS as well as FS, we consider the possible phases after transition might be Anderson glass for BCS superfluid, Fermi glass for unitary superfluid and Bose glass for BEC superfluid.

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Field of dirty crossover is introduced.

- Effect of weak disorder in three dimensional continuum model:
 - Monotonic depletion of order parameter is observed.
 - Nonmonotonic behavior of condensate fraction is discussed.
 - Suppression of sound velocity is presented.
- Study of FS and DOS:
 - The FS looses symmetric nature in presence of disorder, associated skewness and kurtosis approach zero for large disorder strength.
 - Spectral gap is considerably reduced at unitarity where as BCS and BEC extremes remains unaffected.

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