Transmission of Entanglement in Three Coupled Qubit Systems With Heat Bath

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Isfahan Quantum Optics Group

Outline

- Motivations, Basic Concepts.
- The Evolution of a three partite system coupled to thermal reservoir.
- Transmission of Entanglement in the presence of thermal reservoir.
- Monogamy Inequality in Terms of Negativity.
- Effects of External Fields on Transmission of Entanglement and Control of Entanglement.
- Conclusions and Future Directions.

Motivations

- Any observed disappearance of quantum coherence and interference can be attributed to **interaction** with the environment, that is, **Decoherence.** But is it always true to say that Reservoir is a destroyer of Entanglement?

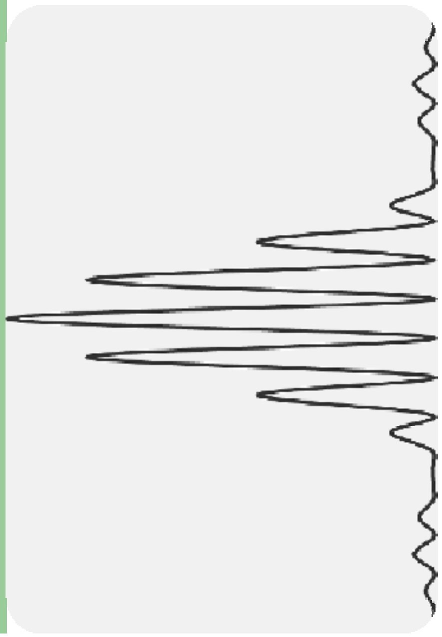
- To understand how to **control and transfer** entanglement as a resource of quantum communication and information, we need to study the mechanism by which this process occurs.

- There has been a lot of interest and work in this regard, but most of this work involves the coupling between **two qubits ,**We will instead concentrate on the coupling between **three Qubits**.

Superposition Principle, Basic Concepts

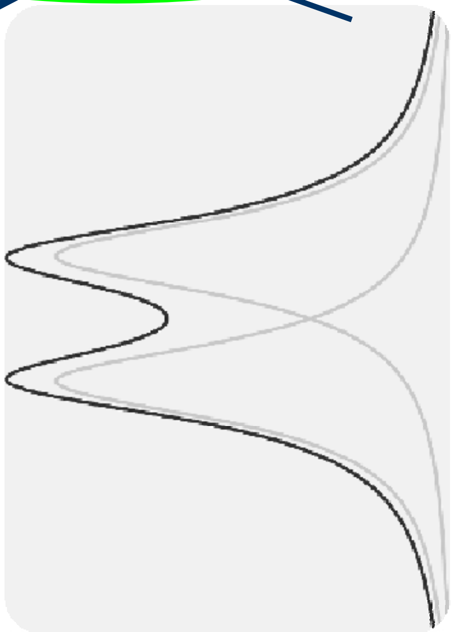
$$|\psi\rangle = \sum C_n |\psi_n\rangle \qquad |\psi\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle)$$

$$\rho(x) = \frac{1}{2}|\psi_1(x)+\psi_2(x)|^2 = \frac{1}{2}|\psi_1(x)|^2 + \frac{1}{2}|\psi_2(x)|^2 + \mathrm{Re}\{\psi_1(x)\psi_2^*(x)\}$$



Interference Term

$$\rho \propto \frac{1}{2}|\psi_1(x)|^2 + \frac{1}{2}|\psi_2(x)|^2$$



Decoherence, Basic Concepts

What is Decoherence ?

- Measurement by the environment
- Caused by the environment, Environment Noise
- hallmark of the crossover from the quantum to the classical world
- Collapse of Wave function
- Destruction of quantum mechanical behavior

Destruction of Entangled state

Quantum Entanglement, Basic Concepts

A pure state is separable if it can be expressed as a tensor product of its subsystems: $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$

A mixed state is separable if it can be represented as a mixture of product states: $\rho = \sum_i p_i |a_i\rangle\langle a_i| \otimes |b_i\rangle\langle b_i|$

State that is not separable is entangled

Bell states (pure entangled states)

Werner states (mixed entangled states)

$$|\psi'\rangle = \frac{1}{\sqrt{2}}|01\rangle + |11\rangle$$ → $$|\psi'\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |1\rangle$$

Separable state

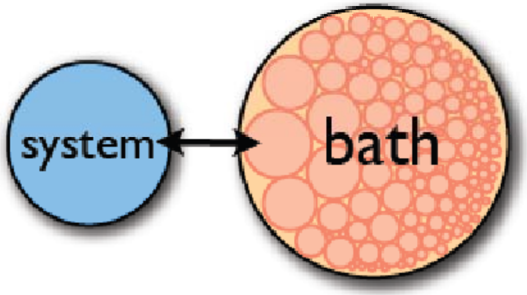
$$|\psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle$$ → **Entangled State**

Decoherence, Basic Concepts

Closed quantum systems

Unitary evolution keep the system in pure state, and prevent from mixed-state.

- Open quantum systems



Real system cannot be perfectly isolated from its surroundings. The open system evolution is characterized by non-Unitary evolution.

Interaction of a two level atom with a single mode field

$$\hat{H} = \hbar\omega\hat{\sigma}_z + \hbar\omega_f(\hat{a}^\dagger\hat{a} + \frac{1}{2}) + \hbar g(\hat{a} + \hat{a}^\dagger)(\hat{\sigma}_x)$$

Janynes-Cumming Model

$$\hat{H}_I(t) = \hbar g(\hat{a}^\dagger\hat{\sigma}^- \exp(i\Delta t) + \hat{a}\hat{\sigma}^+ \exp(-i\Delta t))$$

$$\Delta = \omega_f - \omega$$

Rotation Wave Approximation-Interaction Picture

$$\hat{H}_I(t) = \hbar\sum_k g_k(\hat{a}_k^\dagger\ \hat{\sigma}^- \exp(i\Delta t) + \hat{a}_k\hat{\sigma}^+ \exp(-i\Delta t))$$

Thermal Reservoir-Rotation Wave Approximation

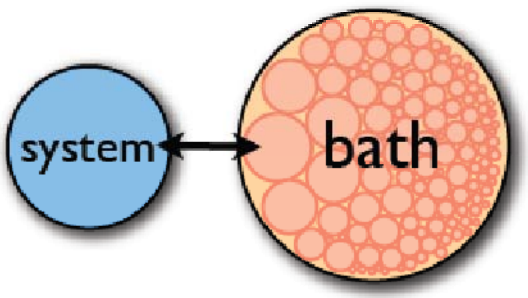
Interaction of a two level atom with a single mode field

$$d\hat{\rho}_S / dt = \hat{L}[\hat{\rho}_S(t)] = -i[\hat{H}'_S, \hat{\rho}_S(t)] + \hat{D}[\hat{\rho}_S(t)]$$

Master Equation

Unitary Evolution

Non-Unitary evolution



Decoherence and possibly also Dissipation

Interaction of a two level atom with a thermal Reservoir

Born-Markov Master Equation

$$d\hat{\rho}/dt = \frac{-i}{\hbar} Tr_R([\hat{H}_I(t), \hat{\rho}_{S(t_i)} \otimes \hat{\rho}_{R(t_i)}] - \frac{1}{\hbar^2} Tr_R \int_{t_i}^{t} \hat{H}_I(t), [\hat{H}_I(t'), \hat{\rho}_{S(t)} \otimes \hat{\rho}_{R(t_i)}]]dt'$$

Born Approximation

Markov Approximation

$$d\hat{\rho}_S/dt = [\hat{H}'_s, \hat{\rho}_S(t)] - n_{th}\frac{\Gamma}{2}[\hat{\sigma}^-\hat{\sigma}^+\hat{\rho}_S(t) + \hat{\rho}_S(t)\hat{\sigma}^-\hat{\sigma}^+ - 2\hat{\sigma}^+\hat{\rho}_S(t)\hat{\sigma}^-]$$

$$-(n_{th}+1)\frac{\Gamma}{2}[\hat{\sigma}^+\hat{\sigma}^-\hat{\rho}_S(t) + \hat{\rho}_S(t)\hat{\sigma}^+\hat{\sigma}^- - 2\hat{\sigma}^-\hat{\rho}_S(t)\hat{\sigma}^+]$$

Absorbing energy from the environment

$$n_{th} = \frac{1}{\exp(\frac{\hbar w_k}{k_B T}) - 1}$$

Emission of energy to the environment

$$T = 0 \Rightarrow n_{th} = 0$$

Interaction of a two level atom with a thermal Reservoir

Results:

- at T=0, the energy of excited atom emitted to the reservoir.
- Reservoir acts as a **disturbing factor** which absorbs energy of the excited atom.
- After the short run, exited atom decay to its lower level.
- Due to **dissipation,** We expect that at T=0 thermal reservoir is **a disadvantage for** atoms.

Interaction of N identical Qubits with a thermal Reservoir at T=0

$$\hat{H} = \sum_{i}^{N} \hbar\omega\hat{\sigma}_i^z + \sum_{l} \hbar\omega_l(\hat{a}_l^\dagger\hat{a}_l + \frac{1}{2}) + \hbar\sum_{l}\sum_{i=1}^{N}(g_l(\hat{\sigma}_i^+\hat{a}_l + \hat{\sigma}_i^-\hat{a}_l^\dagger))$$

$$d\hat{\rho}_S / dt = [\hat{H}_s', \hat{\rho}_S(t)] - \frac{\Gamma}{2}[\hat{\sigma}^+\hat{\sigma}^-\hat{\rho}_S(t) + \hat{\rho}_S(t)\hat{\sigma}^+\hat{\sigma}^- - 2\hat{\sigma}^-\hat{\rho}_S(t)\hat{\sigma}^+]$$

$$\hat{\sigma}^\pm = \sum_{i}^{N}\hat{\sigma}_i^\pm$$

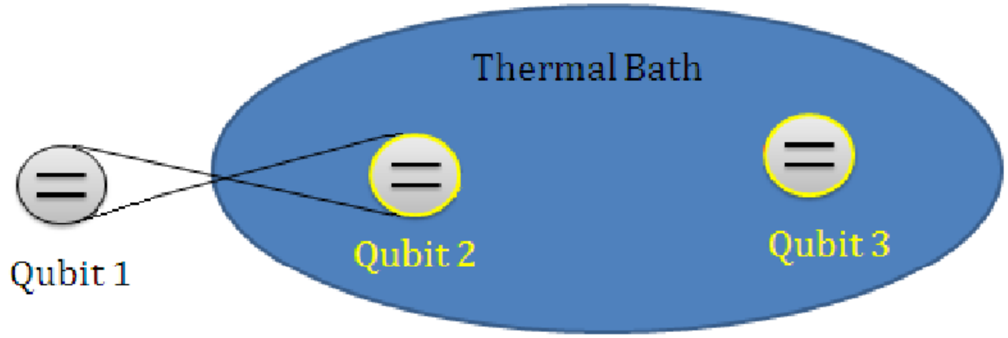
$$\Gamma = \omega^3\mu^2 / (3\pi\varepsilon_0\hbar c^3)$$

Weisskopf-Wigner Approximation

$$d\hat{\rho} / dt = [\hat{H}, \hat{\rho}] / i\hbar + L[\hat{\rho}]$$

*Environment-Induced Entanglement with a single photon, **Physical Review A**, 2009, L. Davidovich et. al.*

Transmission of Entanglement



$$d\hat{\rho}/dt = [\hat{H},\hat{\rho}]/i\hbar + L[\hat{\rho}]$$

Master Equation

$$((|\uparrow\downarrow\rangle_{1,2} - |\downarrow\uparrow\rangle_{1,2})/\sqrt{2}) \otimes |\downarrow\rangle_3$$

Initial State

$\{|\uparrow\rangle, |\downarrow\rangle\}$ Eigenstates σ^z

Transmission of Entanglement, Negativity Measure

$$d\hat{\rho}_S / dt = -\frac{\Gamma}{2}[\hat{\sigma}^+\hat{\sigma}^-\hat{\rho}_S(t) + \hat{\rho}_S(t)\hat{\sigma}^+\hat{\sigma}^- - 2\hat{\sigma}^-\hat{\rho}_S(t)\hat{\sigma}^+]$$

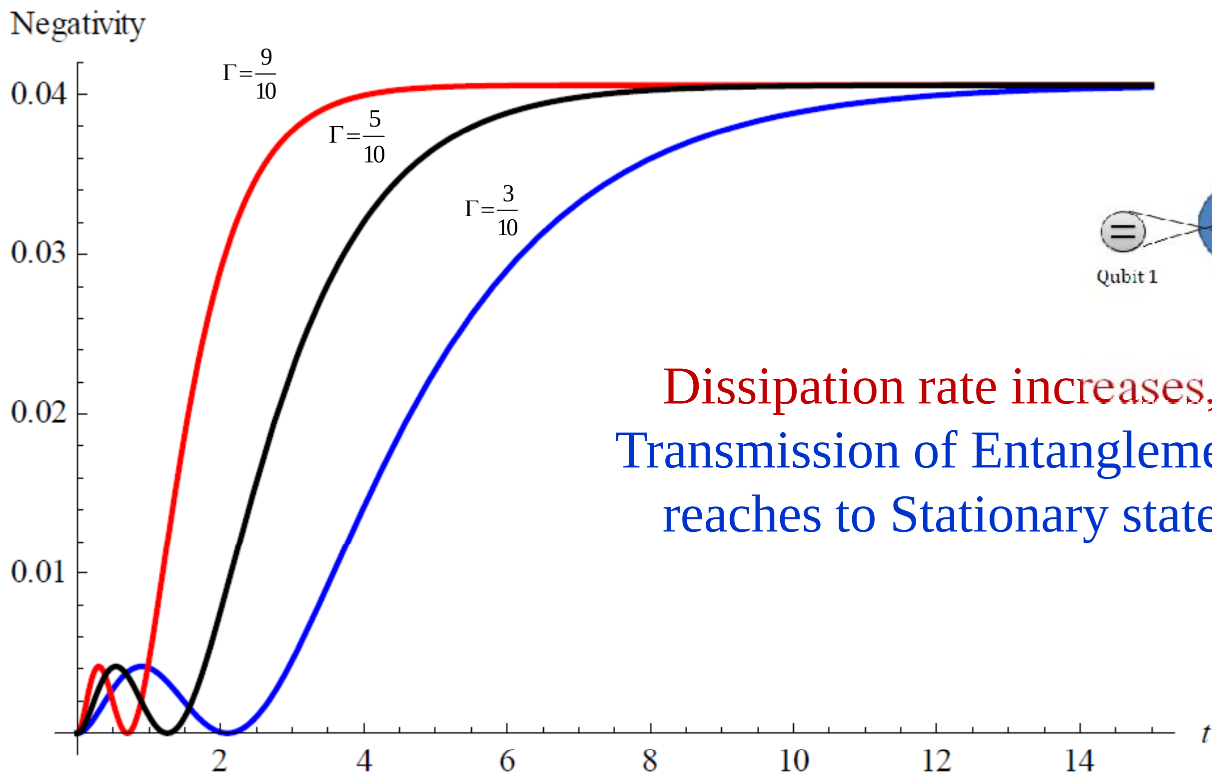
$$\hat{\sigma}^{\pm} = \sum_{i=2}^{3}\hat{\sigma}_i^{\pm}$$

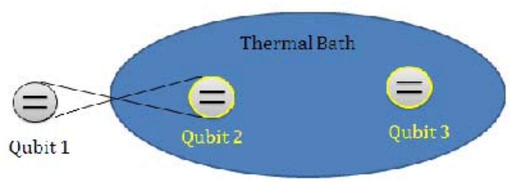
$$\square = \left(\left\|\rho^{T_B}\right\|_1 - 1\right)/2 \qquad \left\|\rho^{T_B}\right\|_1 = Tr\sqrt{\rho^{T_B}\rho^{T_B\dagger}}$$

8-dimensional Hilbert space

→ **64-Coupled differential Equations**

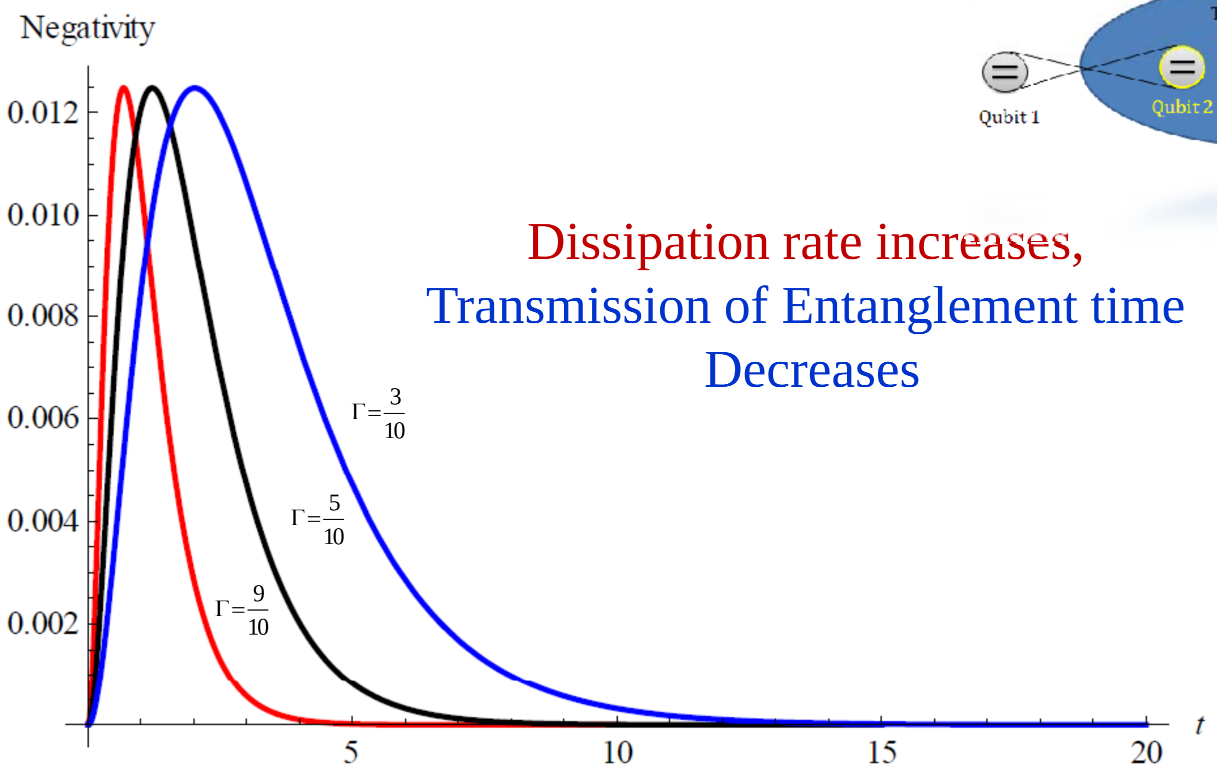
Transmission of Entanglement between Second and Third Qubits

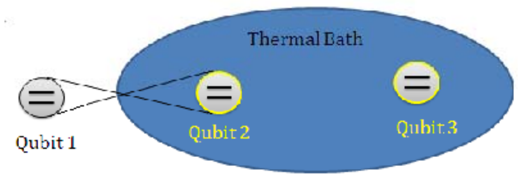




Dissipation rate increases,
Transmission of Entanglement
reaches to Stationary state

Transmission of Entanglement between First and Third Qubits

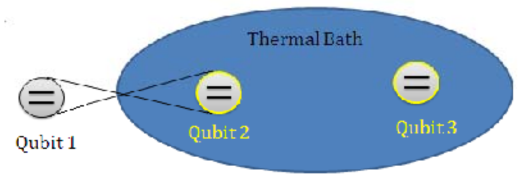


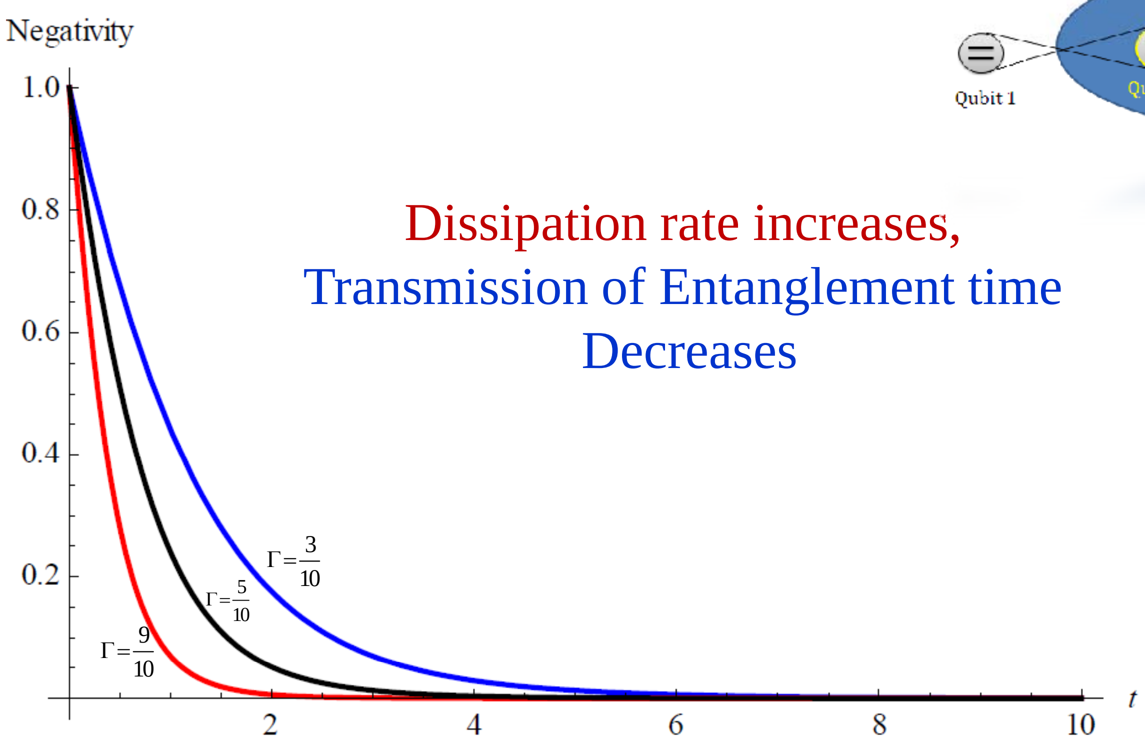


Dissipation rate increases,

Transmission of Entanglement time Decreases

Transmission of Entanglement between First and Second Qubits





Dissipation rate increases,
Transmission of Entanglement time Decreases

Monogamy Inequality in terms of Negativity, Concurrence Measure

$$\rho' = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$$

$$C(\rho') \equiv \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}$$

For Two-Qubit System

$$C_{AB}^2 + C_{AC}^2 \leq C_{A(BC)}^2$$

$$C_{A(BC)} = 2\sqrt{\det \rho_A}$$

Coffman-Kundu-Wootters

CKW

For Three-Qubit System

Monogamy Inequality in terms of Negativity

$$C_{AB}^2 + C_{AC}^2 \le C_{A(BC)}^2$$

$$\square_{AB}^2 + \square_{AC}^2 \le \square_{A(BC)}^2$$

Monogamy Inequality

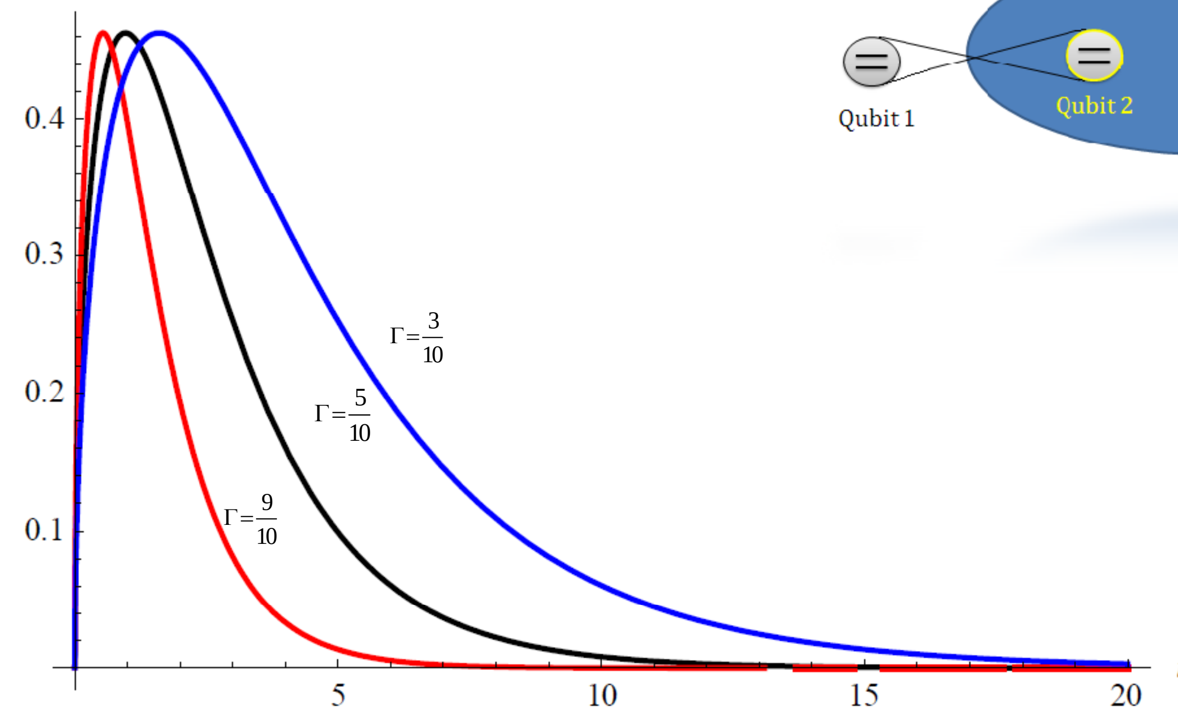
$$\square_{AB} = C_{AB}$$

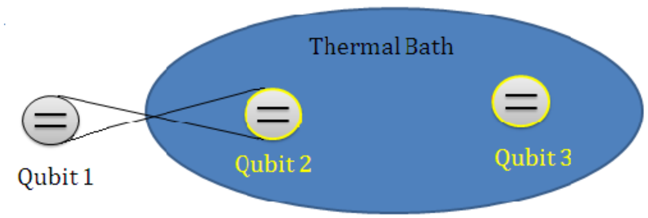
$$\square_{AC} = C_{AC}$$

$$\square_{A(BC)} = C_{A(BC)}$$

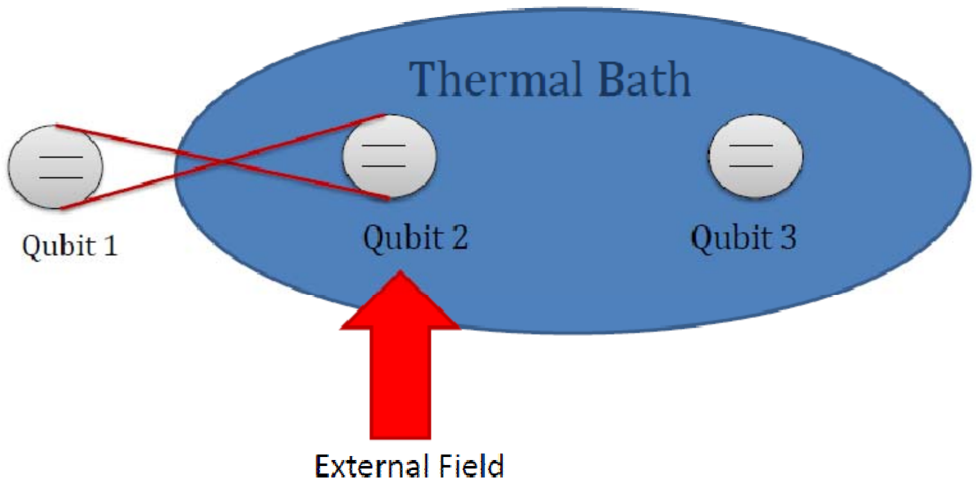
*Monogamy Inequality In terms of Negativity for three-Qubit, **Physical Review A**, 2007, Yong-Cheng Ou and Heng Fan.*

Monogamy Inequality in terms of Negativity





Effects of External Fields on Transmission of Entanglement

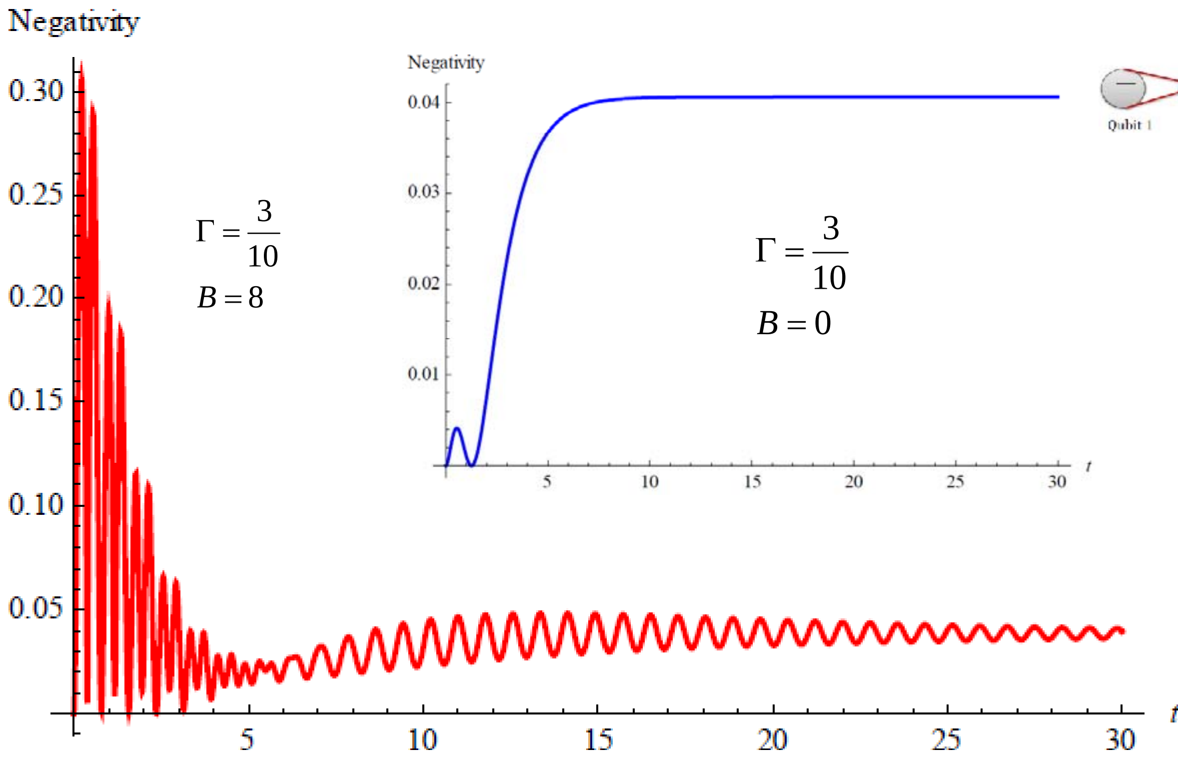


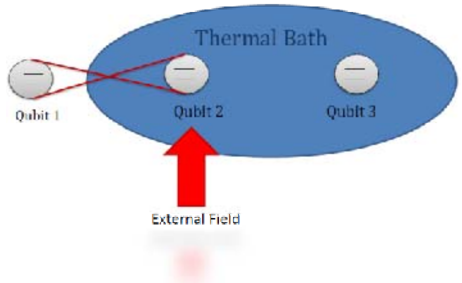
$$d\hat{\rho}_S / dt = [\hat{H}, \hat{\rho}_S] / i\hbar + L[\hat{\rho}_S]$$

$$\hat{H} = B\hat{\sigma}_x$$

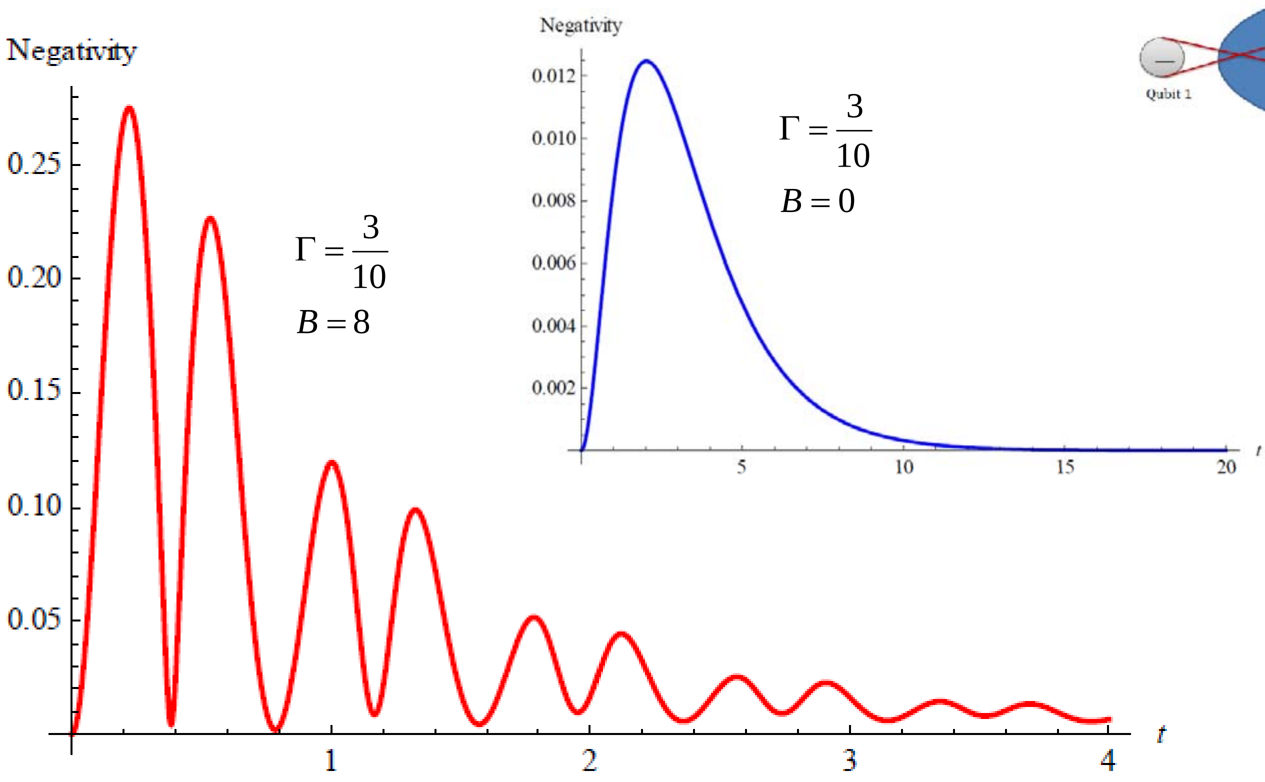
$$L[\hat{\rho}_S] = \frac{\Gamma}{2}[2\hat{\sigma}^- \hat{\rho}_S(t)\hat{\sigma}^+ - \hat{\sigma}^+\hat{\sigma}^- \hat{\rho}_S(t) - \hat{\rho}_S(t)\hat{\sigma}^+\hat{\sigma}^-]$$

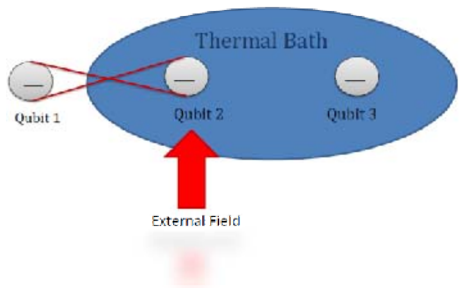
Transmission of Entanglement between Second and Third Qubits



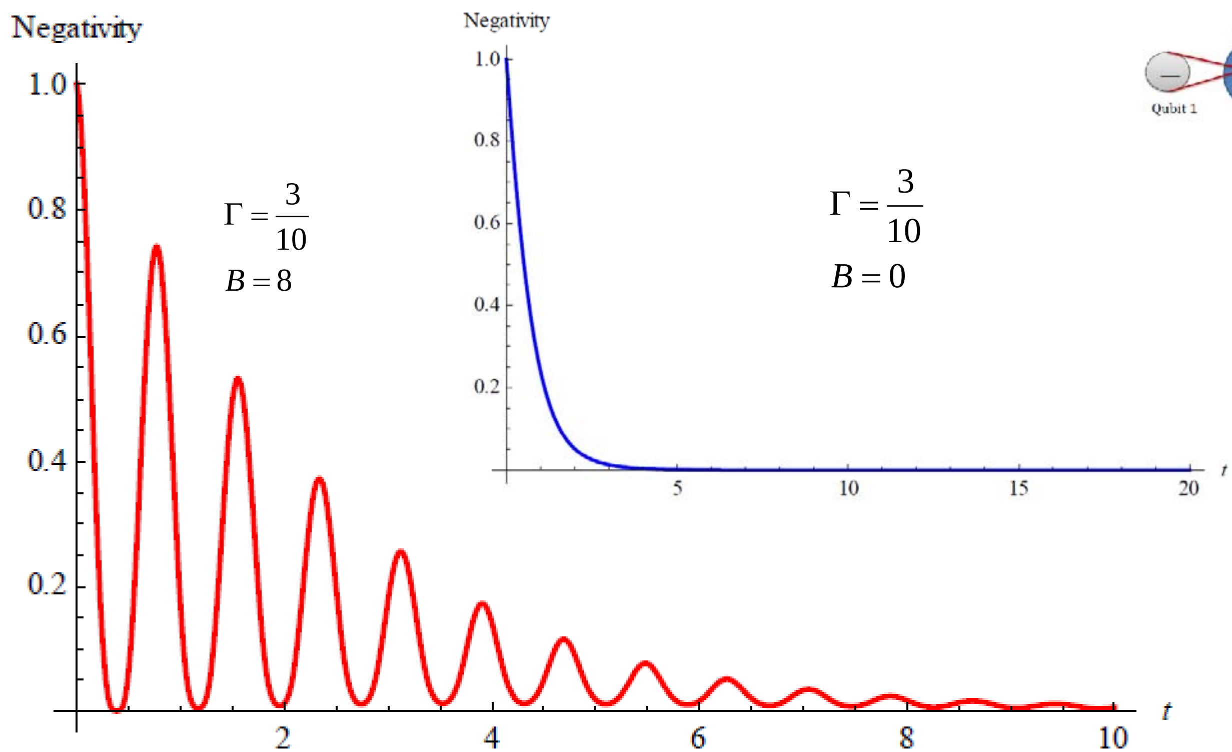


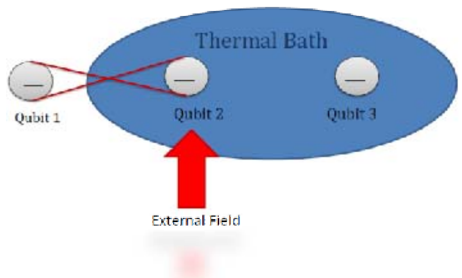
Transmission of Entanglement between First and Third Qubits



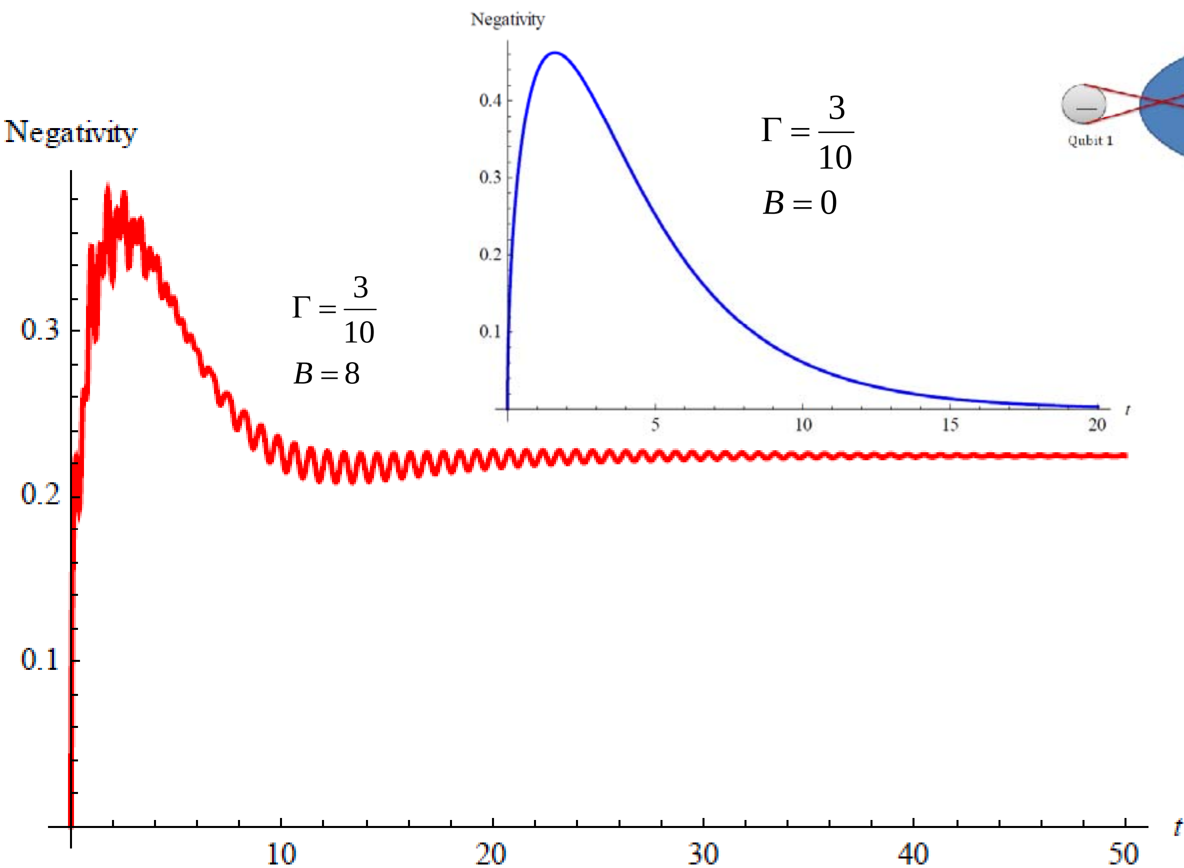


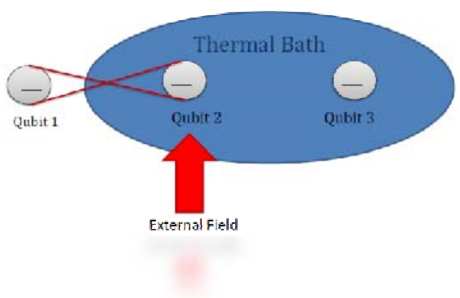
Transmission of Entanglement between First and Second Qubits



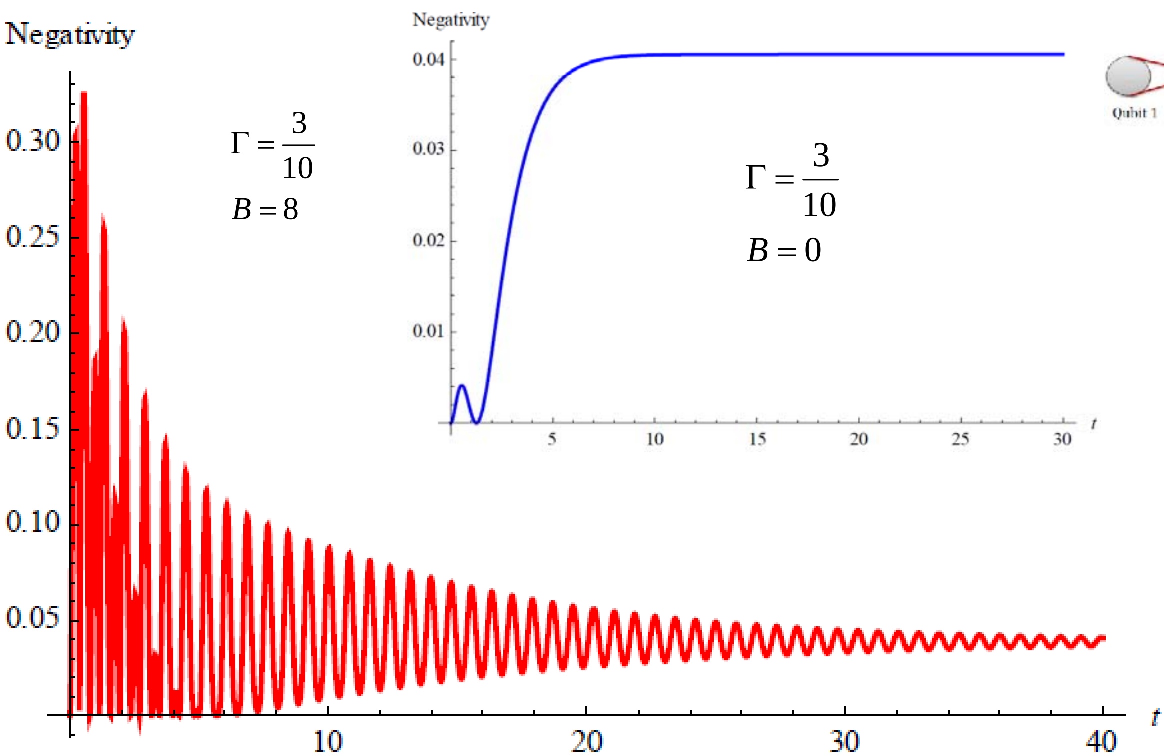


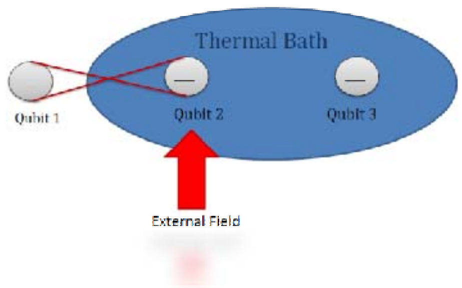
Monogamy Inequality in terms of Negativity





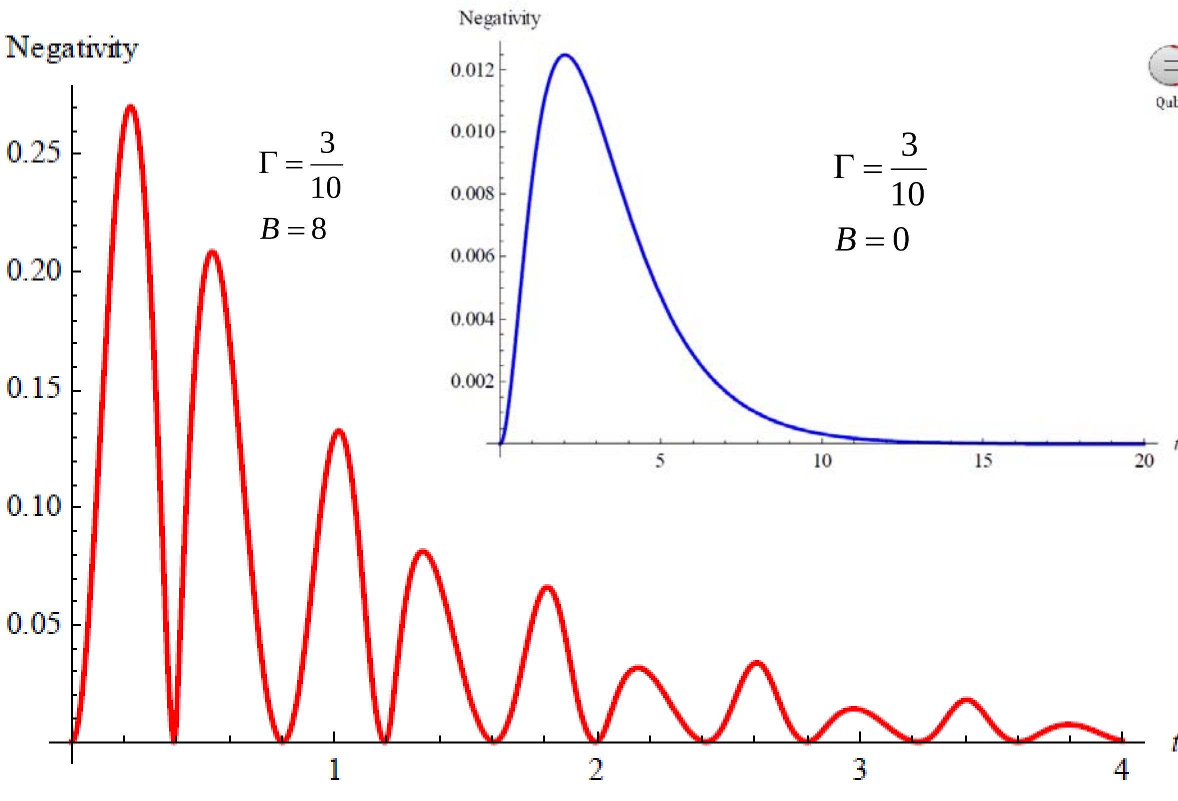
Transmission of Entanglement between Second and Third Qubits

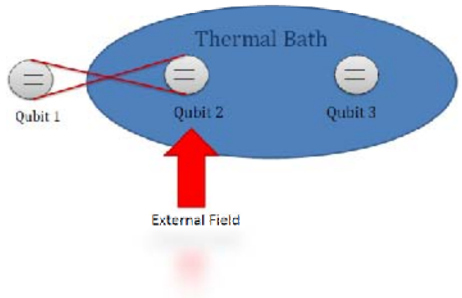




$$\hat{H} = B\hat{\sigma}_y$$

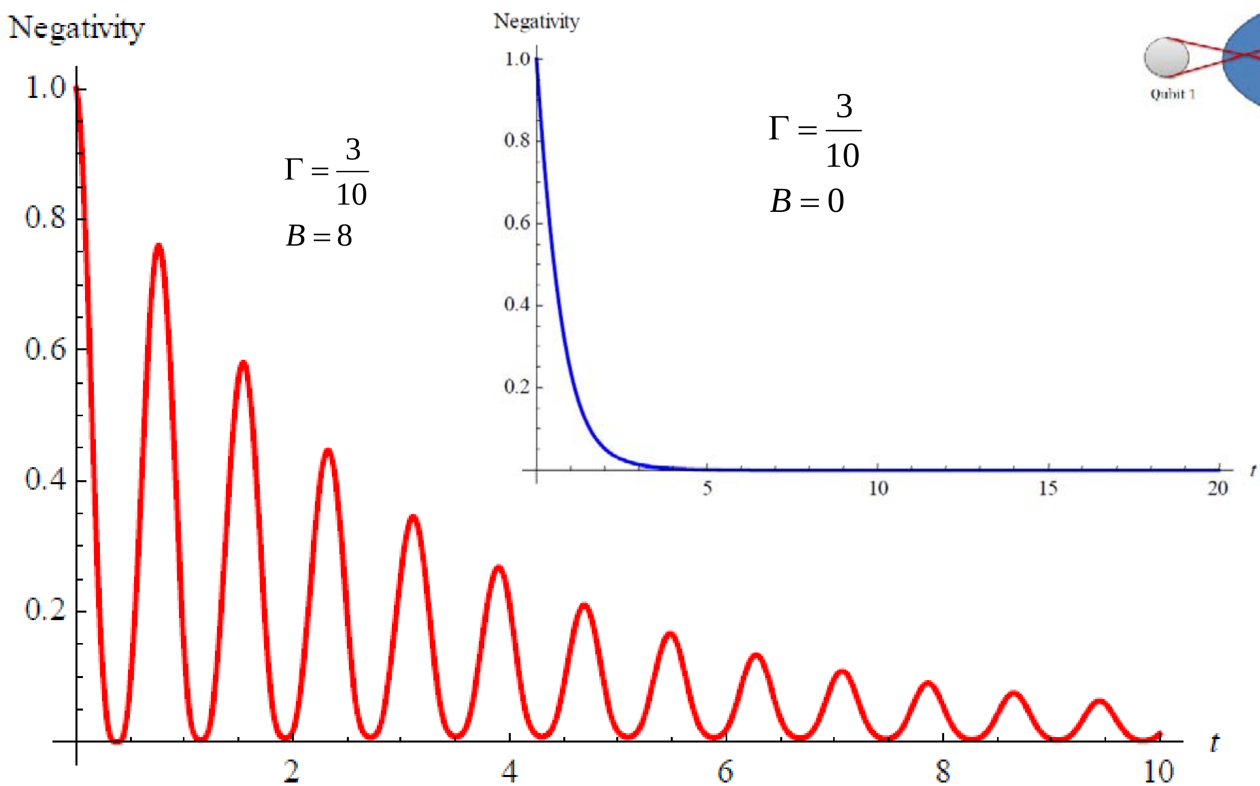
Transmission of Entanglement between First and Third Qubits





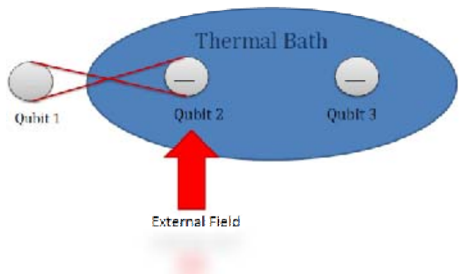
$$\hat{H} = B\hat{\sigma}_y$$

Transmission of Entanglement between First and Second Qubits



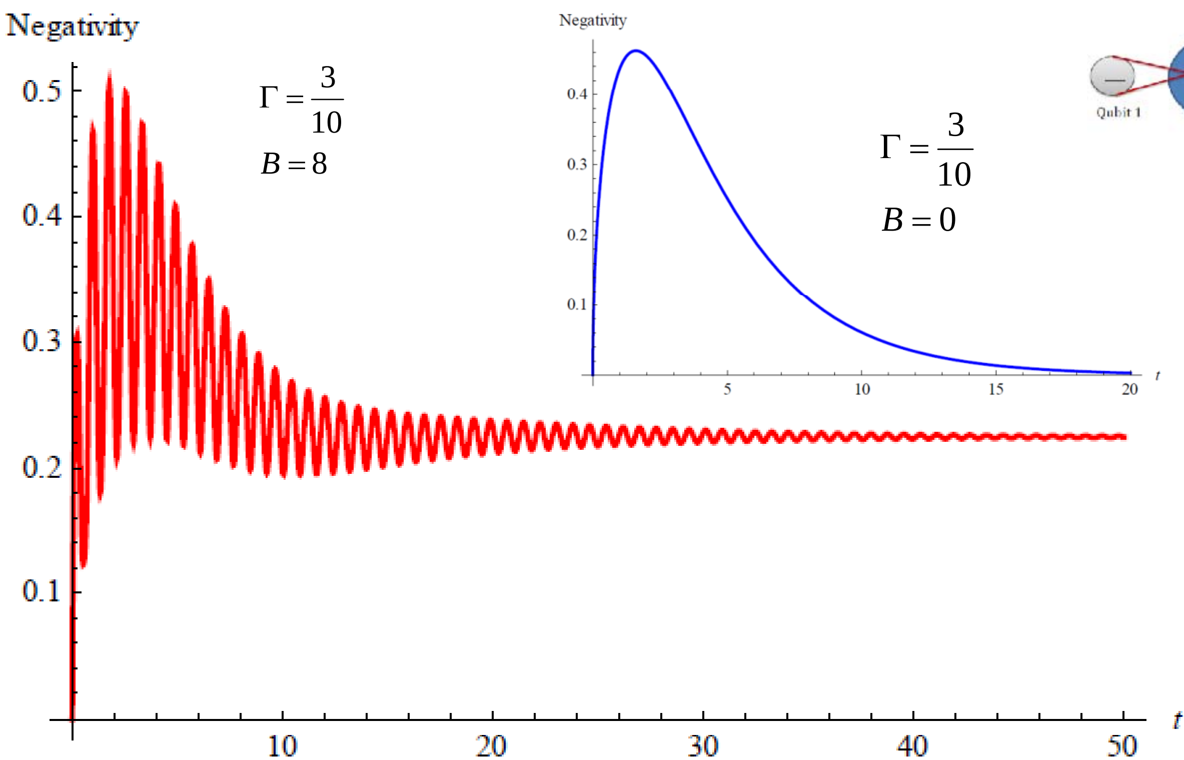
$$\Gamma = \frac{3}{10}$$
$$B = 8$$

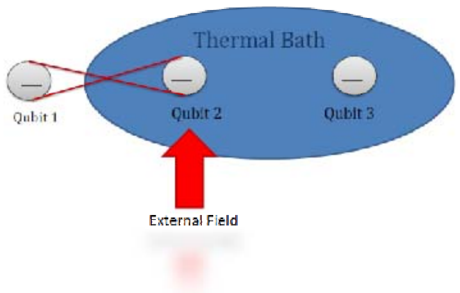
$$\Gamma = \frac{3}{10}$$
$$B = 0$$



$$\hat{H} = B\hat{\sigma}_y$$

Monogamy Inequality in terms of Negativity





$$\hat{H} = B\hat{\sigma}_y$$

Conclusion and Future Direction

- Creation of Entanglement between **Second** and **Third** Qubits in the common thermal reservoir.

- Transmission of entanglement from on subsystem consists of **First** and **Second** Qubits to another one that contains **First** and **Third** Qubits as a result of the above effect.

- Realizing the effect of **External magnetic Field** as an object for control of entanglement.

Future Directions

- To explore a theoretical set-up for **optimum control** and Transmission of Entanglement in three Partite Open Quantum system.
- The mechanism which give rise to the desire Entangled state even in presence of Noise source.
- To change the Nature of Reservoir with memory effect and investigate the effect of it on the **control** and **Transmission of Entanglement.**
- New techniques for generating **long lived Entanglement** by means of Reservoir Engineering.

Thank you for your attention

Experimental Set-Up

- Consider two flux qubit that are coupled to a transition line resonator. An external voltage field is applied to system. It can be shown that we can control the entanglement between qubit and resonator mode by applying detuning between external voltage field and qubit-resonator resonance frequency.