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Fuzzy (r,s)-Totally Semi-Continuous and Fuzzy (r,s)-Semi Totally-Continuous Mappings in Sostak's Sense

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ABSTRACT

In this paper, we introduce the concept of fuzzy (r,s)-totally continuous mappings and its variants totally semi-continuous and semi totally-continuous mappings on intuitionistic fuzzy topological spaces in Sostak's Sense. Their characterizations, examples and relationship with other notions of continuous mappings in this space are provided.

Keywords: intuitionistic fuzzy topological spaces; fuzzy(r,s)-totally continuous; fuzzy(r,s)-totally semi-continuous; fuzzy (r,s)-semi totally-continuous.

INTRODUCTION

The concept of fuzzy set was introduced by Zadeh (1965). Later, Chang (1968) defined fuzzy topological spaces. Then these spaces and its generalizations are studied by several authors, one of which developed by Sostak (1985) when used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by (Chattopadhyay *al.* (1992); Samanta *al.* (2002) and Ramadan (1992)). As a generalization of fuzzy sets. The concept of intuitionistic fuzzy sets was introduced by Atanassov (1986). Recently, Coker and his colleagues (1997) introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Coker and Demirci (1996) defined intuitionistic fuzzy topological spaces in Sostak's sense as a generalization of smooth topological spaces and intuitionistic fuzzy topological spaces. On the other hand Yong Chan Kim and his colleagues (1999) considered this concepts on smooth topological spaces. In (2011) Mukherjee introduced two new classes of mappings, called fuzzy totally continuous and fuzzy totally semi continuous mappings.

In this paper, we introduce and study the notions of fuzzy (r,s)-totally continuous mappings and its variants fuzzy (r,s)-totally semi-continuous and fuzzy (r,s)-semi totally-continuous mappings in intuitionistic fuzzy topological spaces in Sostak's sense are discussed.

PRELIMINARIES

Let I(X) be a family of all intuitionistic fuzzy sets in X and let $I \otimes I$ be the set of the pair (r, s) such that r, s ϵ I and $r + s \leq 1$.

Definition 2.1. (Coker (1996); Lee and Kim. (2007); Lee and Kim (2009))**.** Let X be a nonempty set. An intuitionistic fuzzy topology in-Sostak's sense (SoIFT for short) $\mathcal{T} = (\mathcal{T}_1, \mathcal{T}_2)$ on X is a mapping $\mathcal{T}: I(X) \rightarrow I \otimes I$ which satisfies the following properties:

(1) $\mathcal{T}_1(\underline{0}) = \mathcal{T}_1(\underline{1}) = 1$ and $\mathcal{T}_2(\underline{0}) = \mathcal{T}_2(\underline{1}) = 0$.

(2) $\mathcal{T}_1(A \cap B) \geq \mathcal{T}_1(A) \wedge \mathcal{T}_1(B)$ and $\mathcal{T}_2(A \cap B) \leq \mathcal{T}_2(A) \vee \mathcal{T}_2(B)$.

(3) $\mathcal{T}_1(\cup A_i) \geq \wedge \mathcal{T}_1(A_i)$ and $\mathcal{T}_2(\cup A_i) \leq \vee \mathcal{T}_2(A_i)$.

The $(X,\mathcal{T}) = (X,\mathcal{T}_1,\mathcal{T}_2)$ is said to be an intuitionistic fuzzy topological space in Sostak's sense (SoIFTS for short). Also, we call $\mathcal{T}_1(A)$ a gradation of openness of A and $\mathcal{T}_2(A)$ a gradation of non openness of A.

Definition 2.2. (Lee *and Kim* (2009); Lee and *Kim* (2011))**.** Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and (r, s) ϵ $I \otimes I$. Then A is said to be

(1) fuzzy (r, s)-open if $\mathcal{T}_1(A) \geq r$ and $\mathcal{T}_2(A) \leq s$,

(2) fuzzy (r, s)-closed if $\mathcal{T}_1(A^c) \geq r$ and $\mathcal{T}_2(A^c) \leq s$.

Definition 2.3. (Lee and Kim (2009)). Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS. For each (r,s) ϵ $I \otimes I$ and for each A ϵ I(X), the fuzzy (r,s)-interior and the fuzzy (r,s)-closure is defined by

$\mathrm{int}(A,r,s) = \cup\{B \in I(X) \mid B \subseteq A, B \text{ is fuzzy}(r,s)\text{-open}\};$
$\mathrm{cl}(A,r,s) = \cap\{B \in I(X) \mid A \subseteq B, B \text{ is fuzzy}(r,s)\text{-closed}\}.$

Lemma 2.4. (Lee and Kim (2005)). For an intuitionistic fuzzy set A in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and (r,s) ϵ $I \otimes I$,
(1) $\mathrm{int}(A,r,s)^c = \mathrm{cl}(A^c,r,s)$.
(2) $\mathrm{cl}(A,r,s)^c = \mathrm{int}(A^c,r,s)$.

Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be an intuitionistic fuzzy topological space in Sostak's sense. Then for each (r,s) ϵ $I \otimes I$, the family $\mathcal{T}_{(r,s)}$ defined by

$\mathcal{T}_{(r,s)} = \{A \in I(X) \mid \mathcal{T}_1(A) \geq r \text{ and } \mathcal{T}_2(A) \leq s\}$ is an intuitionistic fuzzy topology on X.

Let $(X,\mathcal{T})$ be an intuitionistic fuzzy topology space and (r,s) ϵ $I \otimes I$. Then the map $\mathcal{T}^{(r,s)}: I(X) \rightarrow I \otimes I$ defined by

$$\mathcal{T}^{(r,s)}(A) = \begin{cases} (1,0) & \text{if } A = \underline{0},\ \underline{1}, \\ (r,s) & \text{if } A \in T - \{\underline{0},\ \underline{1}\}, \\ (0,1) & \text{otherwise} \end{cases}$$

becomes an intuitionistic fuzzy topology in Sostak's sense on X.

Definition 2.5. (Lee and Kim (2009); Lee (2006); Lee and Kim (2007)) Let A be an intuitionistic fuzzy set in a SoIFTS $(X,\mathcal{T}_1,\mathcal{T}_2)$ and $(r,s) \in I \otimes I$. Then A is said to be:

(1) fuzzy (r,s)-regular open if A= int(cl(A,r,s),r,s),
(2) fuzzy (r,s)-regular closed if A=cl(int(A,r,s),r,s),
(3) fuzzy (r,s)-semiopen if A⊆ cl(int(A,r,s),r,s),
(4) fuzzy (r,s)-semiclosed if A⊇ int(cl(A,r,s),r,s),
(5) fuzzy (r,s)-preopen if A⊆ int(cl(A,r,s),r,s),
(6) fuzzy (r,s)-preclosed if A⊇ cl(int(A,r,s),r,s).

Remark 2.6. Let A be an intuitionistic fuzzy set in a SoIFTS $(X,\mathcal{T}_1,\mathcal{T}_2)$ and $(r,s) \in I \otimes I$. Then A is said to be**:**

(1) fuzzy (r,s)-clopen if it is fuzzy (r,s)-open and fuzzy (r,s)-closed set,
(2) fuzzy (r,s)-semi clopen if it is fuzzy (r,s)-semiopen and fuzzy (r,s)-semiclosed set,
(3) fuzzy (r,s)-pre clopen if it is fuzzy (r,s)-preopen and fuzzy (r,s)-preclosed set.

Remark 2.7. If an intuitionistic fuzzy set A in a SoIFTS $(X, \mathcal{T}, \mathcal{T}^*)$ is a fuzzy (r,s)-clopen set, then it is a fuzzy (r,s)-semi clopen and a fuzzy (r,s)-pre clopen set.

Definition 2.8. (Lee and Kim (2009); Lee (2006)) Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then f is called

(1) a fuzzy (r,s)-continuous mapping if $f^{-1}(B)$ is a fuzzy (r,s)-open set in X for each fuzzy (r,s)-open set B in Y.
(2) a fuzzy (r,s)-semi continuous mapping if $f^{-1}(B)$ is a fuzzy (r,s)-semiopen set in X for each fuzzy (r,s)-open set B in Y.

FUZZY(r,s)-TOTALLY CONTINUOUS AND FUZZY (r,s)-TOTALLY SEMI-CONTINUOUS MAPPINGS

Now, we introduce the concept of fuzzy (r,s)-totally continuous and fuzzy (r,s)-totally semi-continuous mappings, and investigate some of their characteristic properties.

Definition 3.1. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then f is called:

(1) a fuzzy (r,s)-totally continuous if f^{-1}(B) is fuzzy (r,s)-clopen in X for each fuzzy (r,s)-open set B in Y.

(2) a fuzzy (r,s)-totally semi (pre)-continuous mapping if f^{-1}(B) is fuzzy (r,s)-semi(pre)clopen, for each fuzzy (r,s)-open set B in Y.

Example 3.2. Let X = {x,y } and let A_1 and A_2 be intuitionistic fuzzy sets in X defined as

$$A_1(x) = A_1(y) = (0.5, 0.5),$$
$$A_2(x) = A_2(y) = (0.5, 0.5);$$

Define $\mathcal{T}$: I(X) →I ⊗ I and $\mathcal{U}$: I(X) →I ⊗ I by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ \left(\frac{1}{2}, \frac{1}{2}\right) & \text{if } A = A1, \\ (0,1) & otherwise. \end{cases}$$

and

$$\mathcal{U}(A) = (\mathcal{U}_1(A), \mathcal{U}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ \left(\frac{1}{2}, \frac{1}{2}\right) & \text{if } A = A2, \\ (0,1) & otherwise. \end{cases}$$

Then $\mathcal{T}$ and $\mathcal{U}$ are SoIFTs on X. Let f : (X, $\mathcal{T}$) → (X, $\mathcal{U}$) defined by

$$f(x) = x, \quad f(y) = y,$$

so

$$f^{-1}(\underline{0}) = \underline{0}, \quad f^{-1}(\underline{1}) = \underline{1}, \text{ and } f^{-1}(A_2) = A_2.$$

Since $\underline{0}$, $\underline{1}$ and A_2 are fuzzy ($\frac{1}{2}, \frac{1}{2}$)-semiopen (clopen) sets in (X, $\mathcal{T}$). so f is fuzzy($\frac{1}{2}, \frac{1}{2}$)-totally continuous mapping. Also f is fuzzy ($\frac{1}{2}, \frac{1}{2}$)-totally semi-continuous mapping.

Theorem 3.3. An intuitionistic fuzzy set A in a SoIFTS (X, $\mathcal{T}$, $\mathcal{T}^*$) is a fuzzy (r,s)-clopen if and only if it is a fuzzy (r,s)-semi clopen and a fuzzy (r,s)-pre clopen set, where A is an intuitionistic fuzzy set in X and (r,s) ϵ I⊗I.

Proof. Follow from the fact that each fuzzy (r,s)-clopen set is fuzzy (r,s)-semi clopen and fuzzy (r,s)-pre clopen set.

Conversely, let A be both a fuzzy (r,s)-semi clopen and a fuzzy (r,s)-pre clopen set, where A is an intuitionistic fuzzy set in X. Since A is a fuzzy (r,s)-semiclosed, then

$$\text{int}(\text{cl}(A, r, s), r, s) \subseteq A,$$

so that

$$\text{int}(\text{int}(\text{cl}(A,r,s), r, s), r, s) = \text{int}(\text{cl}(A, r, s), r,s) \subseteq \text{int}(A, r, s).$$

But A is a fuzzy (r,s)- pre open, then

$$A \subseteq \text{int}(\text{cl}(A, r, s),r,s) \subseteq \text{int}(A, r, s),$$

which implies that A is a fuzzy (r,s)-open. Also A is a fuzzy (r,s)-semiopen, then A is a fuzzy

(r,s)-open set in X, $A \subseteq cl(int(A, r, s), r, s)$, but A is a fuzzy (r,s)-pre closed, then

$$cl(A, r, s) \subseteq cl(cl(int(A, r, s), r, s), r, s) = cl(int(A, r, s), r, s) \subseteq A,$$

and so A is a fuzzy (r,s)-closed set, where $A \in X$. Therefore A is a fuzzy (r,s)-clopen set.

Theorem 3.4. Let $f : (X, \mathcal{T}, \mathcal{T}^*) \rightarrow (Y, \mathcal{U}, \mathcal{U}^*)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then the following statements are equivalent:

(1) f is a fuzzy (r,s)-totally (totally semi)-continuous,
(2) $f^{-1}(B)$ is a fuzzy (r,s)-clopen (semi clopen) set of X for each intuitionistic fuzzy set B^c in Y,
(3) $cl(f^{-1}(B), r, s)) \subseteq f^{-1}(cl(B, r, s))$ and $f^{-1}(B) \subseteq int(f^{-1}(cl(B, r, s), r, s)$ for each intuitionistic fuzzy set B in Y,
(4) $f^{-1}(int(B, r, s)) \subseteq int(f^{-1}(B), r, s)$ and $cl(f^{-1}(int(B, r, s)), r, s) \subseteq f^{-1}(B)$ for each $B \in Y$.

Proof. (1) $\Rightarrow$ (2) : Let $B \in Y$. Then from (1), $f^{-1}(int(B, r, s))$ is fuzzy(r,s)-clopen (semi clopen). But $f^{-1}(B^c) \in X$ i.e., $(f^{-1}(B))^c \in X$, we have $f^{-1}(B)$ is fuzzy (r,s)-closed (semiclosed) in X. From $f^{-1}(B^c)$ is fuzzy (r,s)-closed (semiclosed) in X so, then $f^{-1}(B) \in X$. Thus, $f^{-1}(B)$ is fuzzy (r,s)-clopen (semi clopen).

(2) $\Rightarrow$ (3) : Let B be a fuzzy (r,s) in Y. Then cl(B,r,s) is fuzzy (r,s)-closed (semiclosed) in Y. By (2), $f^{-1}(cl(B,r,s))$ is fuzzy (r,s)-closed (semiclosed) in X.
Hence,

$$cl(f^{-1}(B), r, s)) \subseteq cl(f^{-1}(cl(B, r, s)), r, s) = f^{-1}(cl(B, r, s)).$$

again by (2), $f^{-1}(cl(B, r, s)) \in X$.
Hence,

$$f^{-1}(B) \subseteq f^{-1}(cl(B, r, s)) = int(f^{-1}(cl(B, r, s)), r, s).$$

(3) $\Rightarrow$ (4) : Let B be a fuzzy (r,s)-open set in Y. By (3),we have

$$f^{-1}(cl(B^c, r, s) \supseteq cl(f^{-1}(B^c)), r, s) = cl((f^{-1}(B))^c, r, s).$$

Hence,

$$f^{-1}(int(B, r, s)) = f^{-1}(cl(B^c, r, s))^c = (f^{-1}(cl(B^c, r, s)))$$
$$\subseteq (cl(f^{-1}(B))^c, r, s))^c = int(f^{-1}(B), r, s).$$

By hypothesis of (3), we have

$$f^{-1}(B^c, r, s) \subseteq int(f^{-1}(cl(B^c, r, s)), r, s)$$
$$= int(f^{-1}(int(B, r, s)^c), r, s)$$
$$= int(f^{-1}(int(B, r, s))^c, r, s).$$

Hence,

$$f^{-1}(B) = (f^{-1}(B^c))^c \supseteq (int(f^{-1}(int(B, r, s)), r, s)^c, r, s)^c$$
$$= cl(f^{-1}(int(B, r, s)), r, s).$$

(4) $\Rightarrow$(1) : Let $B \in Y$. Then $B = int(B,r,s)$. By first formula of (4), we have

$$f^{-1}(B) = f^{-1}(int(B, r, s)) \subseteq int(f^{-1}(B), r, s).$$

Hence,

$$f^{-1}(B) = f^{-1}(\text{int}\,(B, r, s)), \text{ i.e., } f^{-1}(\text{ int}\,(B, r, s)) \in X.$$

By second formula of (4), we have

$$f^{-1}(B) \supseteq \text{cl}(f^{-1}(\text{ int}\,(B, r, s))), r, s) = \text{cl}\,(f^{-1}(B), r, s),$$

Hence,

$$f^{-1}(B)=f^{-1}(\text{int}\,(B, r, s)),$$

i.e., $f^{-1}(B)$ is a fuzzy(r,s)-clopen (semi clopen) in X . Thus, f is fuzzy (r,s)-totally (totally semi) continuous.

Theorem 3.5. let $(X, \mathcal{T}, \mathcal{T}^*)$ and $(Y, \mathcal{U}, \mathcal{U}^*)$ be a SoIFTS X and Y respectively. A mapping f: $(X, \mathcal{T}, \mathcal{T}^*) \to (Y, \mathcal{U}, \mathcal{U}^*)$ is an intuitionistic fuzzy (r,s)-totally continuous if and only if f is a fuzzy (r,s)-totally semi-continuous and a fuzzy (r,s)-totally pre-continuous.

Proof: let B be a fuzzy (r,s)-open set in $(Y, \mathcal{U}, \mathcal{U}^*)$, then $f^{-1}(B)$ is fuzzy (r,s)-semi clopen set. Since f is an intuitionistic fuzzy (r,s)-totally semi-continuous and $f^{-1}(B)$ is fuzzy (r,s)-pre clopen set because f is an intuitionistic fuzzy (r,s)-totally pre-continuous. from Remark 2.6, $f^{-1}(B)$ is fuzzy (r,s)-clopen set in $(X, \mathcal{T}, \mathcal{T}^*)$. Therefore f is an intuitionistic fuzzy (r,s)-totally continuous.

The completion of the proof is straightforward .

Theorem 3.6. (1) Every intuitionistic fuzzy (r,s)-totally continuous mapping is intuitionistic fuzzy (r,s)-totally semi continuous mapping.
(2) Every intuitionistic fuzzy (r,s)-totally semi continuous mapping is intuitionistic fuzzy (r,s)-semi continuous mapping.

Proof . (1)- let f: $(X, \mathcal{T}, \mathcal{T}^*) \to (Y, \mathcal{U}, \mathcal{U}^*)$ is an intuitionistic fuzzy (r,s)-totally continuous and B be a fuzzy (r,s) in Y. By hypothesis $f^{-1}(B)$ is an intuitionistic fuzzy (r,s)-semi clopen set in X. Since every fuzzy (r,s)-open set and every fuzzy (r,s)-closed set is fuzzy (r,s)-semi clopen set, so f is a fuzzy (r,s)-totally semi continuous mapping.

(2)- let B be a fuzzy (r,s)-open set in $(Y, \mathcal{U}, \mathcal{U}^*)$ By the hypothesis $f^{-1}(B)$ is fuzzy (r,s)-semi open set and fuzzy (r,s)-semiclosed set in X. Hence f is a fuzzy (r,s)-semi-continuous mapping.

However, the following examples show that the converse need not be true.

Example 3.7. (1)- Let X = {x,y,z} and let A_1 and A_2 be intuitionistic fuzzy sets in X defined as

$$A_1(x) = (0.1, 0.3),\ A_1(y) = (0.2, 0.7),\ A_1(z) = (0.1, 0.5);$$

and

$$A_2(x) = (0.1,0.3),\ A_2(y) = (0.3, 0.7),\ A_2(z) = (0.1,0.5).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A1, \\ (0,1) & otherwise. \end{cases}$$

and

$$\mathcal{U}(A) = (\mathcal{U}_1(A), \mathcal{U}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A2, \\ (0,1) & otherwise. \end{cases}$$

Then $\mathcal{T}$ and $\mathcal{U}$ are SoIFTs on X. Let f : (X, $\mathcal{T}$) → (X, $\mathcal{U}$) defined by

$$f(x) = x,\ f(y) = y \text{ and } f(z) = z,$$
$$f^{-1}(\underline{0}) = \underline{0},\ f^{-1}(\underline{1}) = \underline{1}, \text{ and } f^{-1}(A_2) = A_2.$$

So $\underline{0}$, $\underline{1}$ and A_2 are fuzzy ($\frac{1}{2}, \frac{1}{3}$)-semi clopen sets in (X, $\mathfrak{T}$). But f is not fuzzy (r,s)-totally continuous mapping because $f^{-1}(A_2) = A_2$ is not fuzzy ($\frac{1}{2}, \frac{1}{3}$)-clopen in X.

(2)- See(1), and defined A_1 as follows:

$$A_1(x) = A_2(x) = (0.1, 0.3),$$
$$A_1(y) = A_2(y) = (0.3, 0.7),$$
$$A_1(z) = A_2(z) = (0.1, 0.5),$$

so

$$f^{-1}(\underline{0}) = \underline{0},\ f^{-1}(\underline{1}) = \underline{1}, \text{ and } f^{-1}(A_2) = A_2.$$

Since $\underline{0}$, $\underline{1}$ and A_2 are fuzzy ($\frac{1}{2}, \frac{1}{3}$)-semi open sets, but not fuzzy ($\frac{1}{2}, \frac{1}{3}$)-semi clopen set in (X,$\mathcal{T}$). then f is fuzzy(r,s)-semi continuous mapping, but not fuzzy (r,s)-totally semi-continuous mapping.

FUZZY(r,s)-SEMI TOTALLY-CONTINUOUS MAPPINGS

Definition 4.1. Let f : (X, $\mathcal{T}$, $\mathcal{T}^*$) → (Y, $\mathcal{U}$, $\mathcal{U}^*$) be a mapping from a SoIFTS X to a SoIFTS Y and (r, s) ϵ I ⊗ I, then f is called a fuzzy (r,s)-semi totally-continuous mapping if f^{-1} (B) is fuzzy (r,s)-clopen, for each fuzzy (r,s)-semi open B in Y.

Theorem 4.2. Let f : (X, $\mathcal{T}$, $\mathcal{T}^*$) → (Y, $\mathcal{U}$, $\mathcal{U}^*$) be a mapping from a SoIFTS X to a SoIFTS Y and (r,s) ϵ I ⊗ I. Then the following statements are equivalent:

(1) f is a fuzzy (r,s)-semi totally-continuous,

(2) f^{-1}(B) is a fuzzy (r,s)-clopen set of X for each intuitionistic fuzzy (r,s)-semiclosed set B in Y.

Proof. (1) ⟹ (2) : Let B be any intuitionistic fuzzy (r,s)-semiclosed set in Y. Then Y−B is fuzzy (r,s)-semiopen set in Y. By definition f^{-1}(Y−B) is fuzzy (r,s)-clopen in X. That is X-$f^{-1}(B)$ is fuzzy (r,s)-clopen in X. This implies f^{-1} (B) is fuzzy (r,s)-clopen in X.

(2) ⇒ (1) : Let A is fuzzy (r,s)-semiopen in Y, then Y−A is fuzzy (r,s)-semiclosed in Y, but f^{-1}(Y –A) =X $-f^{-1}$ (A) is fuzzy (r,s)-clopen in X, which implies f^{-1}(A) is fuzzy (r,s)-clopen in X. Thus, inverse image of every fuzzy (r,s)-semiopen set in Y is fuzzy (r,s)-clopen in X. Therefore f is fuzzy(r,s)-semi totally continuous mapping.

Proposition 4.3. Every fuzzy (r,s)-semi totally-continuous mapping is a fuzzy (r,s)-totally (totally semi)-continuous mapping.

Proof. Suppose $f : (X, \mathcal{T}, \mathcal{T}^*) \rightarrow (Y, \mathcal{U}, \mathcal{U}^*)$ is a fuzzy (r,s)-semi totally-continuous mapping and B ϵ Y. Since B ϵ Y, so B is fuzzy (r,s)-semiopen in Y and f is a fuzzy (r,s)-semi totally continuous, it follows f^{-1}(B) is fuzzy (r,s)-clopen (and hence a fuzzy (r,s)-semi clopen) in X. Thus inverse image of each B ϵY which is a fuzzy (r,s)-semiopen (semi clopen) set in Y is fuzzy (r,s)-clopen (semi clopen) in X. Therefore f is a fuzzy (r,s)-totally (totally semi)-continuous mapping.

The converse of the proposition need not to be true in general as shown by the following examples.

Example 4.4. (1) See the Example 3.7. (1) and let

$$A_2(x) = (0.3, 0.1),\ A_2(y) = (0.7, 0.2),\ A_2(z) = (0.5, 0.1),$$

and

$$A_3(x) = (0.1, 0.3),\ A_3(y) = (0.3, 0.7),\ A_3(z) = (0.1,0.5).$$

So $f^{-1}(\underline{0}) = \underline{0}$, $f^{-1}(\underline{1}) = \underline{1}$, and $f^{-1}(A_2) = A_2$ are fuzzy $(\frac{1}{2},\frac{1}{3})$-clopen sets, but A_3 is fuzzy $(\frac{1}{2},\frac{1}{3})$-semiopen set in Y and $f^{-1}(A_3) = A_3$ is not a fuzzy $(\frac{1}{2},\frac{1}{3})$-clopen set in X. Thus f *is* a fuzzy $(\frac{1}{2},\frac{1}{3})$-totally-continuous mapping, but not a fuzzy $(\frac{1}{2},\frac{1}{3})$-semi totally-continuous mapping.

(2) Let X = {x,y,z} , A,B and C be intuitionistic fuzzy sets in X defined as

$$A(x) = (0.3, 0.6),\ A(y) = (0.1, 0.8),\ A(z) = (0.7, 0.2);$$
$$B(x) = (0.8, 0.1),\ B(y) = (0.8, 0.1),\ B(z) = (0.4,0.5);$$

and

$$C(x) = (0.7, 0.4),\ C(y) = (0.7, 0.2),\ C(z) = (0.6,0.4),$$

Define $\mathcal{T} : I(X) \rightarrow I \otimes I$ and $\mathcal{U} : I(X) \rightarrow I \otimes I$ by

$$\mathcal{T}(D) = \begin{cases} (1,0) & \text{if } D = \underline{0}, \underline{1}, \\ \left(\frac{1}{2},\frac{1}{3}\right) & \text{if } D = A, B, A\cap B, A\cup B, \\ (0,1) & otherwise; \end{cases}$$

and

$$\mathcal{U}(D) = \begin{cases} (1,0) & \text{if } D = \underline{0}, \underline{1}, \\ \left(\frac{1}{2},\frac{1}{3}\right) & \text{if } D = C, \\ (0,1) & otherwise; \end{cases}$$

Then $\mathcal{J}$ and $\mathcal{U}$ are SoIFTs on X. Let f : (X, $\mathcal{T}$) → (X, $\mathcal{U}$) defined by

$$f(x) = x,\ f(y) = y \text{ and } f(z) = z,$$

so

$$f^{-1}(\underline{0}) = \underline{0},\ f^{-1}(\underline{1}) = \underline{1}, \text{ so } f^{-1}(C) = C.$$

Since $\underline{0}$, $\underline{1}$ and C are fuzzy ($\frac{1}{2}, \frac{1}{3}$)-semi clopen sets in (X, $\mathcal{J}$), so f is a fuzzy($\frac{1}{2}, \frac{1}{3}$)-totally semi-continuous mapping. But f is not fuzzy ($\frac{1}{2}, \frac{1}{3}$)-semi totally-continuous mapping because f^{-1}(C) = C is not fuzzy ($\frac{1}{2}, \frac{1}{3}$)-clopen in X.

Theorem 4.5. Every fuzzy (r,s)-semi totally-continuous mapping is fuzzy (r,s)-semi continuous mapping.

Proof. Suppose f :(X, $\mathcal{T}$, $\mathcal{T}$*) → (Y, $\mathcal{U}$,$\mathcal{U}$*) is a fuzzy (r,s)-semi totally-continuous mapping and B ϵ Y. Since f is a fuzzy (r,s)-semi-continuous mapping, f^{-1}(B) is a fuzzy (r,s)-clopen and hence a fuzzy (r,s)-semi clopen in X. This implies f^{-1}(B) is a fuzzy (r,s)-semiopen in X. Therefore f is a fuzzy (r,s)-semi-totally continuous mapping.

The converse of the above need not to be true as shown by the following example.

Example 4.6. Let (X, $\mathcal{T}$) be the SoIFTS as described in Example 4.4. (2) and C be an intuitionistic fuzzy set defined as

$$C(x) = (0.8, 0.1),\ C(y) = (0.7, 0.2),\ C(z) = (0.6, 0.4).$$

Since $\underline{0}$, $\underline{1}$ and C are fuzzy ($\frac{1}{2}, \frac{1}{3}$)-semiopen sets in X, and it is easy to see that C is a fuzzy ($\frac{1}{2}, \frac{1}{3}$) -semiopen, so f is a fuzzy ($\frac{1}{2}, \frac{1}{3}$)-semi-continuous mapping. But it is not fuzzy ($\frac{1}{2}, \frac{1}{3}$)-semi totally-continuous mapping because f^{-1}(C) = C is not fuzzy ($\frac{1}{2}, \frac{1}{3}$)-clopen in X.

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