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Flow and Heat Transfer over a Stretching Cylinder

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ABSTRACT

A mathematical model is analyzed to study the effect of stretching cylinder in a boundary layer flow and heat transfer in an incompressible viscous fluid of two-dimensional steady flow. The governing nonlinear partial differential equations are reduced to nonlinear ordinary differential equations using a similarity transformation. The resulting system of equation solved numerically using a shooting method in Maple 14 software. The results indicate that the skin friction coefficient and the Nusselt number increase as curvature parameter γ increases.

Keywords: Boundary layer, Heat transfer, Stretching cylinder, Similarity solution

INTRODUCTION

The study of hydrodynamic flow and heat transfer over a stretching cylinders has gained considerable attention due to its applications in industries and important bearings on several technologies processes. Viscoelastic boundary layer flow and heat transfer over an exponentially stretching continuous sheet were examined by Khan et al. (2005). Then, Fang (2008) proposed the boundary layers over a continuously shrinking sheet with a power-law surface velocity and mass transfer. Besides that, Yao et al. (2011) analyzed the heat transfer of a generalized stretching/shrinking wall problem with convective boundary conditions. The effect of uniform suction/blowing on flow and heat transfer due to a stretching cylinder was studied by Ishak et al. (2008). The boundary layer flow due to a vertical cylinder in a quiescent viscous and incompressible fluid has been considered in the papers by Ishak (2009) and Bachok and Ishak (2009). Lin and Shih (1981) studied the laminar boundary layer and heat transfer along moving cylinders. The effect of heat transfer on the steady laminar compressible boundary layer flow past a horizontal circular cylinder was studied by Hossain et al. (1998). Then, Bachok and Ishak (2010) investigated the steady laminar flow caused by a stretching cylinder immersed in an incompressible viscous fluid with prescribed surface heat flux. Besides that, Mukhopadhyay (2012) analyzed axi-symmetric laminar boundary layer flow of a viscous incompressible fluid and heat transfer towards a stretching cylinder embedded in a porous medium. In the same year, the problem of boundary layer flow past a stretching cylinder and heat transfer with variable thermal conductivity was analyzed by Rangi et al. (2012).

PROBLEM FORMULATIONS

Consider a steady-state flow and make the standard boundary layer approximations, based on the governing equations given by Bhattacharyya (2011):

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial r}(rv)=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=\nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\alpha\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right) \tag{3}$$

where x and r are coordinates measured along the surface of the cylinder and in the radial direction, respectively, with u and v being corresponding velocity components. The boundary conditions of Equations. $(1)-(3)$ are taken to be

$$u=u_w(x),\quad v=0,\quad T=T_w \quad \text{at} \quad r=R$$

$$u\to 0,\quad T\to T_\infty \qquad \text{as} \quad r\to\infty \tag{4}$$

where T is the temperature in the boundary layer, ν is the kinematic viscosity coefficient and α is the thermal diffusivity. By looking for similarity solutions of equations (1)-(3), subject to the boundary conditions (4), by writing

$$\eta=\frac{r^2-R^2}{2R}\left(\frac{c}{\nu L}\right)^{1/2},\quad \psi=\left(\frac{\nu c}{L}\right)^{1/2} x\,R\,f(\eta),\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{5}$$

where η is the similarity variable, ψ is the stream function defined as $u=r^{-1}\,\partial\psi/\partial y$ and $v=-r^{-1}\,\partial\psi/\partial x$, which identically satisfies equation (1). By defining η in this form, the boundary conditions at $r=R$ reduce to the boundary conditions at $\eta=0$, which is more convenient for numerical computations.

Substituting (5) into equations (2) and (3), we obtain the following nonlinear ordinary differential equations:

$$\left(1+2\gamma\eta\right)f'''+2\gamma f''+ff''-f'^2=0 \tag{6}$$

$$\left(1+2\gamma\eta\right)\theta''+2\gamma\theta'+\Pr f\theta'=0 \tag{7}$$

subjected to the boundary conditions (4) which become

$$\begin{aligned} &f(0)=0, \quad f'(0)=1, \quad \theta(0)=1, \\ &f'(\infty)\to 0, \quad \theta(\infty)\to 0, \end{aligned} \tag{8}$$

where $\Pr = \nu/\alpha$ is the Prandtl number and γ is the curvature parameter defined as

$$\gamma = \left(\frac{\nu L}{cR^2}\right)^{1/2} . \tag{9}$$

RESULTS AND DISCUSSION

Numerical solutions to the equations (6) and (7) with boundary conditions (8) were obtained by using a shooting method. In this method, the solutions are obtained by setting different initial guesses for the value of $f''(0)$ and $-\theta'(0)$, where all profiles satisfy the far field boundary conditions (8) asymptotically but different shapes. The problem for a regular (Newtonian) fluid involves two parameters, namely Prandtl number and the curvature parameter. According to its definition, the Prandtl number is equal to the ratio of two quantities; viscosity and thermal diffusivity. It is clear that the dimensionless heat transfer rates increase with the increase in Prandtl number as shown in Table 1. The dimensionless heat transfer rates increase with the increase in Prandtl number as shown in Figure 1. From Figure 2, the heat transfer rate at fixed values of γ increases with the Prandtl number. The horizontal velocity curves in Figure 3 show that the rate of transport decreases with the increasing distance (η) of the sheet. The effect of the curvature parameter γ on the temperature distribution are presented in Figure 4. The effect of Prandtl number Pr on the dimensionless temperature profiles are presented on Figure 5 and Figure 6 with the different values of γ.

Table 1. Values of $f''(0)$ and $-\theta'(0)$ for various values of γ when $Pr = 0.7$, $Pr = 1.0$ and $Pr = 7.0$.

γ / Pr		$Pr = 0.7$	$Pr = 1.0$	$Pr = 7.0$
	$f''(0)$	$-\theta'(0)$	$-\theta'(0)$	$-\theta'(0)$
0.0	-1.0000	0.4544	0.5820	1.8954
0.1	-1.0370	0.4890	0.6122	1.9281
0.2	-1.0741	0.5290	0.6458	1.9605
0.3	-1.1112	0.5708	0.6818	1.9927
0.4	-1.1480	0.6131	0.7190	2.0246
0.5	-1.1846	0.6553	0.7569	2.0563

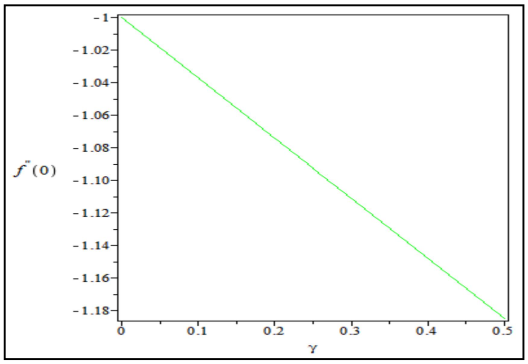

Figure 1: Skin-friction coefficient $f''(0)$ for a various values of γ.

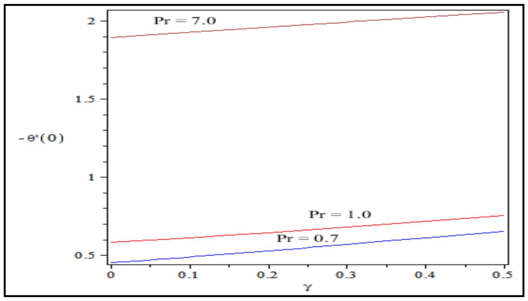

Figure 2: Temperature gradient $-\theta'(0)$ for a various values of γ.

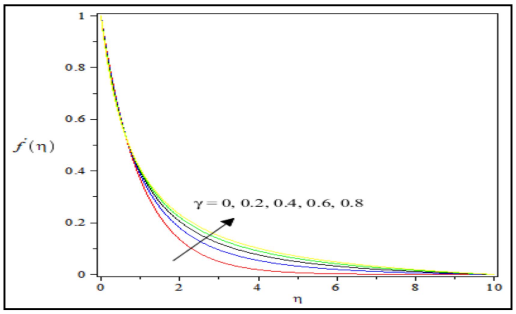

Figure 3: Velocity profiles $f'(\eta)$ with η for various values of γ.

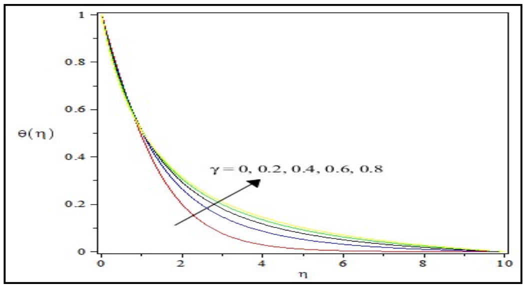

Figure 4: Temperature profiles $\theta(\eta)$ with η for several values of γ.

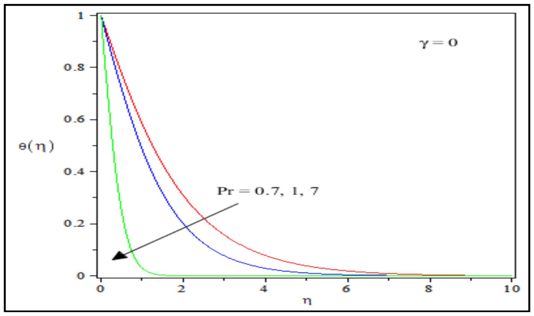

Figure 5: Temperature profiles $\theta(\eta)$ with η for various values of Pr when $\gamma = 0$.

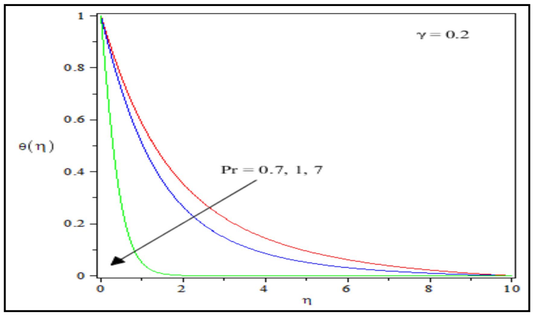

Figure 6: Temperature profiles $\theta(\eta)$ with η for various values of Pr when $\gamma = 0.2$.

CONCLUSIONS

We have theoretically studied the boundary layer flow and heat transfer over a stretching cylinder. The governing boundary layer equations were solved numerically using a shooting technique. The conclusion can be drawn from the study are; The dimensionless heat transfer rates decrease with decrease in Pr number, the thermal boundary layer thickness becomes thinner due to the increasing Prandtl number, the thermal boundary layer thickness decreases as increasing γ which implies increasing in the wall temperature gradient and in turn increase the surface heat transfer rate.

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