QUANTUM ENTANGLEMENT: CRITERIA AND MYSTERY

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QUANTUM ENTANGLEMENT

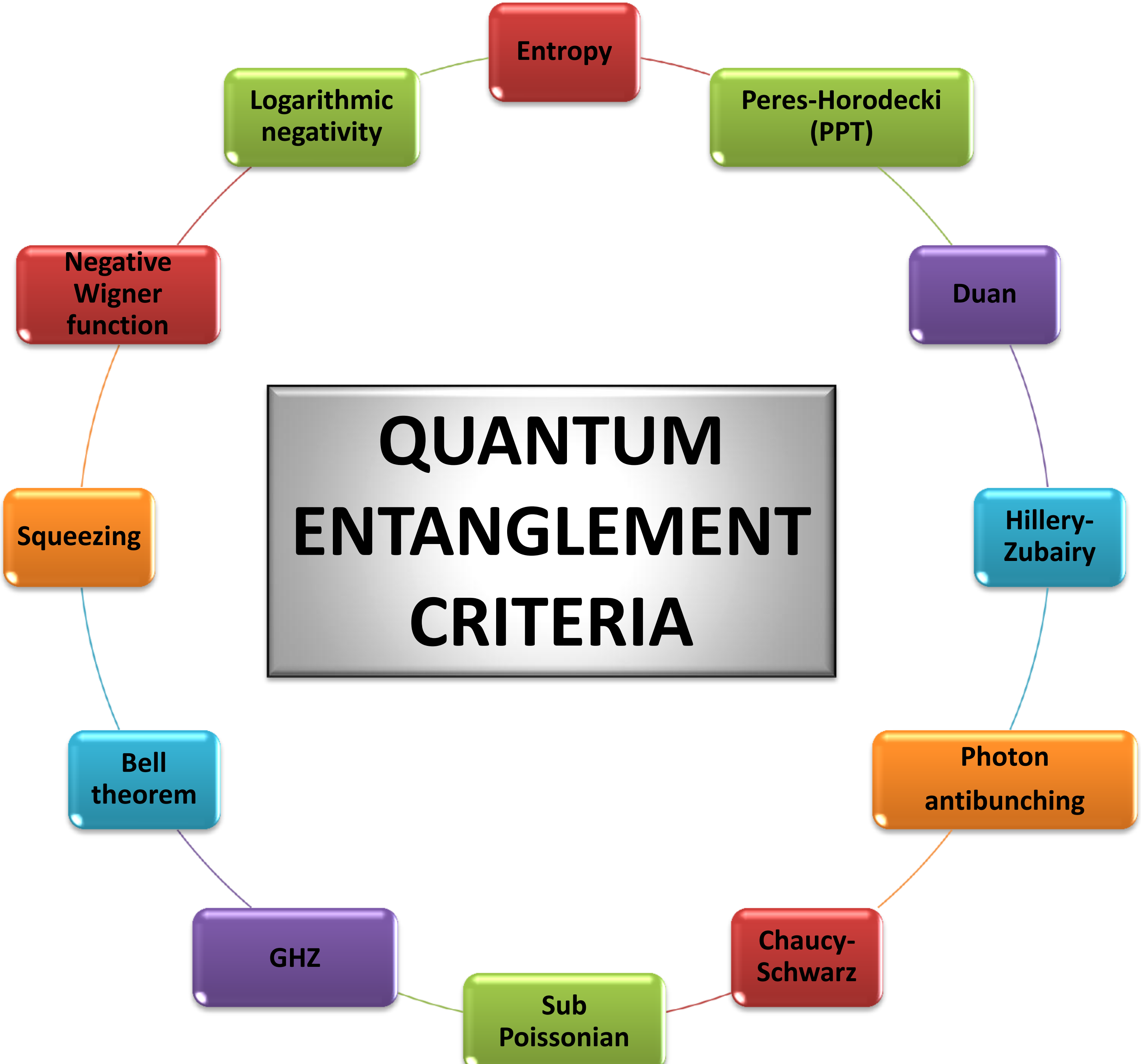
The system of two constituting particles in quantum state linked together

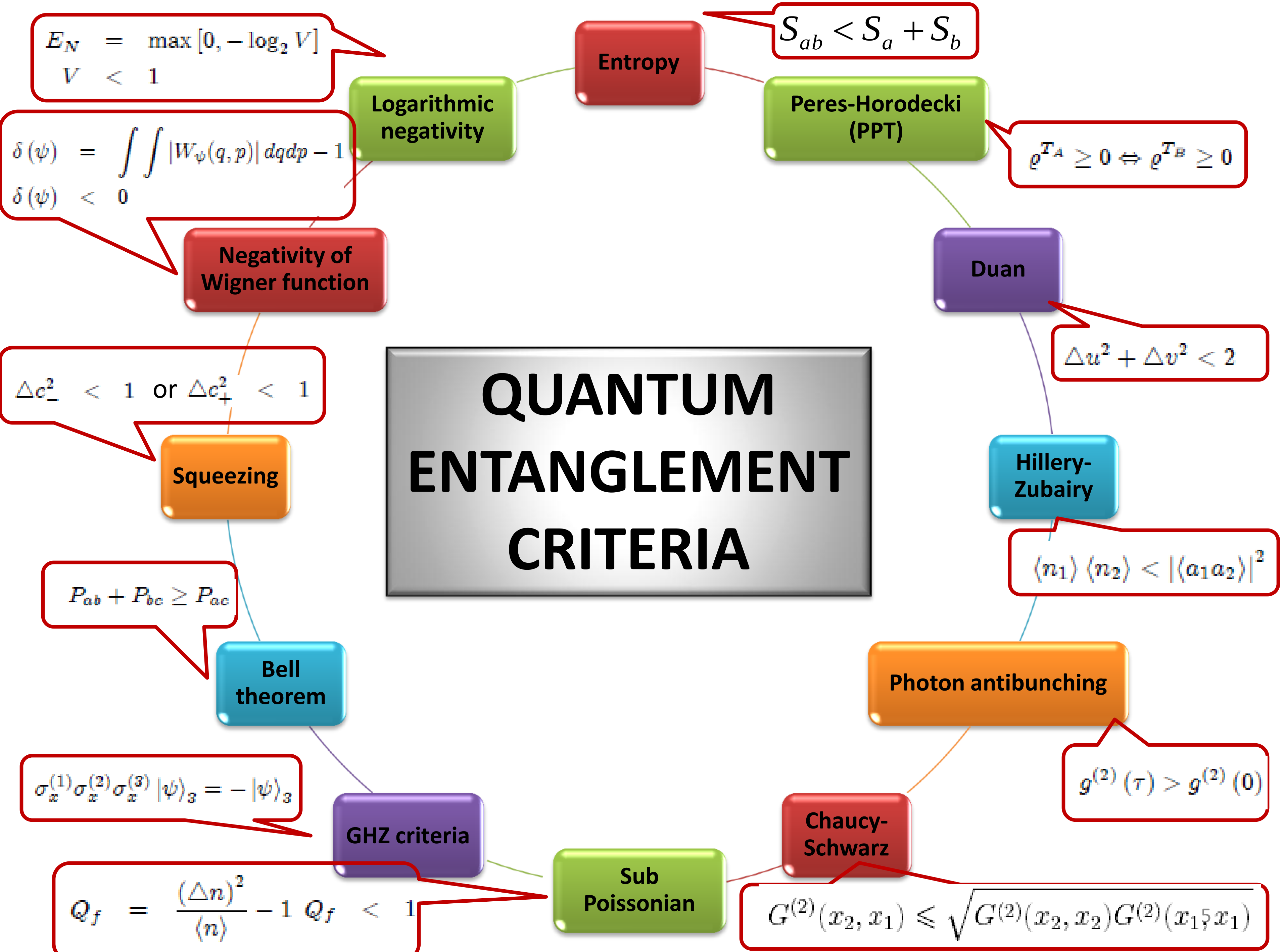
Describe through several mathematical theories

Apply into information technology fields

DEFINITIONS

$G^{(2)}$ correlation	$g^{(2)} = \frac{\langle a^\dagger(t) a^\dagger(t+\tau) a(t+\tau) a(t) \rangle}{\langle a^\dagger a \rangle^2}$
Bell state	$\lvert\psi\rangle_2 = \frac{1}{\sqrt{2}} \left(\lvert\uparrow_1, \downarrow_2\rangle - \lvert\downarrow_1, \uparrow_2\rangle \right)$
GHZ state	$\lvert\psi\rangle_3 = \frac{1}{\sqrt{2}} \left(\lvert\uparrow_1, \uparrow_2, \uparrow_3\rangle - \lvert\downarrow_1, \downarrow_2, \downarrow_3\rangle \right)$
Von Neumann Entropy	$S(\rho) = -k_B Tr(\rho \ln \rho)$
Partial transpose	$\varrho^{T_A} = \sum_k p_k \left(\varrho_k^A\right)^T \otimes \varrho_k^B$
Density operator	$\hat{\rho} = \sum_i p_i \lvert\psi_i\rangle \langle\psi_i\rvert$
Variance	$(\triangle n)^2$

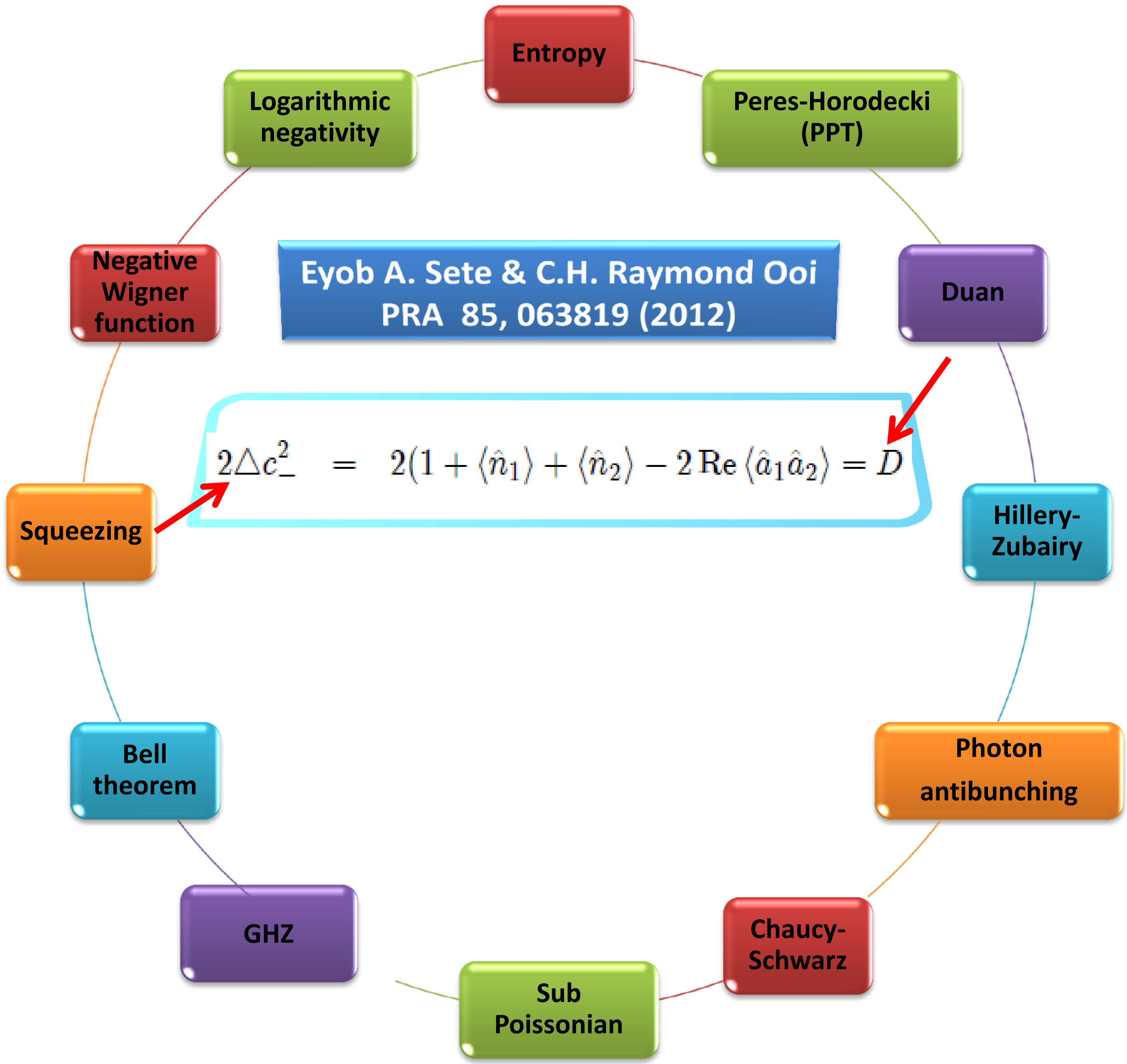

QUANTUM ENTANGLEMENT CRITERIA
Entropy
Peres-Horodecki (PPT)
Duan
Hillery-Zubairy
Photon antibunching
Chaucy-Schwarz
Sub Poissonian
GHZ
Bell theorem
Squeezing
Negative Wigner function
Logarithmic negativity



QUANTUM ENTANGLEMENT CRITERIA
Entropy
$S_{ab} < S_a + S_b$
Peres-Horodecki (PPT)
$\varrho^{T_A} \geq 0 \Leftrightarrow \varrho^{T_B} \geq 0$
Duan
$\Delta u^2 + \Delta v^2 < 2$
Hillery-Zubairy
$\langle n_1 \rangle \langle n_2 \rangle < |\langle a_1 a_2 \rangle|^2$
Photon antibunching
$g^{(2)}(\tau) > g^{(2)}(0)$
Chaucy-Schwarz
$G^{(2)}(x_2, x_1) \leqslant \sqrt{G^{(2)}(x_2, x_2) G^{(2)}(x_1, x_1)}$
Sub Poissonian
$Q_f = \frac{(\Delta n)^2}{\langle n \rangle} - 1 \quad Q_f < 1$
GHZ criteria
$\sigma_x^{(1)} \sigma_x^{(2)} \sigma_x^{(3)} |\psi\rangle_3 = -|\psi\rangle_3$
Bell theorem
$P_{ab} + P_{bc} \geq P_{ac}$
Squeezing
$\Delta c_-^2 < 1$ or $\Delta c_+^2 < 1$
Negativity of Wigner function
$\delta(\psi) = \int\int |W_\psi(q,p)|\, dq dp - 1$
$\delta(\psi) < 0$
Logarithmic negativity
$E_N = \max[0, -\log_2 V]$
$V < 1$

CHEACKLISTS PROPERTIES OF QUANTUM ENTANGLEMENT

Properties / Criteria	Necessary condition	Sufficient condition	Density operator	No. of Photon	Correlation	Phase sensitive
Entropy	✓		✓			
PH	✓		✓			
Duan		✓		✓	✓	✓
HZ		✓		✓	✓	✓
AB	✓			✓	✓	✓
CS	✓			✓		
SubP	✓			✓		✓
GHZ		✓		✓		
Bell	✓			✓		
Sq	✓			✓	✓	✓
N. Wigner		✓	✓			✓
Log N		✓		✓		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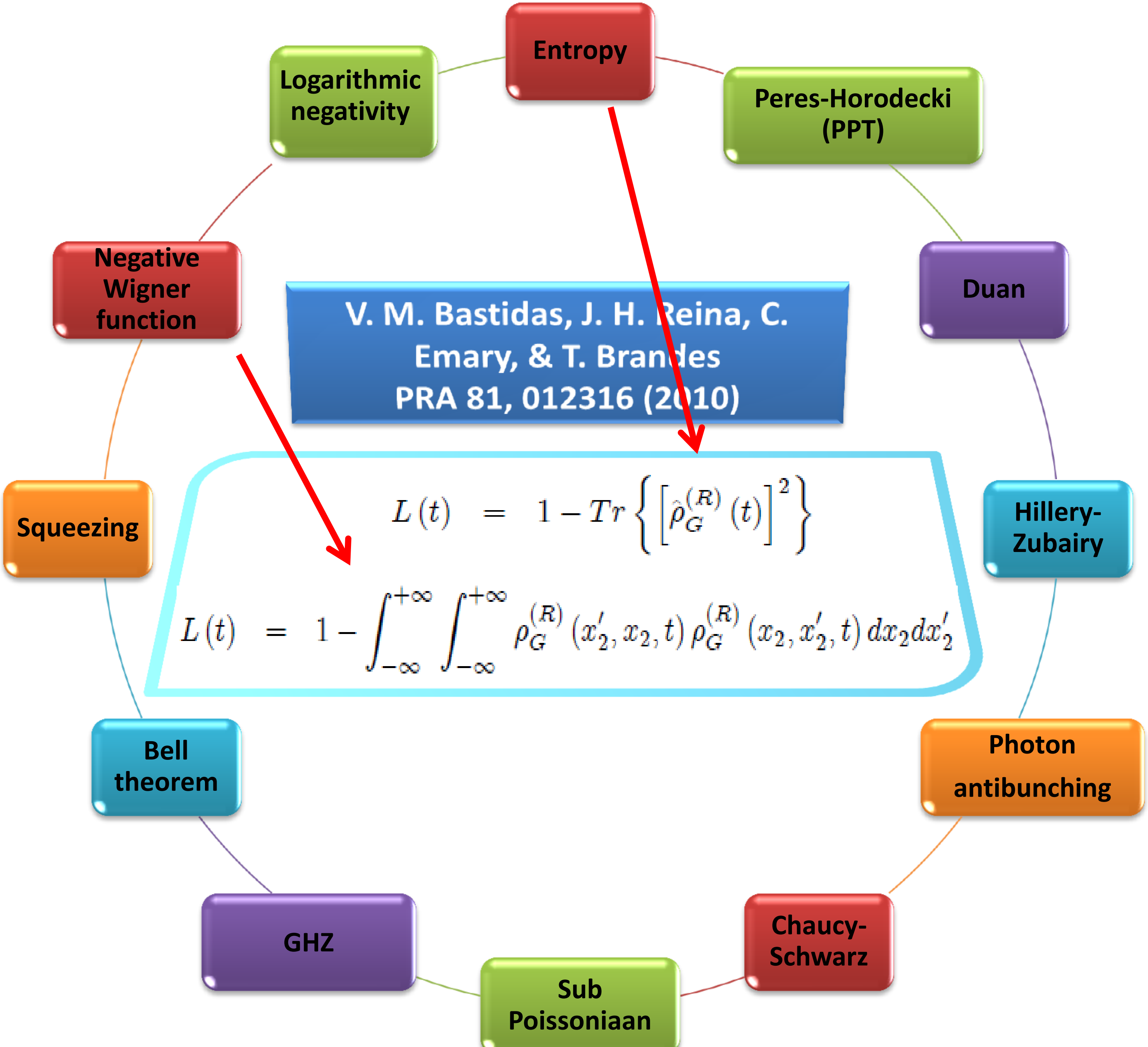
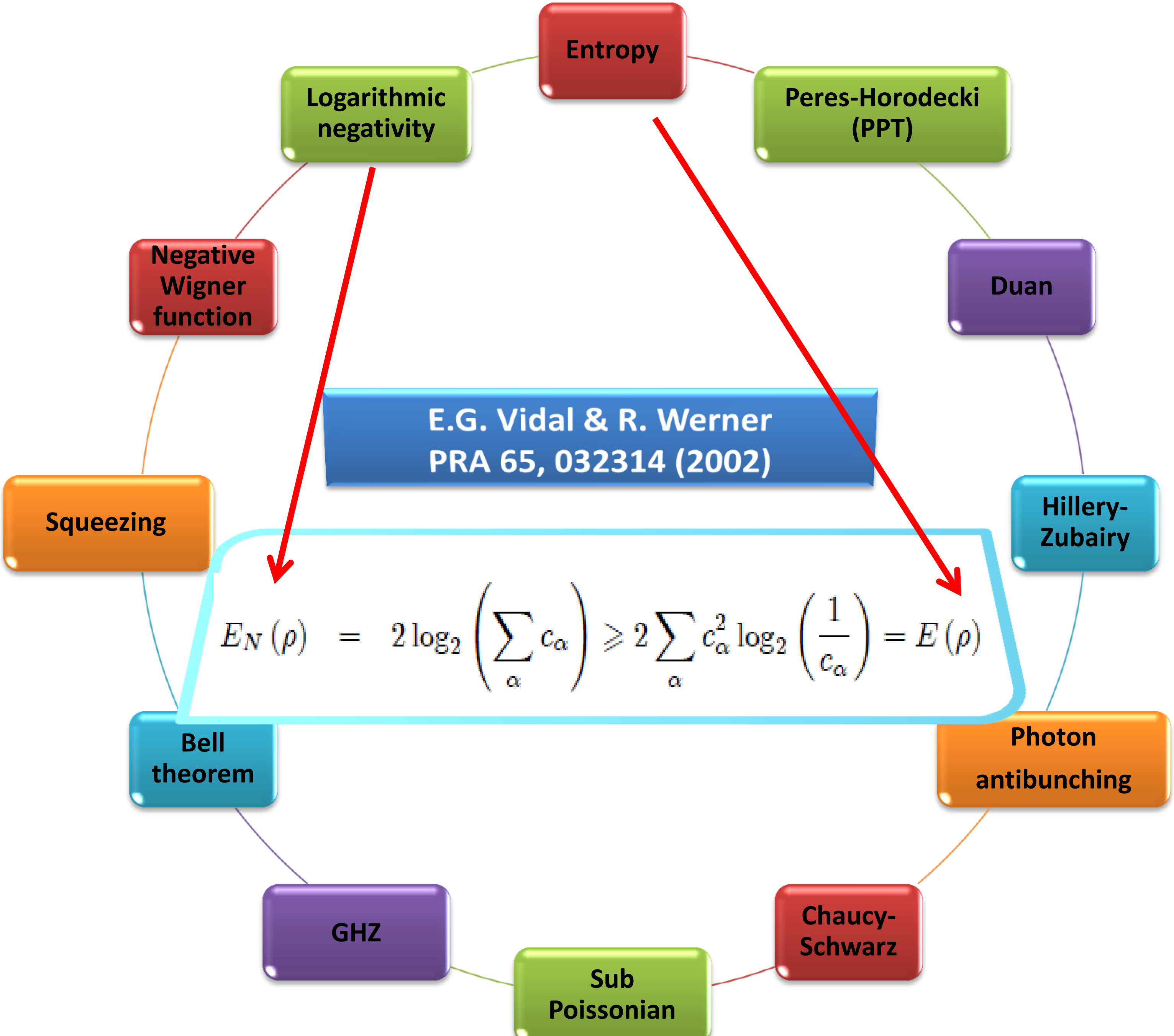
Squeezing
Negative Wigner function
Logarithmic negativity
Eyob A. Sete & C.H. Raymond Ooi
PRA 85, 063819 (2012)
$2\Delta c_-^2 = 2(1+\langle\hat{n}_1\rangle+\langle\hat{n}_2\rangle-2\,\mathrm{Re}\,\langle\hat{a}_1\hat{a}_2\rangle = D$

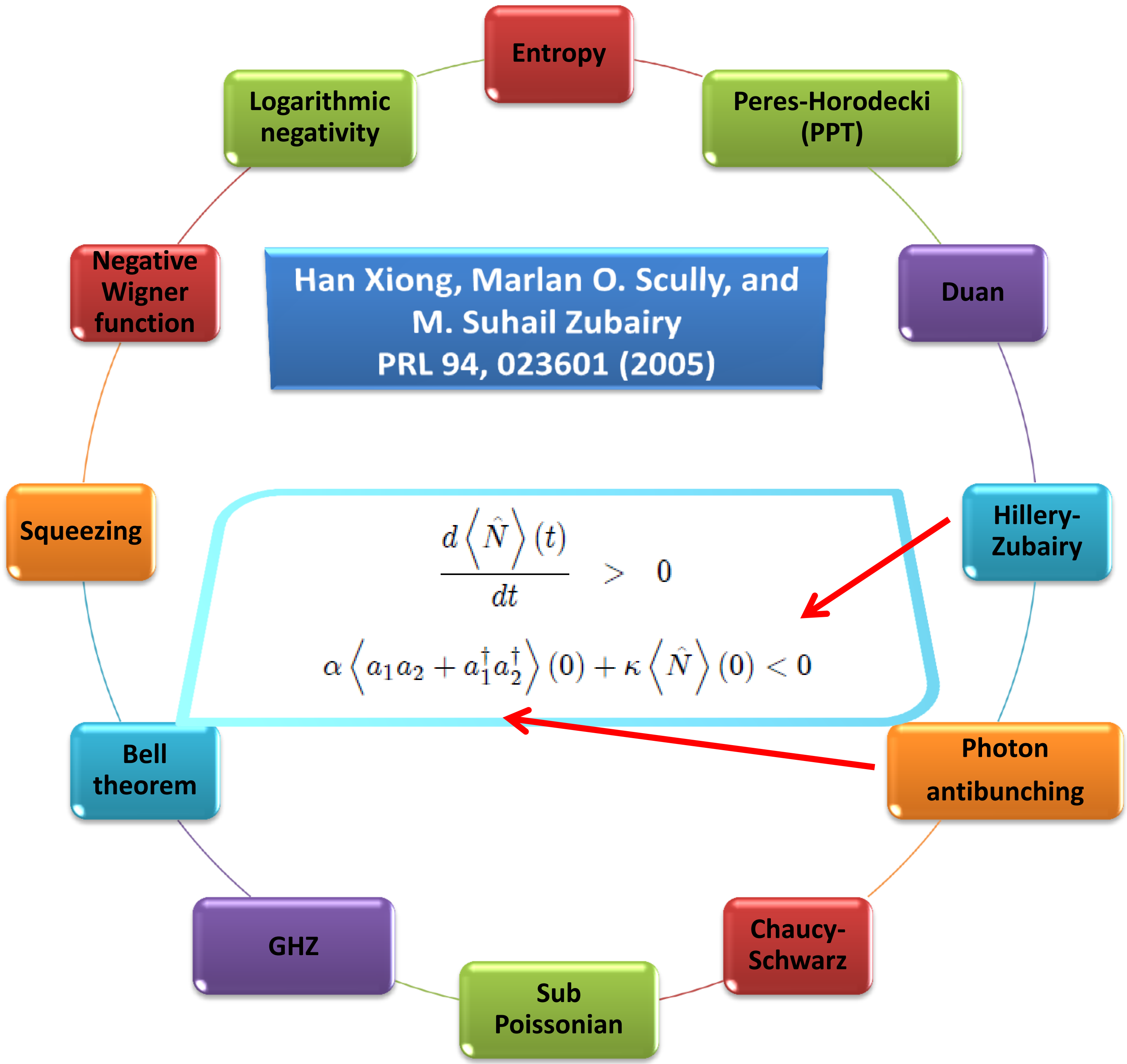


V. M. Bastidas, J. H. Reina, C. Emary, & T. Brandes
PRA 81, 012316 (2010)

$$L(t) = 1 - Tr\left\{\left[\hat{\rho}_G^{(R)}(t)\right]^2\right\}$$

$$L(t) = 1 - \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \rho_G^{(R)}(x_2', x_2, t)\, \rho_G^{(R)}(x_2, x_2', t)\, dx_2 dx_2'$$


Entropy
Logarithmic negativity
Peres-Horodecki (PPT)
Negative Wigner function
Duan
E.G. Vidal & R. Werner
PRA 65, 032314 (2002)
Squeezing
Hillery-Zubairy
$E_N(\rho) = 2\log_2\left(\sum_\alpha c_\alpha\right) \geqslant 2\sum_\alpha c_\alpha^2 \log_2\left(\frac{1}{c_\alpha}\right) = E(\rho)$
Bell theorem
Photon antibunching
GHZ
Sub Poissonian
Chaucy-Schwarz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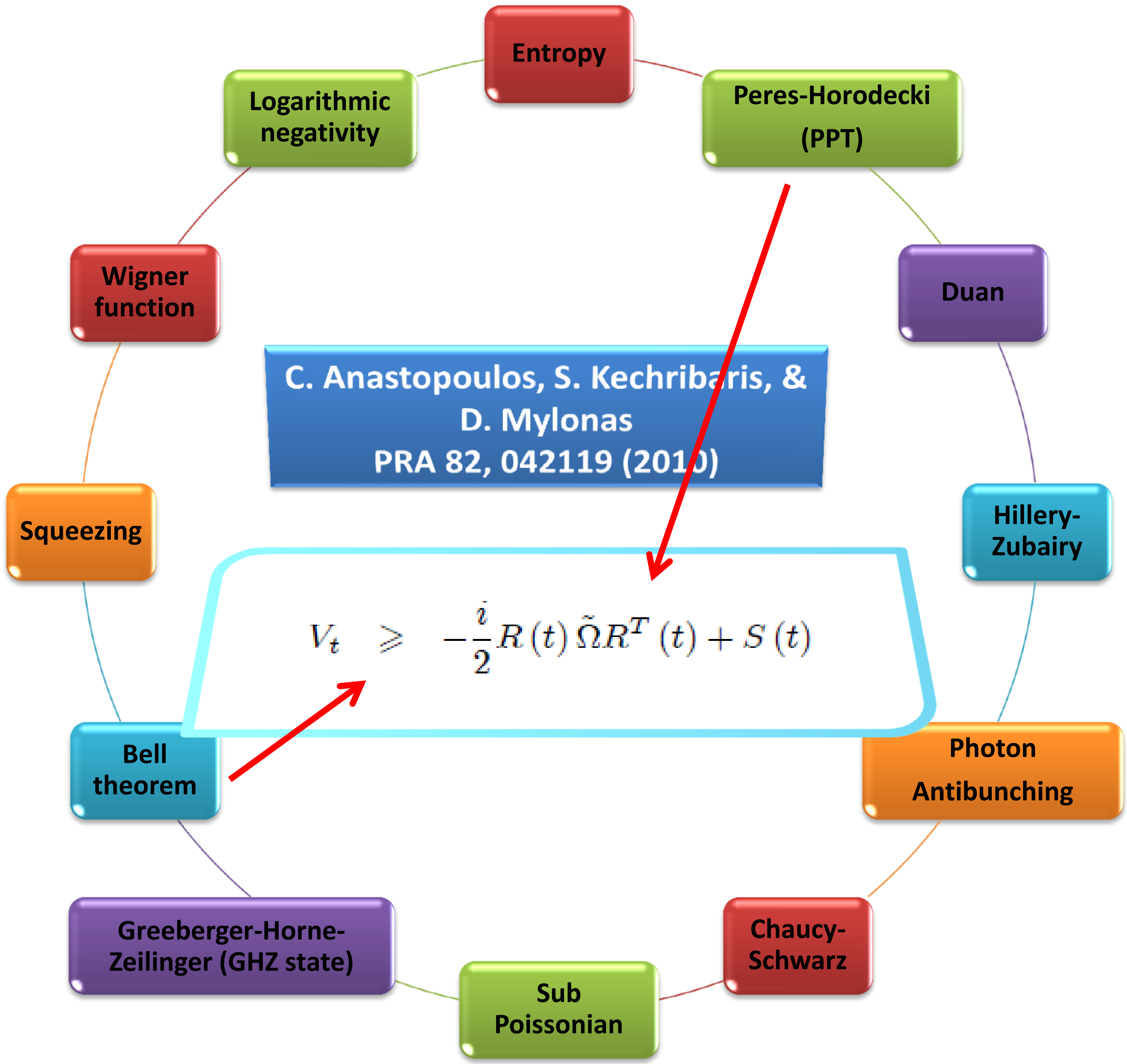
Logarithmic negativity
Han Xiong, Marlan O. Scully, and M. Suhail Zubairy
PRL 94, 023601 (2005)
$\frac{d\left\langle \hat{N}\right\rangle (t)}{dt} > 0$
$\alpha\left\langle a_1 a_2 + a_1^\dagger a_2^\dagger\right\rangle (0) + \kappa\left\langle \hat{N}\right\rangle (0) < 0$



C. Anastopoulos, S. Kechribaris, & D. Mylonas
PRA 82, 042119 (2010)

$$V_t \geqslant -\frac{i}{2} R(t)\, \tilde{\Omega} R^T(t) + S(t)$$

QUANTUM ENTANGLEMENT: MYSTERY

Glory be to the One, who created in pairs all things that the earth produces, as well as themselves, and other things they do not know. (Holy Quran, 36: 36)

And We created pairs of everything that you may complete. (Holy Quran, 51:49)

So many entanglement criteria

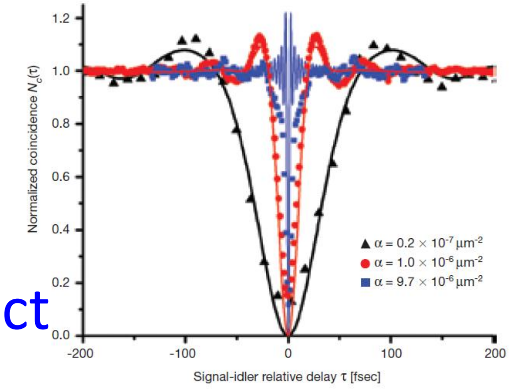
- Violation of a criteria indicates:

nonclassicality, counterintuitive, beyond daily experience

- Rooted in the wave nature:

Destructive interference, e.g. Hong-Ou-Mandel effect

Hard to think of statistical system as waves



Equally/more mysterious

- Quantum coherence ρ_{ab}

Scully, **From lasers and masers to *phaseonium* and phasers**, Phys Rep. 219, 191 (1992)

- Quantum correlation $\langle AB\rangle \neq \langle A\rangle \langle B\rangle$

Outline

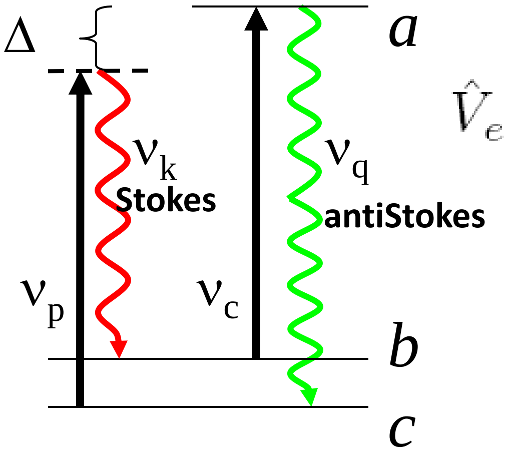
Some features & mysteries of quantum entanglement/correlation

Provide insight on nature of entanglement

How the mysteries can be resolved

Coherent control of Quantum Correlation

Marlan O. Scully and C. H. Raymond Ooi, J. Opt. B: Quant. Semiclass. Opt. **6**, S816(2004)



Effective Hamiltonian

$$\hat{V}_{eff} = -\sum_{\mathbf{k}} \hbar G_{\mathbf{k}} \hat{a}^{\dagger}_{\mathbf{k}} |b\rangle\langle c| e^{i(\mathbf{k}_p-\mathbf{k})\cdot\mathbf{r}} e^{i(\Delta_{\mathbf{k}}-\Delta)t}$$

Spontaneous (off-resonant) Raman

$$-\hbar\Omega_c e^{i\mathbf{k}_c\cdot\mathbf{r}}|a\rangle\langle b| - \sum_{\mathbf{q}} \hbar g_{\mathbf{q}} \hat{a}^{\dagger}_{\mathbf{q}} |c\rangle\langle a| e^{-i\mathbf{q}\cdot\mathbf{r}} e^{i\Delta_{\mathbf{q}}t} + \text{adj}.$$

resonant Raman

$\Omega_p \ll \Delta_p$

$$G_{\mathbf{k}} = \Omega_p g_{\mathbf{k}}/\Delta_p$$

$\Omega_p \ll \Delta_p$

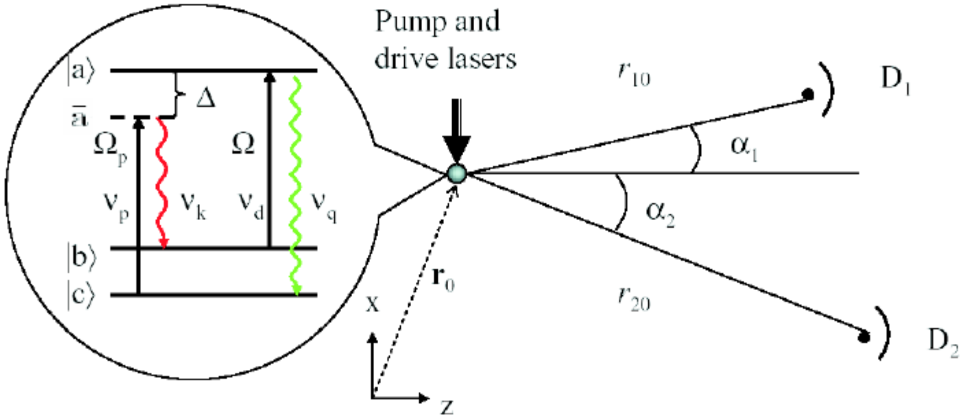
Atom-field state Vector

$$|\Phi\rangle = C_0|c,0\rangle + \sum_{\mathbf{k}} B_{\mathbf{k}}|b,1_{\mathbf{k}}\rangle + \sum_{\mathbf{k}} A_{\mathbf{k}}|a,1_{\mathbf{k}}\rangle + \sum_{\mathbf{k},\mathbf{q}} C_{\mathbf{kq}}|c,1_{\mathbf{k}},1_{\mathbf{q}}\rangle$$

For c.w. lasers, we have exact solutions for C_0, B_k, A_k, and C_{kq}.

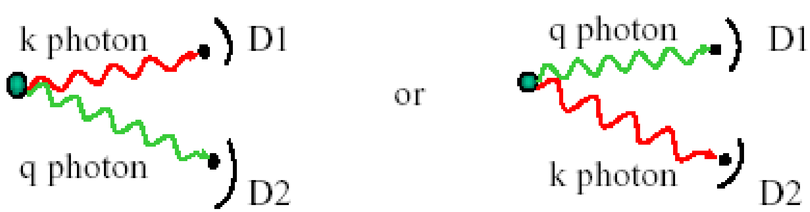
Two-photon amplitude

$$G^{(2)} = |\psi(1,2) + \psi(2,1)|^2$$



$$\langle 0|\hat{E}^{(+)}(\mathbf{r}_2, t_2)\hat{E}^{(+)}(\mathbf{r}_1, t_1)|\Psi\rangle \equiv \psi(1,2) + \psi(2,1)$$

(Emission times $\tau_i = t_i - n_i r_i / c$)



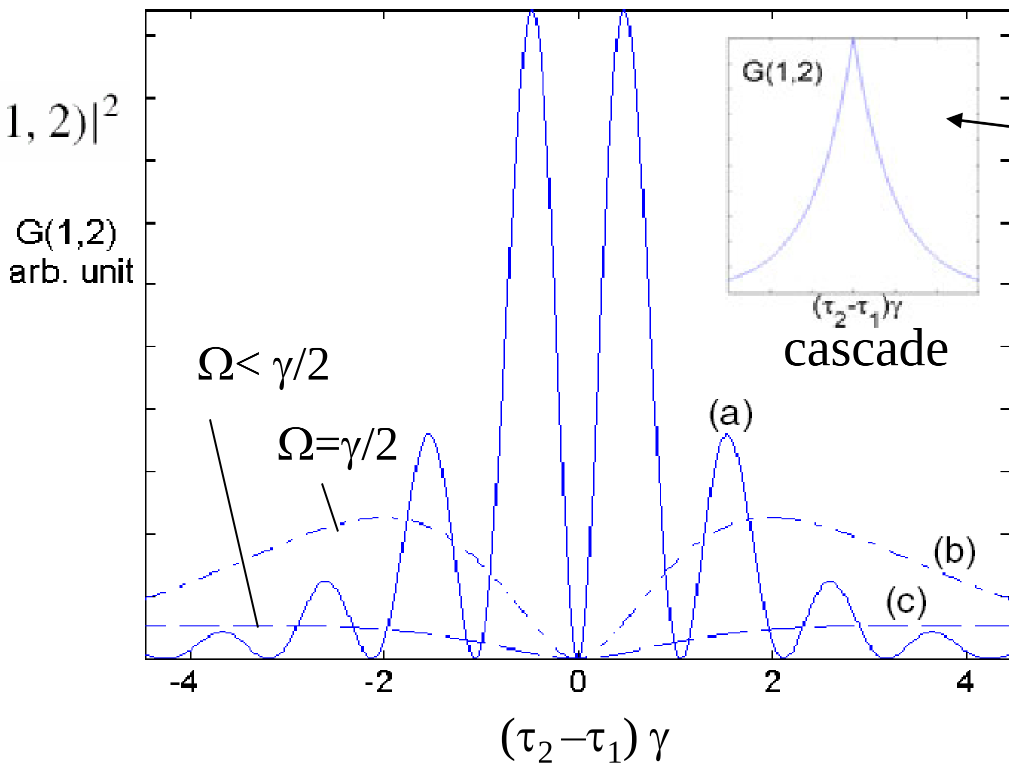
$$\psi(1,2) = -\frac{\mathcal{C}_{12}}{2r_1 r_2}\Theta(\tau_2 - \tau_1)\Theta(\tau_1)e^{i(\mathbf{k}_c + \mathbf{k}_p)\cdot\mathbf{r}_0}e^{-i\nu\tau_1}e^{-i\omega\tau_2} \times$$
$$e^{-(\Gamma/2 - \gamma/4)\tau_1}e^{-(\gamma/4)\tau_2}\{e^{i\tilde{\Omega}(\tau_2 - \tau_1)} - e^{-i\tilde{\Omega}(\tau_2 - \tau_1)}\}$$

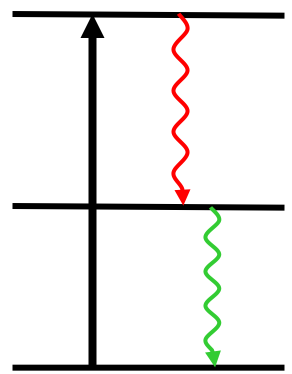
τ_j is emission time of the photon to detector j

Put dichroic filter: photon k goes to D1 ; photon q goes to D2 $G^{(2)}(1,2) = |\Psi(1,2)|^2$

$$\psi(1,2) = -i\frac{\mathcal{C}_{12}}{r_1 r_2}\Theta(\tau_1)\Theta(\tau_2-\tau_1)\mathrm{e}^{-[i\omega+\gamma/2]\tau_2}\mathrm{e}^{-[i\nu+\Gamma/2-\gamma/2]\tau_1}\sin\tilde{\Omega}(\tau_2-\tau_1)$$

$$G^{(2)}(1,2) = |\Psi(1,2)|^2$$



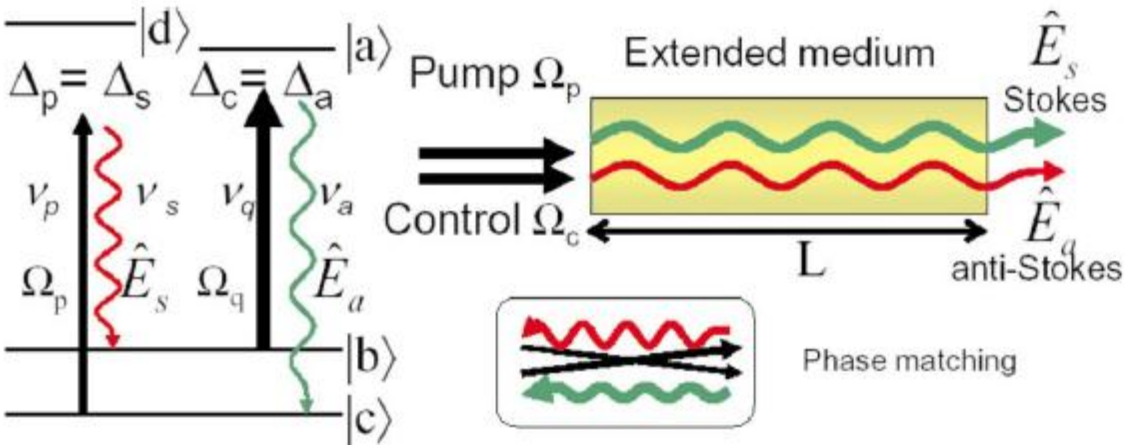


Quantum Interference + Antibunching (Nonclassical)

Effects of Noise on Quantum Correlation

C. H. Raymond Ooi, Q Sun, M. S Zubairy, M O. Scully. PRA **75**, 013820 (2007)

C. H. Raymond Ooi & M. S Zubairy, PRA **75**, 053822 (2007)



$\hat{E}_f'$

Boundary operators $\hat{B}_f$ ← $\hat{E}_f'$ → **Noise operators** $\hat{N}_f$

Fields amplification/dissipation

due to

- input (vacuum $\hat{E}_f(0)$) fields
- atomic coherence/dissipation Ψ_g^f

Fields fluctuations (Noise)

due to

- vacuum fluctuations $\hat{a}_{\mathbf{k}}(0)$
- atomic fluctuations $\hat{\sigma}_{\alpha\beta}(0)$
- atomic coherence/dissipation Ψ_g^f

Coupled equations for Field Operators

$$\hat{E}_a = \hat{A}/g_a$$
$$\hat{E}_s = \hat{S}/g_s$$

$$(\frac{\partial}{\partial z} + \mathcal{G}_s)\hat{S} + \mathcal{K}_s\hat{A}^\dagger = \bar{F}_s$$

$$(\frac{\partial}{\partial z} + \mathcal{G}_a)\hat{A}^\dagger + \mathcal{K}_a\hat{S} = \bar{F}_a^\dagger$$

of parametric oscillator form

$$g_s = \wp_{db}/\hbar \text{ and } g_a = \wp_{ac}/\hbar$$

Solutions are composed of

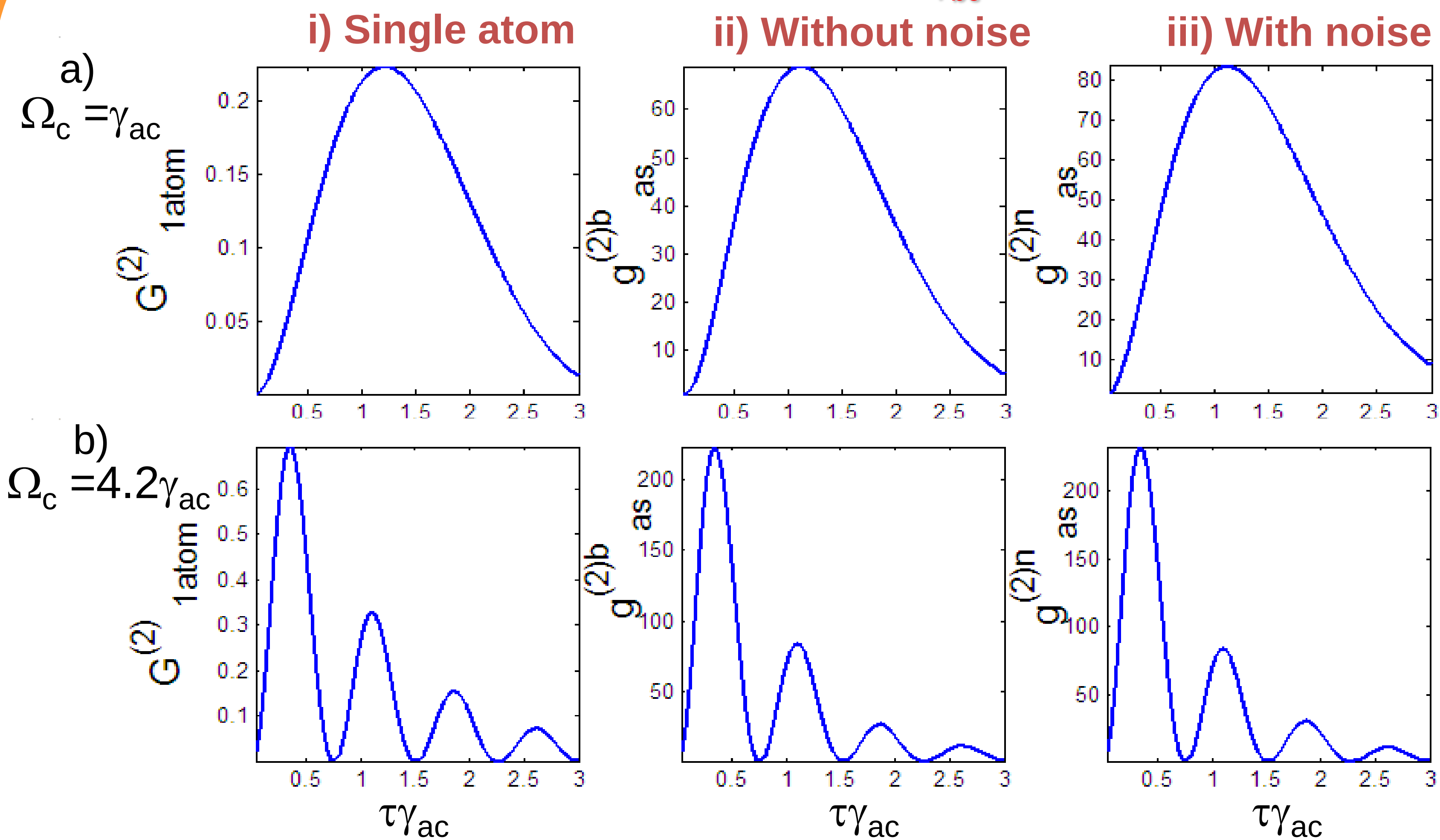
noise part and boundary operators

$$\hat{E}_s(z,\nu) = \hat{N}_s(z,\nu) + \hat{B}_s(z,\nu)$$

$$\hat{E}_a^\dagger(z,\nu) = \hat{N}_a^\dagger(z,\nu) + \hat{B}_a^\dagger(z,\nu)$$

Correlation for Raman-EIT Scheme

No decoherence, $\gamma_{bc} = 0$



Small dipole moment=5×10^{-30}Cm $\quad \Delta = -7.5\gamma_{ac}, \Omega_p = 0.2\gamma_{ac} \quad N = 8 \times 10^{16} m^{-3}$

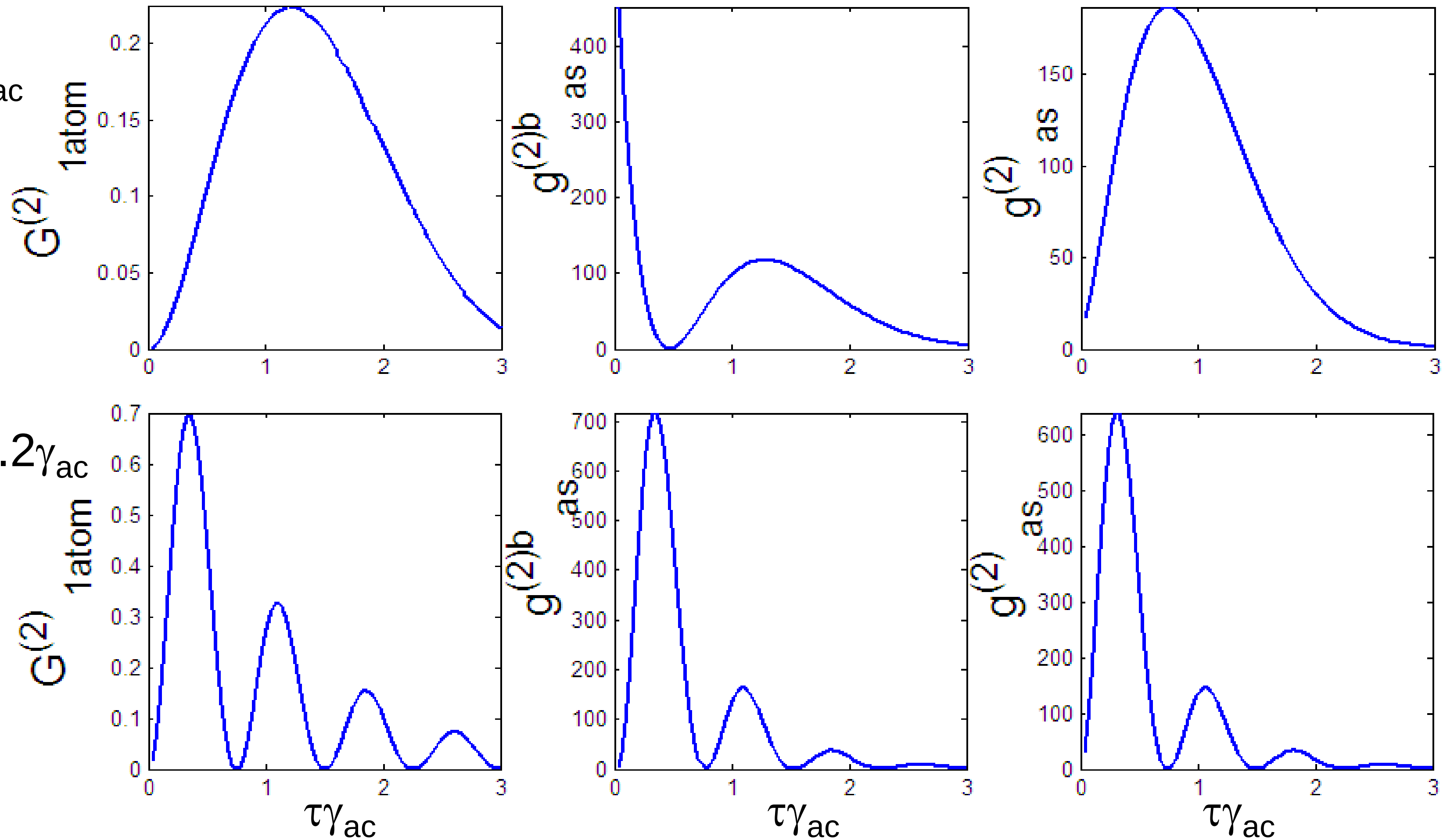
Noise operators are not needed for correct qualitative description for zero decoherence and short sample

With decoherence, $\gamma_{bc} = 0.6\gamma_{ac}$

i) Single atom | ii) Without noise | iii) With noise

a) $\Omega_c = \gamma_{ac}$

b) $\Omega_c = 4.2\gamma_{ac}$



Small dipole moment=5×10^{-30}Cm $\Delta = -7.5\gamma_{ac}, \Omega_p = 0.2\gamma_{ac}$ $\alpha L = 0.008\ (L = 1.5mm)$

$\alpha = g_a\kappa_a/\gamma_{ac} = 235.1m^{-1}$ $N = 8 \times 10^{16} m^{-3}$

<u>When there is decoherence in short sample, noise operators are not needed only when the control field is strong</u>

Summary

Weak field Ω_c

Need noise operators?	Optically thin	Optically thick
$\gamma_{bc}=0$	No	Yes
γ_{bc} finite	Yes	Yes

Strong field Ω_c

Need noise operators?	Optically thin	Optically thick
$\gamma_{bc}=0$	No	Mostly no
γ_{bc} finite	No	Perhaps no

Quantum theory without quantum noise has been widely used to describe SPDC, OPA and OPO.

This is NOT always correct.

The theory fails to describe correlation for:

- Weak control field
- Finite decoherence
- Finite sample

Contribution of noise grows linearly with optical density

$$\sqrt{\frac{G_{as}^{n}}{G_{as}^{b}}} \sim N\lambda^2 L \frac{3\Gamma}{2\gamma_{ac}}$$

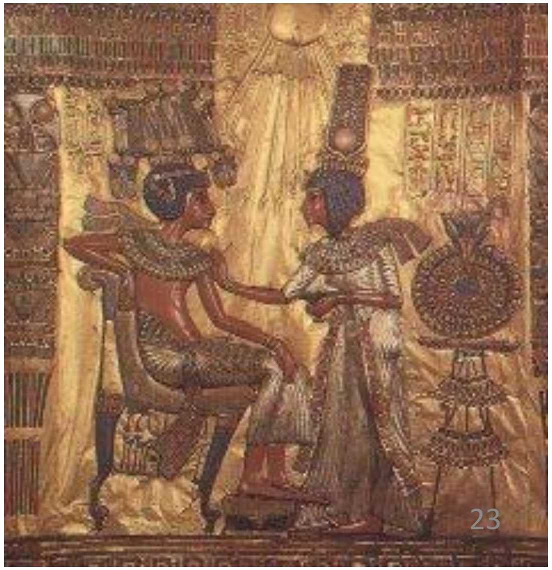
Postulate

Decoherence/ death of entanglement is due to the disturbance of the harmonic vibrations by quantum noise in the environment.

Entanglement would survive (immortal) if harmonious vibrations are maintained/preserved.

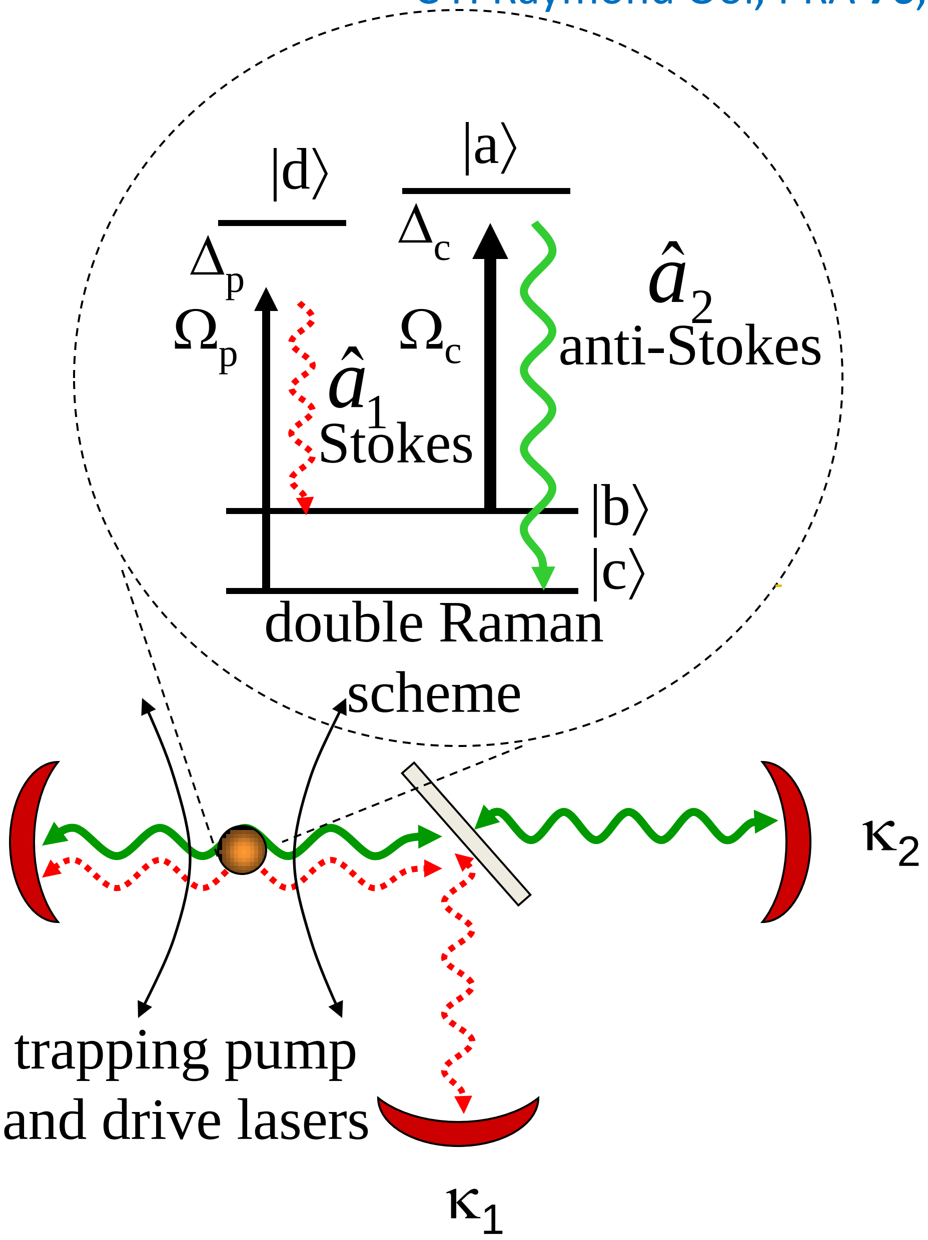
Coherent field can prolong quantum correlation against noise

Mummification is to shield from degradation due to environmental

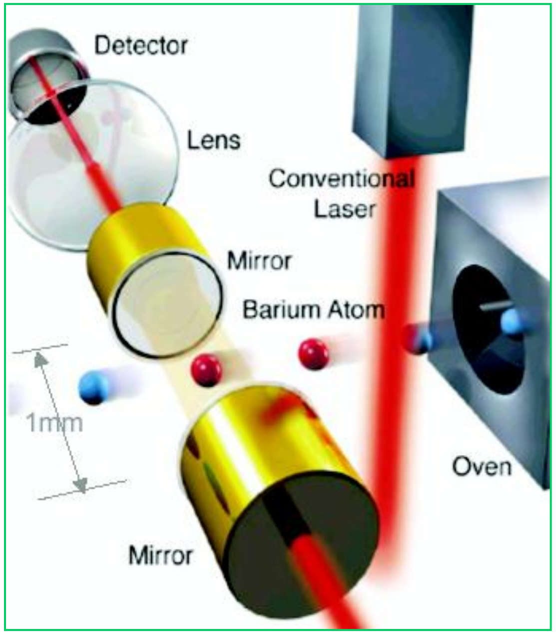


Controllable entanglement of two-photon laser

C H Raymond Ooi, PRA **76**, 013809 2007



can be extended using the single atom microlaser setup



K. An et. al., PRL **73**, 3375,(1994); Sc. Am. 1998

Quantum optical approach

Single atom Hamiltonian

$$\hat{V} = -\hbar[\Omega_p \hat{\sigma}_{dc} e^{-i\Delta_p t} + g_1 \hat{\sigma}_{db} \hat{a}_1 e^{-i\Delta_1 t} + \Omega_c \hat{\sigma}_{ab} e^{-i\Delta_c t} + g_2 \hat{\sigma}_{ac} \hat{a}_2 e^{-i\Delta_2 t}] + adj.$$

$$\hat{\sigma}_{\alpha\beta} = |\alpha\rangle\langle\beta|,\ \Omega_q = |\Omega_q| e^{i\varphi_q} (q = p, c),\ g_j = |g_j| e^{i\varphi_j} (j = \text{1-Stokes, 2-anti-Stokes})$$

laser phase

Density matrix equation for the field

$$\frac{d}{dt}\hat{\rho} = i[g_1 \hat{a}_1 \hat{\rho}_{bd} + g_2 \hat{a}_2 \hat{\rho}_{ca} + g_1^* \hat{a}_1^\dagger \hat{\rho}_{db} + g_2^* \hat{a}_2^\dagger \hat{\rho}_{ac}] + \text{adj.}$$

The coherence operators are obtained under adiabatic approximation using the steady state solutions for the atomic operators up to first order in lasing field operators

$$0 = \frac{d}{dt}\hat{\rho}_\beta \ (\beta = ac, ad, bc, bd)$$

Entanglement of two modes

L.-M. Duan, G. Giedke, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. 84, 2722 (2000).

$$D(t) = \langle(\Delta\hat{u})^2\rangle + \langle(\Delta\hat{v})^2\rangle < 2$$

$$\hat{u} = \hat{x}_1 + \hat{x}_2 \text{ and } \hat{v} = \hat{p}_1 - \hat{p}_2$$

$$\hat{x}_j = \frac{1}{\sqrt{2}}(\hat{a}_j + \hat{a}_j^{+}) \text{ and } \hat{p}_j = \frac{1}{i\sqrt{2}}(\hat{a}_j - \hat{a}_j^{+})$$

Transient correlation at zero delay

$$g^{(2)}(t) \doteq \frac{|\langle\hat{a}_2\hat{a}_1\rangle|^2}{\langle\hat{a}_2^{\dagger}\hat{a}_2\rangle\langle\hat{a}_1^{\dagger}\hat{a}_1\rangle} + 1$$

$$\bar{n}_j = \langle\hat{a}_j^{\dagger}\hat{a}_j\rangle$$

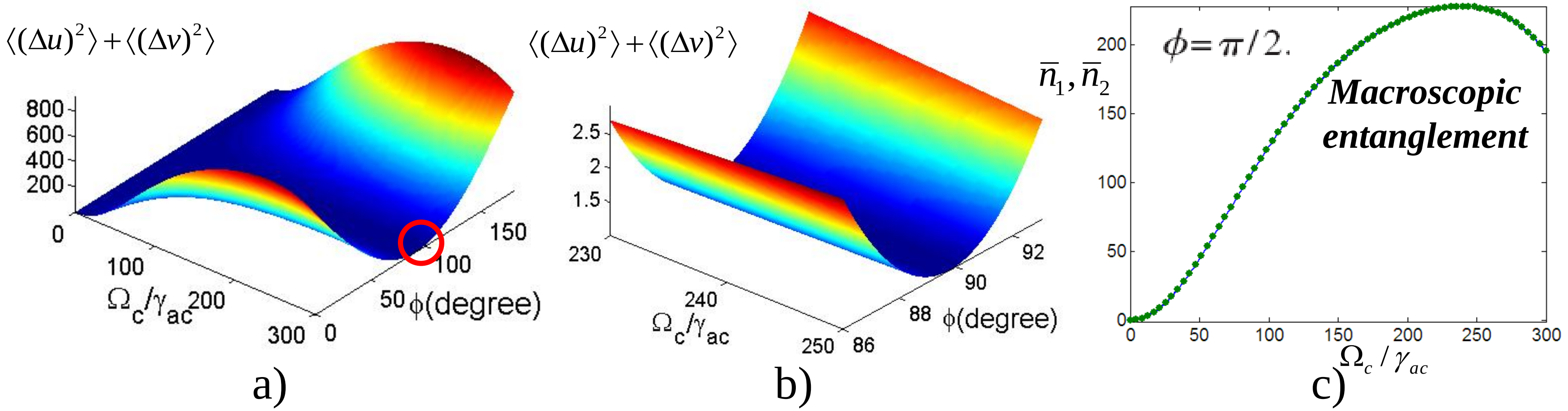
Relationship between entanglement and correlation

$$\underbrace{\langle\hat{a}_2\hat{a}_1\rangle + \langle\hat{a}_1^{+}\hat{a}_2^{+}\rangle}$$

$$\begin{aligned} D(t) = {} & 2[1 + \bar{n}_1 + \bar{n}_2 + 2\sqrt{\bar{n}_1\bar{n}_2(g^{(2)}(t) - 1)}\cos\phi_{21} \\ & - |\langle\hat{a}_2\rangle|^2 - |\langle\hat{a}_1\rangle|^2 - \langle\hat{a}_2\rangle\langle\hat{a}_1\rangle - \langle\hat{a}_2^{\dagger}\rangle\langle\hat{a}_1^{\dagger}\rangle]. \end{aligned}$$

Larger pump field and detuning

$$\Omega_p = 10\gamma_{ac},\ \Delta_p = 400\gamma_{ac},$$



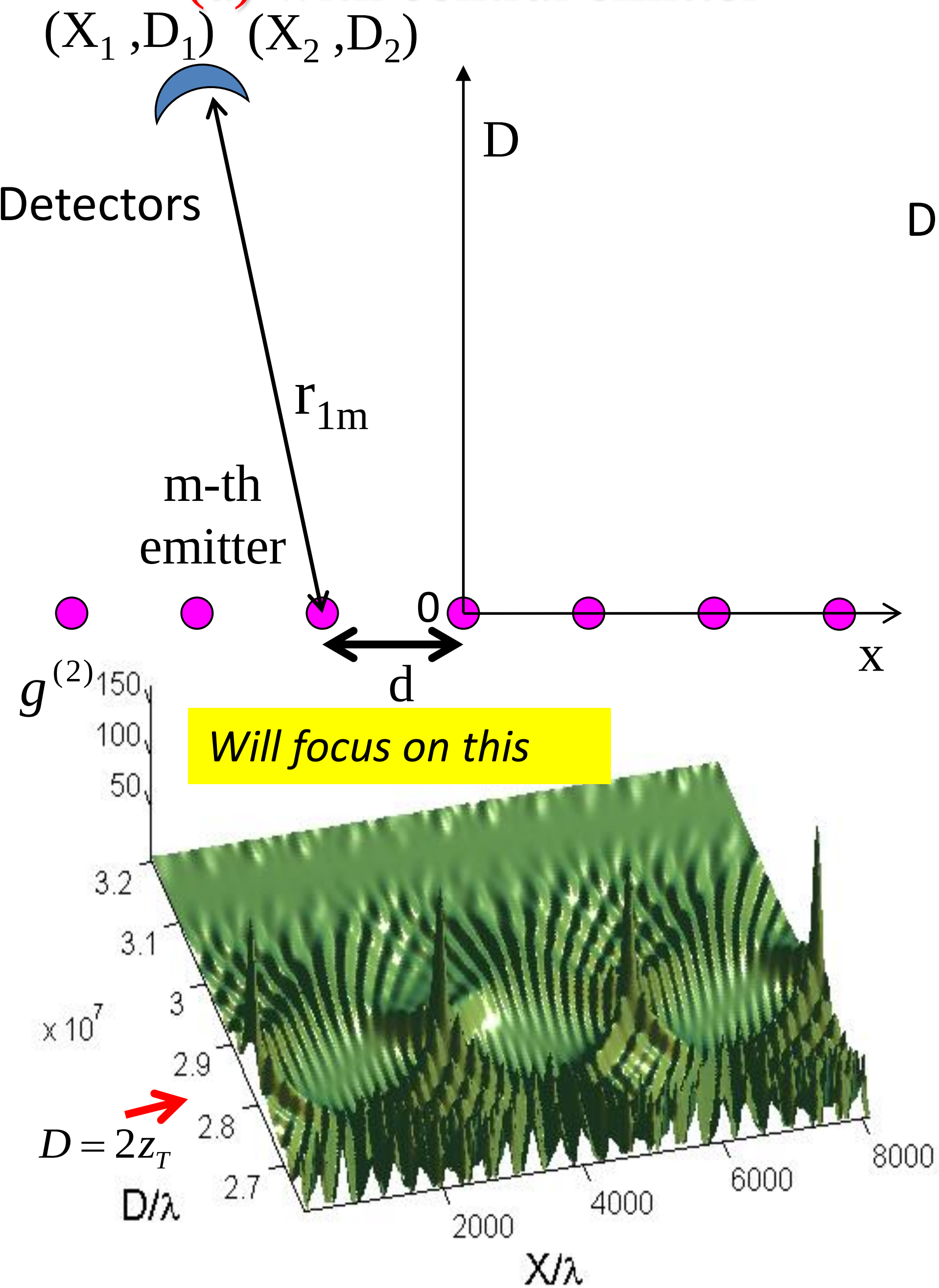
$$\kappa_1 = \kappa_2 = 1.001|C_2|$$

Entanglement around $\phi\sim\pi/2$ with **macroscopic** number of photons at a large field $\Omega_c\sim250\,\gamma_{ac}$!

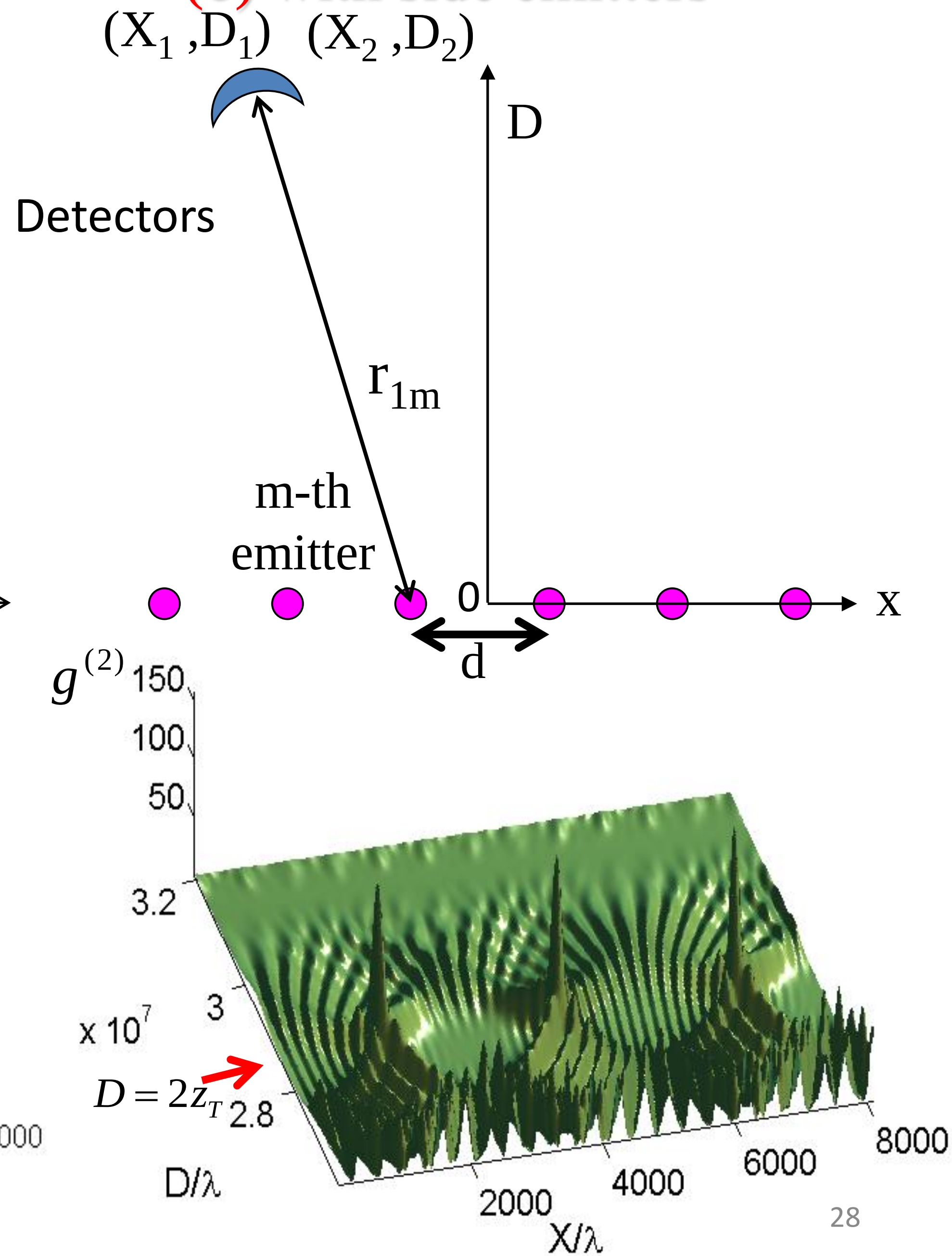
Why hard to have macroscopic entanglement ?

CHRaymond Ooi & BL Lan, PRA 81, 063832 (2010)

(a) with central emitter



(b) with side emitters

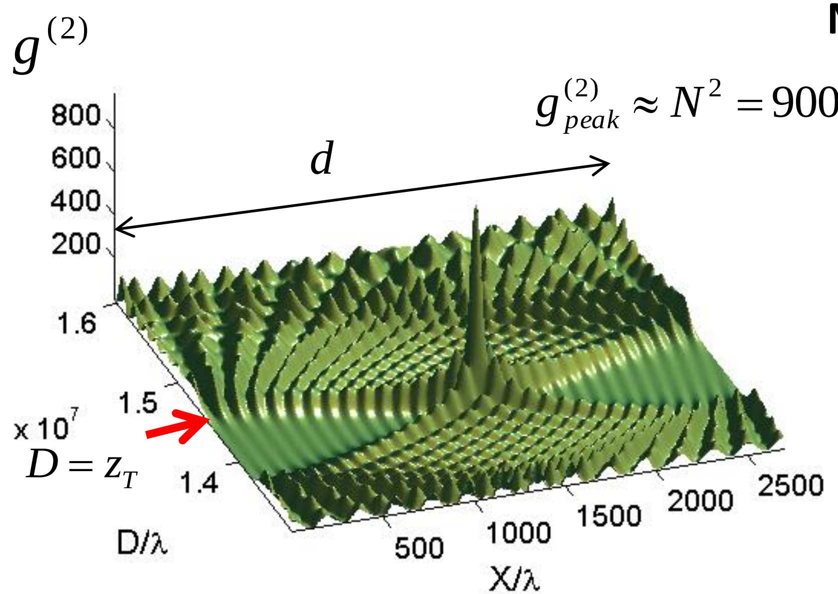


Interference effect

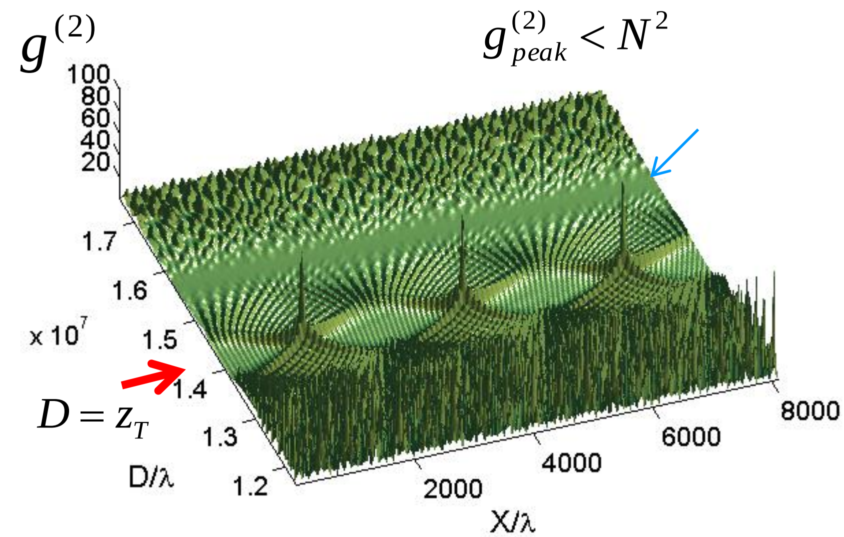
Without sine term

With sine term

N=30



(a)



(b)

At half the two-photon Talbot length the peaks are shifted half-period compared the pattern at $D=2z_T$

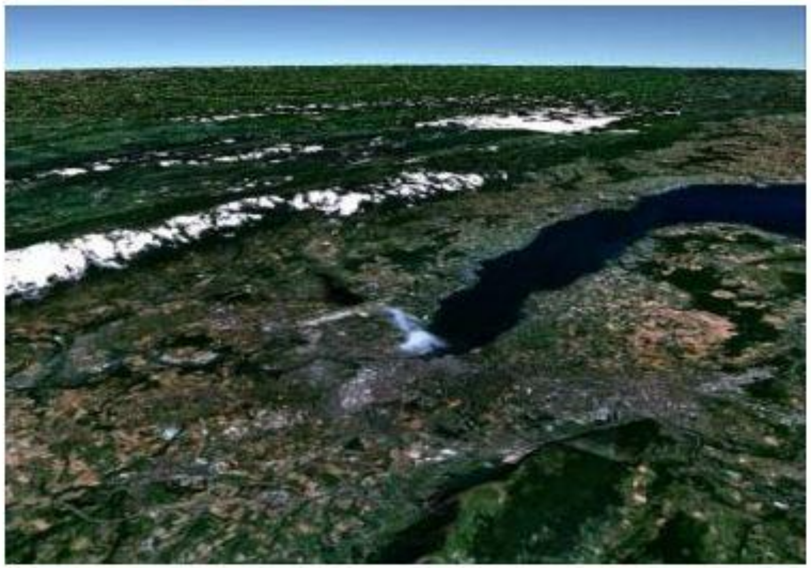
Evidence of nonlocality

Anton Zeilinger of the University of Vienna : transmitted a quantum key wirelessly over a distance of 144 km, between two of the Canary Islands - the longest distance quantum information has been transported through the air.

He likened the **entangled photons to a pair of "quantum dice,"** that would always **show the same number no matter how far they are separated**.

Entanglement remains a mystery

Aug 13, 2008 38 comments



If two particles are "entangled", so quantum mechanics says, any tinkering with one can cause an instantaneous change in the other, no matter how separated they are.

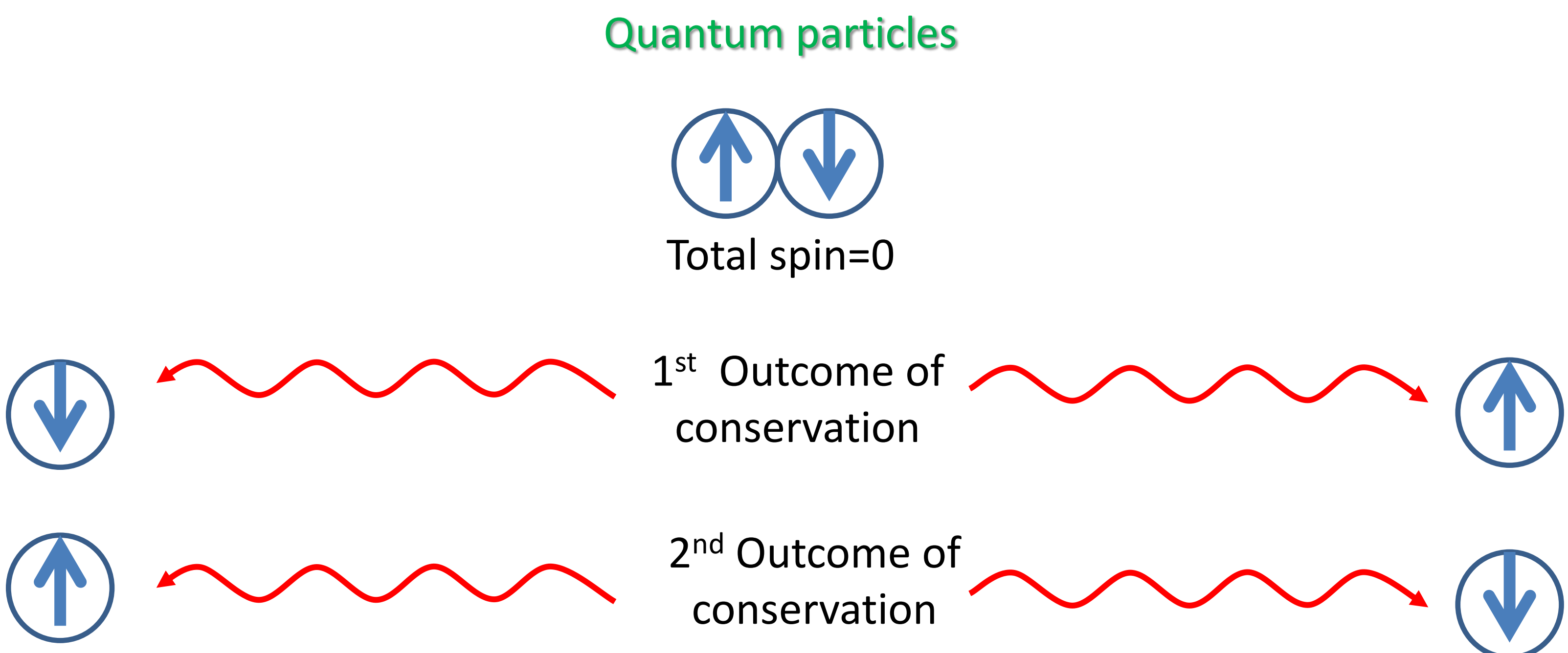
Einstein rejected this notion as "spooky action at a distance". But what if quantum mechanics is not quite right — that the change is not instantaneous, but instigated by a signal transmitted between the two entangled particles? Now an experiment performed in Switzerland has showed that, if such a signal does exist, it would have to travel at least as fast as light, and probably thousands of times faster.

— it implies the **photons are changing their properties instantaneously** to suit each other.

— the signal would have **to travel even faster, at more than 10,000 times the speed of light** (*Nature* **454** 861)

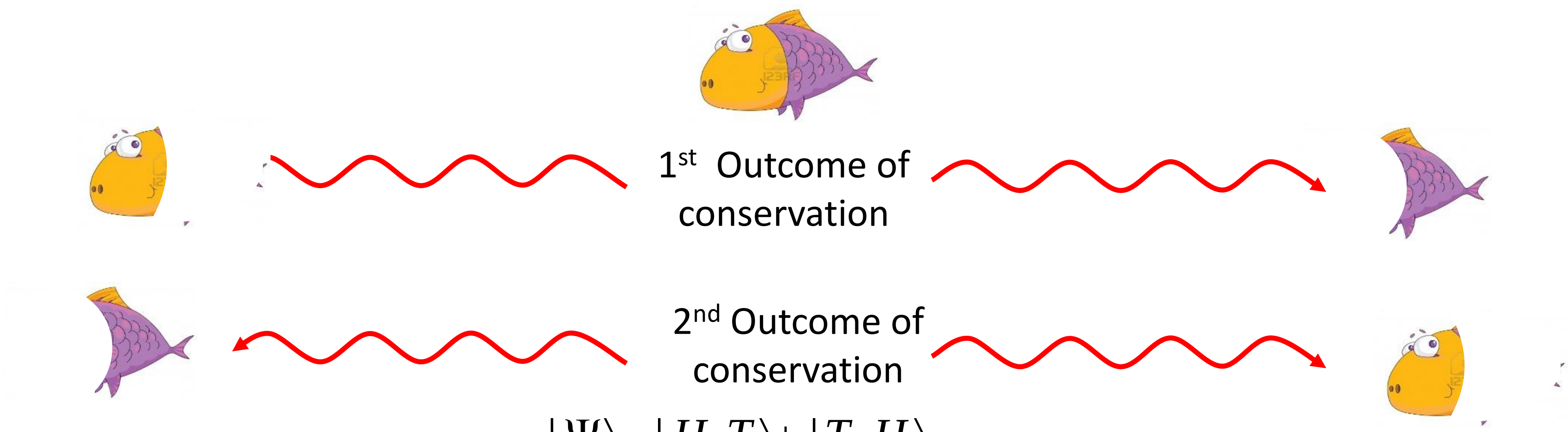
Gisin told *physicsworld.com* that his team's work, which is the first time the possibility of any hypothetical reference frame has been taken into account, "**confirms the predictions of quantum theory".** He also hopes it will enable other researchers to find a more palatable explanation for the mysteries of entanglement.

The Real Essence of Quantum Entanglement



$|\Psi\rangle = |\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle$ $|\Psi\rangle = |\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle$ $|\Psi\rangle = |\uparrow,\downarrow\rangle + e^{i\theta}|\downarrow,\uparrow\rangle$

Classical system/ a fish



$$|\Psi\rangle = |H, T\rangle + |T, H\rangle$$

There is no phase for classical system
Therefore, there is no interference

Phase is the Key
Phase gives interference
Phase gives due to Wave Nature

Demystefying entanglement

It is easy to understand entanglement as the consequence of oneness & conservation of a physical system (e.g. spin just like the head/tail of a fish cannot simply disappear.)

Entanglement is NOT the "Mystery"

The "mystery" arises from the misunderstanding that the two particles are separated and communicate through a superluminal signal that violates Einstein's special relativity.

Einstein's inability to grasp nonlocality of Nature has confused the scientific world for many years

Time to abandon Einstein's belief & hidden variable concept.

What is behind QM?

- Nature/Quantum entities are composed of waves or harmonic vibrations
  wave nature of matter.

- Quantization, discreteness , digital rather than analog come from the wave nature.
  (comes from the wave equation & the deBroglie equation)

- Wave nature gives rise to : the Heisenberg Uncertainty Principle
  (can be derived from FT of two canonical variables)

$$\hat{p} = -i\hbar \frac{\partial}{\partial x} \qquad p = h/\lambda \qquad \frac{\partial}{\partial x} \rightarrow i\frac{2\pi}{\lambda}$$

Particle

Wave

$$\Delta p \Delta x > \hbar$$

Uncertainty principle

$$[\hat{a}(t), \hat{a}^{\dagger}(t)] = 1$$

Noncommutivity

$$n\hbar\omega, l(l+1)\hbar$$

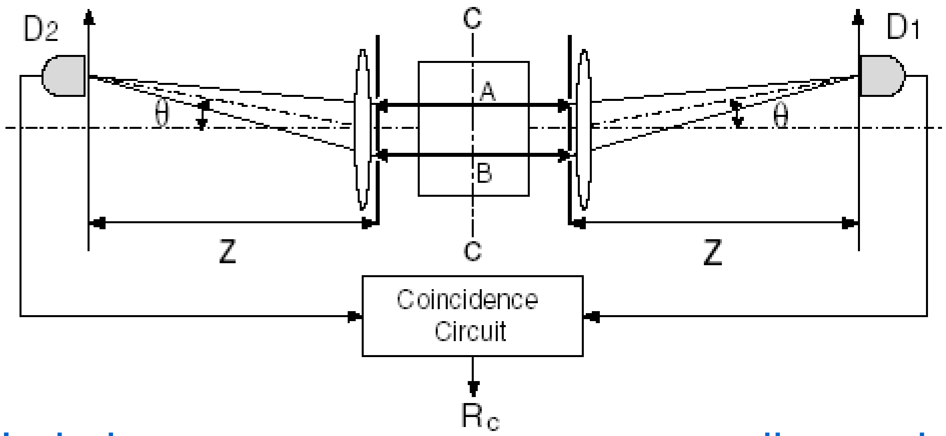
Quantization

CH Raymond Ooi & PR Berman, Physica E 42 (2010) 407–410

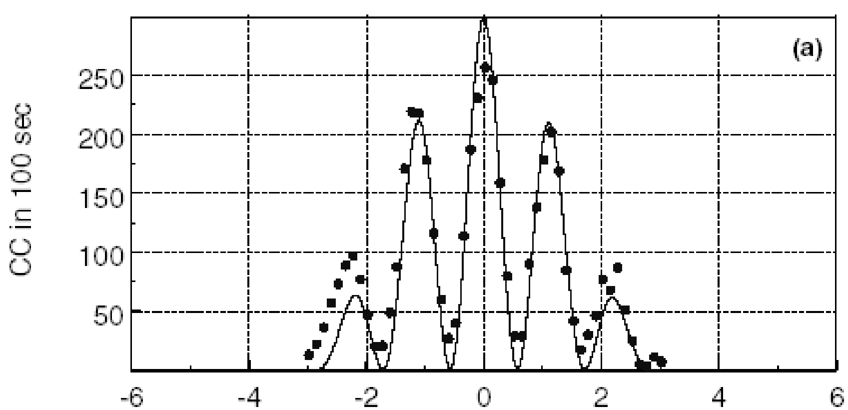
QM is more than just nonclassicality or entanglement

Entangled photons improves resolution – quantum lithography

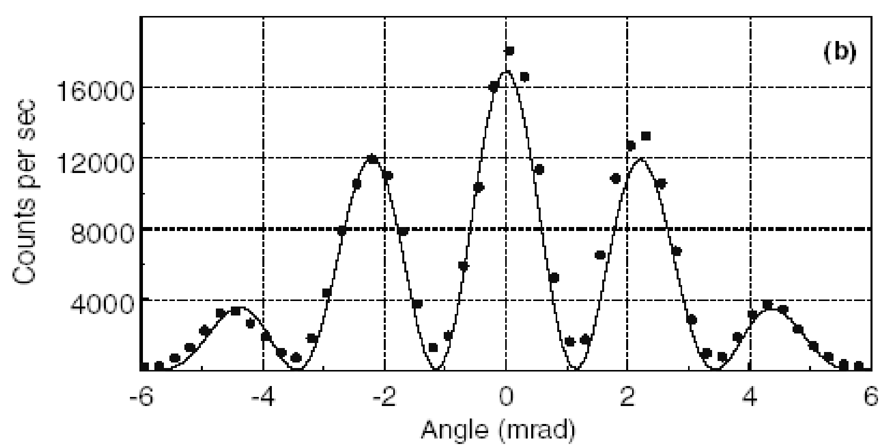
M. D. Angelo, M. V. Chekhova, and Y. Shih, *PRL* 87, 013602, 2001.



entangled photons



ordinary photons



resolution is doubled

2 entangled photons are like 1 frequency-doubled photon - Oneness

Nature of Entanglement

Oneness

Beyond space

Conserved
& Transforming

Susceptible to noise
& environment

Interconnectedness

Conclusions

Entanglement Demystefied:

Understand nature as a whole rather than the sum of parts.

Quantum Mechanics Demystefied:

The wave nature in particle (wave-particle duality) is the crux of the "Mystery" in QM
Normal human beings are not used to think/experience wave nature of matter which manifest only at submicron level.

Understanding these is Primal Virtue.
Primal Virtue is deep and far.
It leads all things back
Toward the great oneness.

Thank You