Chemically Reacting Radiative Heat and Mass Transfer with Heat Generation Over an Inclined Plate at Prescribed Heat Flux

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ABSTRACT

A steady laminar flow of viscous electrically conducting incompressible fluid, over a semi-infinite inclined plate, which is at prescribed heat flux with radiation, heat generation and chemical reaction is presented in the manuscript. The numerical solutions using an implicit finite difference scheme known as Keller-box method for velocity, concentration and temperature are found. The effects of various parameters like Schmidt number, chemical reaction, thermal Grashof number, solutal Grashof number, radiation parameter, angle of inclination etc. on the velocity, temperature and concentration are presented graphically.

Keywords: Chemical reaction, Heat Transfer, Mass Transfer, Heat flux, Concentration, Inclined wall.

INTRODUCTION

Convection flow driven by temperature and concentration differences has been the objective of extensive research because such processes exist in nature and has engineering applications. The process occurring in nature includes photo-synthetic mechanism, calm-day evaporation and vaporization of mist and fog, while the engineering application includes the chemical reaction in a reactor chamber consisting of rectangular ducts, chemical vapor deposition on surfaces and cooling of electronic equipment. Also, the study of natural convection flow for an incompressible viscous fluid past a heated surface has important applications such as cooling of nuclear reactors, the boundary layer control in aerodynamics, crystal growth and food processing and cooling towers.

It is well known that natural convection heat transfer occurs as a result of temperature differences in an enclosure or near a heated or cooled flat plate. Natural convection along an inclined plate has received less attention than the cases of vertical and horizontal plates. However, this configuration is frequently encountered in engineering devices and in the natural environment. A number of researchers have considered an inclined, semi-infinite flat plate in their research because of its engineering applications. The study of natural convection flow of inclined plate is presented by the authors (Ganesan and Palani, 2003, 2004; Sparrow and Husar, 1969). Chen (2004) presented the analysis to study natural convection flow over a permeable inclined surface with variable wall temperature and concentration. Hossain et al. (1996) studied the free convection flow from an isothermal plate inclined at a small angle to the horizontal. Anghel et al. (2001) presented a numerical solution of free convection flow past an inclined surface. Bhuvaneswari et al. (2010) studied exact analysis of radiation convective flow heat and mass transfer over an inclined plate in a porous medium. Sivasankaran (2006) presented a Lie group analysis of natural convection heat and mass transfer in an inclined surface. Shit and Haldar (2011) studied MHD flow, heat and mass transfer over an inclined permeable stretching sheet with thermal radiation and Hall Effect.

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years.

In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Natural convection processes involving the combined mechanisms are also encountered in many natural processes, such as evaporation, condensation and agricultural drying, and in many industrial applications, such as the curing of plastics, cleaning and chemical processing of materials relevant to the manufacture of printed circuitry, manufacture of pulp-insulated cables, etc. Diffusion rates can be altered tremendously by chemical reactions. The Effect of a chemical reaction depends whether the reaction is homogeneous or heterogeneous. This depends on whether they occur in an interface or as a single phase volume reaction. In a well-mixed system, the reaction is heterogeneous if the reactants are in multiple phase, and homogeneous if the reactants are in the same phase. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. Kandasamy et al. (2009) studied thermophoresis and variable viscosity effects on MHD mixed convective heat and mass transfer past a porous wedge in the presence of chemical reaction. Kandasamy and Devi (2004) studied the effects of chemical reaction, heat and mass transfer on non-linear laminar boundary-layer flow over a wedge with suction or injection.

The study of the heat generation or absorption in moving fluids is important while dealing with chemical reactions and those concerned with dissociating fluids. Specifically, the effects of heat generation may alter the temperature distribution, consequently affecting the particle deposition rate in nuclear reactors, electronic chips and semiconductor wafers. In fact, the literature is replete with examples of heat transfer in the laminar flow of viscous fluids. For instance, Vajravelu and Hadjinicolaou (1993) studied heat transfer characteristics in the laminar boundary-layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. Kumar (2011) investigated heat transfer over a stretching porous sheet subjected to power law heat flux in the presence of a heat source.

Moreover, when the temperature of surrounding fluid is high, the radiation effects play an important role that cannot be ignored (Modest, 2003; Siegel and Howell, 1992). The effects of radiation on temperature have become more important industrialized. Many processes in engineering areas occur at high temperature and acknowledge radiation heat transfer become very important for the design of pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. In such cases one has to take into account the effects of radiation and free convection. For an impulsively started infinite vertical isothermal plate, Ganesan, Loganathan and Soundalgekar (2001) studied the effects of radiation and free convection, by using Rosseland approximation, (Brewster, 1992). Problem of radiative heat transfer with hydromagnetic flow and viscous dissipation over a stretching surface in the presence of variable heat flux is solved analytically by Kumar (2009).

In the present work, radiative heat and mass transfer over an inclined plate in the presence of chemical reaction is studied when the wall is at prescribed heat flux. The boundary layer equations are solved numerically using an implicit finite-difference scheme which is the Keller-box method (Cebeci and Bradshaw, 1977, 1988). The effects of Schmidt number, Chemical reaction, Prandtl Number, Heat generation parameter, Radiation parameter, Magnetic field, Thermal Grashof number, Solutal Grashof number and angle of inclination is studied on velocity, concentration and temperature field.

MATHEMATICAL ANALYSIS

Consider a steady laminar flow of an incompressible viscous electrically conducting fluid past a semi-infinite inclined wall with an acute angle ϕ from the vertical in the presence of chemical reaction and radiation. The wall is at prescribed heat flux, and at a constant concentration gradient. The flow is assumed to be in the *x*-direction, which is taken along the semi-infinite inclined plate and *y*-axis normal to it. Since the motion is two dimensional and length of the plate is large, therefore all physical quantities are assumed to be independent of *x*. A magnetic field of uniform strength B_0 is introduced normal to the direction of the flow. In the analysis, we assume that the magnetic Reynolds number is much less than unity so that the induced magnetic field is neglected in comparison to the applied magnetic field. It is also assumed that all fluid properties are constant. Then, under the usual Boussinesq's and boundary layer approximations, the governing equations of the mass, momentum, energy and concentration for the steady flow can be written as,
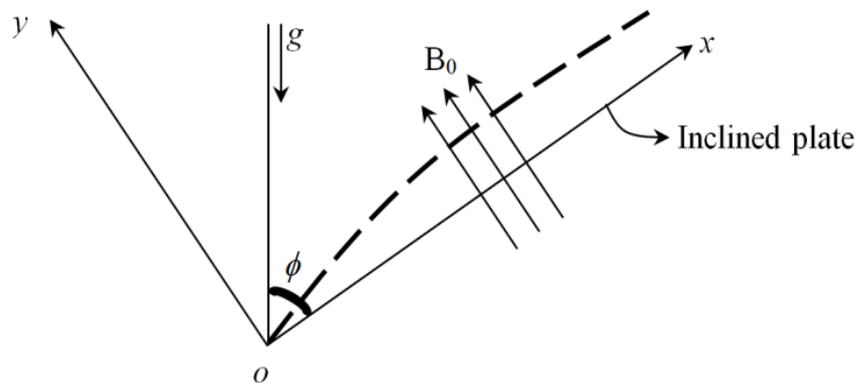

Fig.1: Physical Model and Coordinate System

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$v\frac{\partial u}{\partial y} = \vartheta\frac{\partial^2 u}{\partial y^2} + g\,\beta_T\,(T - T_\infty)\cos\phi + g\,\beta_C\,(c - c_\infty)\cos\phi - \frac{\sigma B_0^2 u}{\rho} \tag{2}$$

$$v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{Q}{\rho c_p}\left(T - T_\infty\right) \tag{3}$$

$$v\frac{\partial c}{\partial y} = D\frac{\partial^2 c}{\partial y^2} - K_l\left(c - c_\infty\right) \tag{4}$$

with boundary conditions

$$u = 0,\ v = -v_w,\ \frac{\partial T}{\partial y} = -\frac{q}{k},\ -D\frac{\partial c}{\partial y} = m_w,\ \text{at}\ y = 0$$

$$u \to 0,\ T \to T_\infty,\ c \to c_\infty,\ \text{as}\ y \to \infty \tag{5}$$

The equation of continuity (1) with boundary condition (5) changes to:

$$v = -v_w \tag{6}$$

We assume the Rosseland approximation (Brewster, 1992)for radiative heat flux, which leads to

$$q_r = -\frac{4\sigma'}{3\kappa^*}\frac{\partial T^4}{\partial y},$$

If the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature, then the Taylor series for T^4 about T_∞, after ignoring higher order terms, is given by

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 .$$

On assuming the non-dimensional variables as follows:

$$Y = \frac{y v_w}{\vartheta} \;,\quad U = \frac{u}{u_w} \;,\quad \theta = \frac{T-T_\infty}{q\vartheta/k v_w} \;,\quad C = \frac{c-c_\infty}{(m_w\vartheta/v_w D)} \;,\quad Gr_T = \frac{g\beta_T\, q\vartheta^2}{k\, u_w v_w^3} \;,\quad Gr_c = \frac{g\beta_c\, m_w\, \vartheta^2}{u_w v_w^3\, D} \;,$$

$$M^2 = \frac{B_0^2\vartheta\sigma}{v_w^2\rho},\; \Pr = \frac{\rho\vartheta c_p}{k} = \frac{\vartheta}{\alpha},\; N = \frac{\kappa^* k}{4\sigma' T_\infty^3},\; S = \frac{Q\vartheta}{\rho c_p v_w^2},\; Sc = \frac{\vartheta}{D},\; K_c = \frac{\vartheta K_l}{v_w^2}.$$

On using the above non-dimensional parameters, the equations (2) to (4) reduces to:

$$\frac{d^2U}{dY^2} + \frac{dU}{dY} + Gr_T\,\theta\cos\phi + Gr_c\,C\cos\phi - M^2 U = 0 \tag{7}$$

$$\left(1+\frac{4}{3N}\right)\frac{d^2\theta}{dY^2} + \Pr\frac{d\theta}{dY} + \Pr S\,\theta = 0 \tag{8}$$

$$\frac{d^2C}{dY^2} + Sc\frac{dC}{dY} - K_c\,Sc\,C = 0 \tag{9}$$

With the corresponding boundary conditions

$$U = 0,\; \frac{\partial\theta}{\partial Y} = -1,\; \frac{\partial C}{\partial Y} = -1,\;\; \text{at}\;\; Y = 0$$

$$U \to 0,\;\; \theta \to 0,\;\; C \to 0,\;\; \text{as}\;\; Y \to \infty \tag{10}$$

Wall shear stress is defined as

$$\tau_f = \mu\left(\frac{du}{dy}\right)_{y=0}$$

and the skin friction coefficient is defined as

$$C_f = \frac{\tau_f}{\rho\, u_w\, v_w} = \left(\frac{dU}{dY}\right)_{Y=0}$$

The recovery factor can be written as

$$R_f = \theta(0)$$

The wall mass transfer rate is

$$J_w = -D\left(\frac{\partial C}{\partial Y}\right)_{Y=0}$$

The dimensionless wall concentration $= C(0)$.

RESULTS AND DISCUSSION

Eqs. (7), (8) and (9) subject to the boundary conditions (10) are solved numerically using the Keller-box method as described by Cebeci and Bradshaw (1977, 1988). A study of velocity, temperature, concentration, skin friction, recovery factor and Sherwood number of the steady laminar flow of an incompressible viscous electrically conducting fluid past a semi-infinite inclined wall, has been carried out in the present research. The results are obtained in the cases (i) $Gr_T > 0$ (cooling of the plate) and (ii) $Gr_T < 0$ (heating of the plate).

In the case of $Gr_T > 0$, fig.2 is plotted for velocity with the variations of Sc and K_c, and the effects of these variables on concentration are presented in fig.3. The obtained results are compared with the results of Kandasamy and Devi (2004) and found in good agreement. Velocity and concentration decreases as Sc increases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reductions in the velocity and concentration profiles are accompanied by simultaneous reductions in the momentum and concentration boundary layer thickness. Physically, the increase of Sc means decrease of molecular diffusivity (D). That results in decrease of concentration boundary layer. Hence, the concentration of the species is higher for small values of Sc and lower for larger values of Sc. It is seen from the figures that the velocity and concentration of the fluid decreases with increase of chemical reaction. It is considered to be a homogeneous first-order chemical reaction. The diffusing species either can be destroyed or generated in the homogeneous reaction. The chemical reaction parameter can be adjusted to meet these circumstances if one takes $K_c > 0$ for a destructive reaction, $K_c < 0$ for a generative reaction and $K_c = 0$ for no reaction. The destructive chemical reaction is assumed here. Consequently, the concentration falls off for the increments of the chemical reaction parameter. This shows that the diffusion rates can be tremendously altered by the chemical reaction parameter.

Figure-4 shows the effects of S, M and ϕ on the velocity in the case of $Gr_T > 0$. The velocity increases with an increase in S. It is observed that when heat is generated the buoyancy force increases, which induces the flow rate to increase, giving rise to the increase in the velocity profiles. It is seen that the velocity decreases with the increase of M. The application of a transverse magnetic field of an electrically conducting fluid is to raise a resistive type force called Lorentz force. This force has the tendency to slow down the motion of the fluid. The velocity decreases as the angle of inclination (ϕ) increases. The fluid has higher velocity when the surface is vertical than, when inclined because of the fact that the buoyancy effect decreases due to gravity components (g cos ϕ) as the plate is inclined.

The velocity in the variations in Gr_T and Gr_c in the case of $Gr_T > 0$ is drawn in fig.5. The thermal Grashof number Gr_T signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. Also, as Gr_T increases, the peak values of the velocity increase rapidly near the plate and then decays smoothly to the free stream velocity. It is also noticed that the velocity increases with increasing values of Gr_c. The solutal Grashof number Gr_c defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases due to increase in the species buoyancy force. The velocity distribution attains a maximum value in the vicinity of the plate and then decreases to approach the free stream value.

In the case of $Gr_T < 0$ the velocity is drawn against the variations of S and ϕ in fig.6. It is observed that S decreases; and ϕ increases the velocity. This is because the negative Gr_T

makes the rate of heat transfer also negative and at wall, heat flux become positive and that accelerate the convection effect at the wall; Stream function for lower values of thermal Grashof number become more thinner (diluted) due to stronger convection, hence the velocity reduces.

Figure-7 shows the effects of Pr, S and ω on temperature. Pr reduces the temperature field. This is in agreement with the physical fact that the thermal boundary layer thickness decreases with increase in Prandtl number. The reason for such behavior is that the high Prandtl number fluid has a relatively low thermal conductivity. It is noticed that θ increases as S increases. This is because of the heat source which increases the amount of heat and consequently increases the thermal boundary layer. Figure-7 also shows that θ increases as ω increases or N decreases (because $\omega = \left(1 + \dfrac{4}{3N}\right)$). Numerically increasing the radiation parameter reduces the radiation effect, physically increasing the radiation parameter leads to decrease in the thermal boundary-layer thickness.

The skin friction coefficient C_f is plotted against Pr for different values of parameter Sc, K_c, M and ϕ in fig.8 and fig.9, and it is concluded from these figures that C_f decreases as any of these parameter increases. This is due to the fact that Sc or K_c or M or ϕ reduces the velocity consequently the shearing stress decreases. C_f is drawn against Pr for different variations of S, ω, Gr_T and Gr_c in fig.10. As it is seen that the velocity increases with S, ω, Gr_T, or Gr_c due to this increase the difference in velocity also rises and hence the skin friction increases. It is also noticed that the shearing stress falls rapidly for lower values of Pr, and it decreases as Pr increases. Physically, this is because for higher values of Pr the viscosity is more, and fluid becomes thicker and that reduces the shearing stress.

The Recovery factor is drawn in fig.11 for different values of S and ω. The recovery factor increases as S or ω increases, it is obvious as the velocity also increases with these parameters and because the high speed flow always have higher frictional heat and that increases the recovery temperature. It is observed that R_f reduces as Pr increases and for small values of Pr, R_f falls rapidly due to larger values of thermal diffusivity. With higher values of Pr, it detracts slowly.

In fig.12 the Sherwood number is plotted against Sc for different values of K_c. Sh increases as Sc or K_c increases. Increase in Sc or K_c reduce the concentration boundary layer and that further decrease the diffusivity of mass transfer, and consequently reduces Sh.

CONCLUSION

In the present problem, numerical solution (using Keller-box method) of steady hydromagnetic boundary-layer flow over a semi-infinite inclined plate, which is at prescribed heat flux, in the presence of chemical reaction, buoyancy, heat generation and thermal radiation. The obtained results were compared with earlier published work and found to be in good agreement. Schmidt number and the chemical reaction decrease the concentration boundary layer. In the case of cooling plate the velocity increases with increasing heat generation, thermal Grashof number and solutal Grashof number; whereas reverse trend is seen with the inclusion of Schmidt number, chemical reaction, magnetic field and angle of inclination. In the case of heating of the plate the heat generation and angle of inclination shows the reverse effects from the case of cooling plate. Increasing Prandtl number results in a thinner thermal boundary layer and more heat transfers from the surface to the fluid; however, radiation (increasing with 1/N) and heat generation extends the thermal boundary layer and decreases heat transfer from the surface to the fluid. Skin friction coefficient decrease with the increase of Schmidt number, chemical reaction, magnetic

field, Prandtl number, or angle of inclination and the phenomena reverses for heat generation, radiation, thermal Grashof number and solutal Grashof number. The recovery factor increases with heat generation and radiation as more heat is added; and decreases with Pr. The Sherwoon number increase with Schmidt number and chemical reaction.
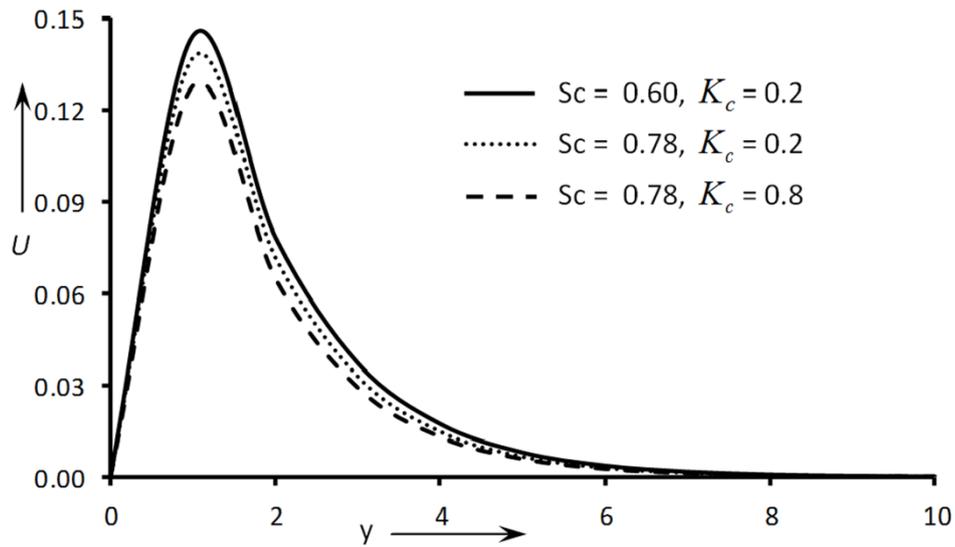

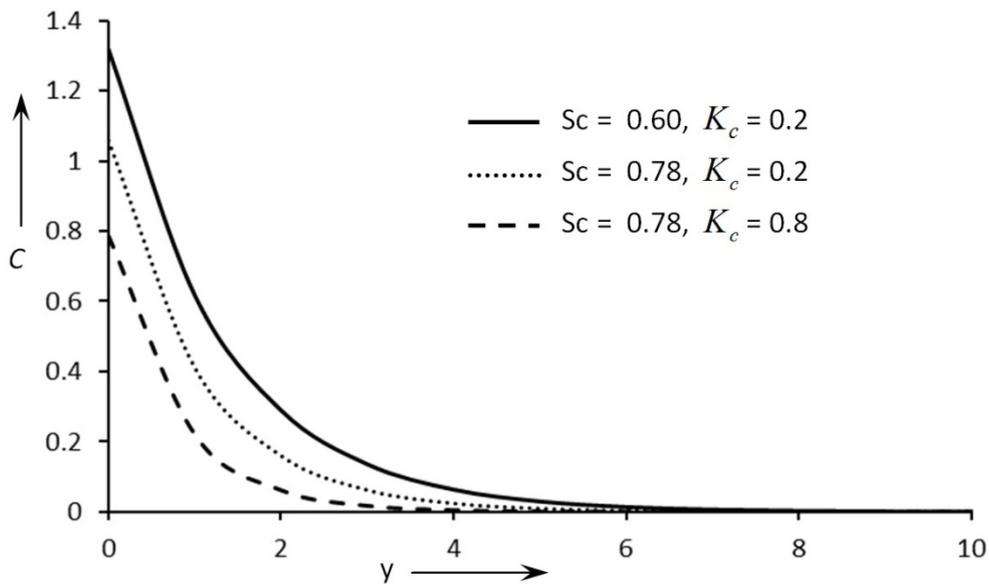**Fig.2**: Velocity against y for different values of Sc and K_c when Pr = 1.0, S = 0.1, $\omega = 1.13$, M = 2.0, $Gr_T = 1.0$, $\phi = 30^o$ and $Gr_C = 0.5$.

Fig.3: Concentration against y for different values of Sc and K_c.
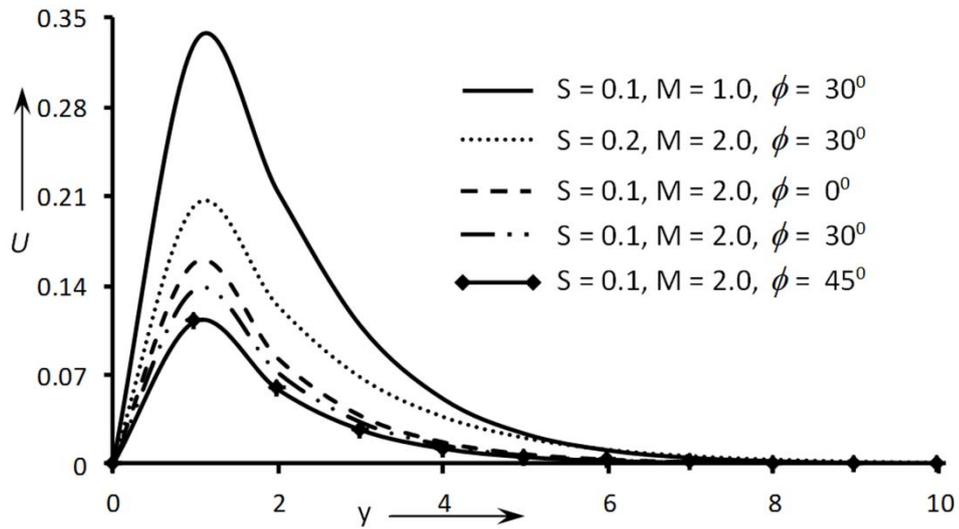
Fig.4: Velocity against y for different values of S, M and ϕ when Sc = 0.78, K_c = 0.2, Pr = 1.0, ω = 1.13, Gr_T = 1.0, and Gr_c = 0.5.
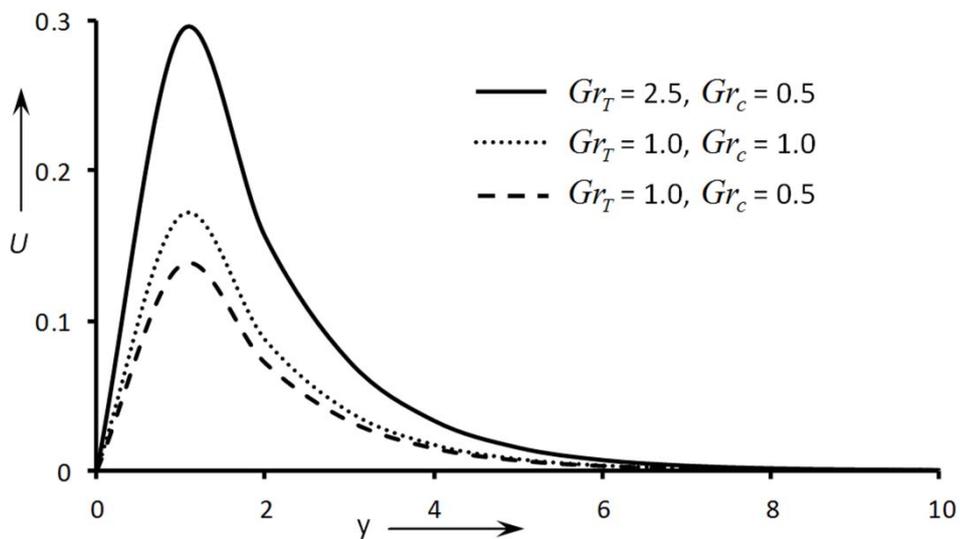
Fig.5: Velocity against y for different values of Gr_T and Gr_c when Sc = 0.78, K_c = 0.2, Pr = 1.0, S = 0.1, ω = 1.13, M = 2.0 and ϕ = 30°.

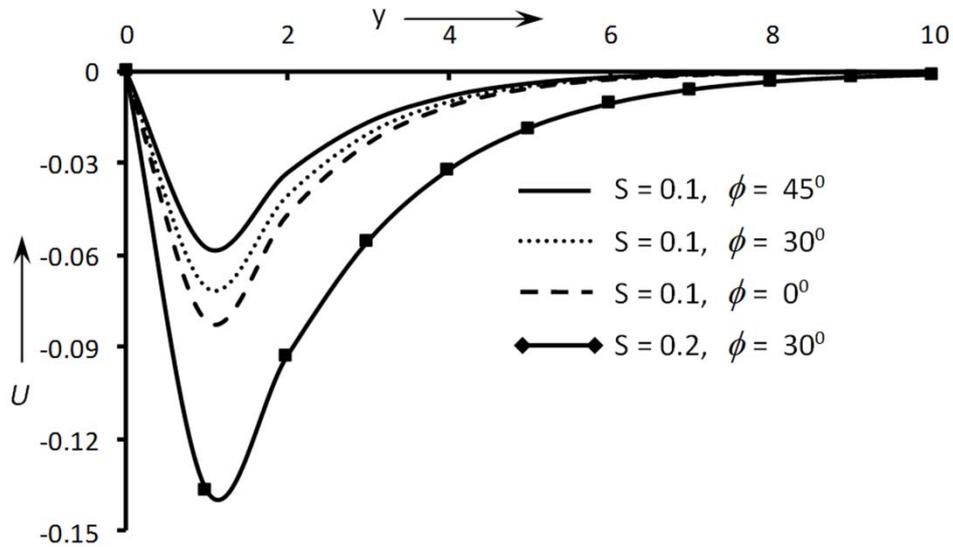**Fig.6**: Velocity against y for different values of S and ϕ when Sc = 0.78, K_c = 0.2, Pr = 1.0, S = 0.1, ω = 1.13, M = 2.0, Gr_T = -1.0 and Gr_C = 0.5.
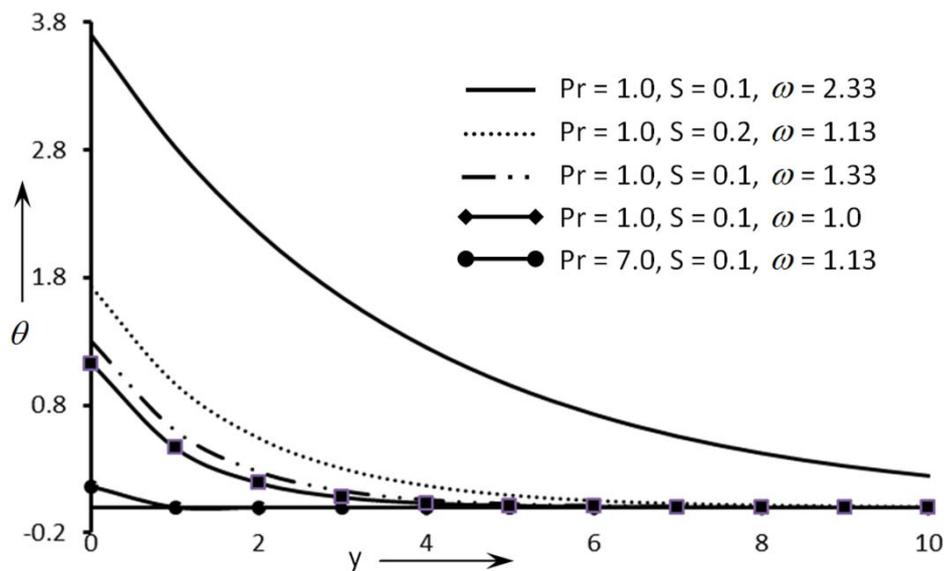
Fig.7: Temperature against y for different values of Pr, S and ω.
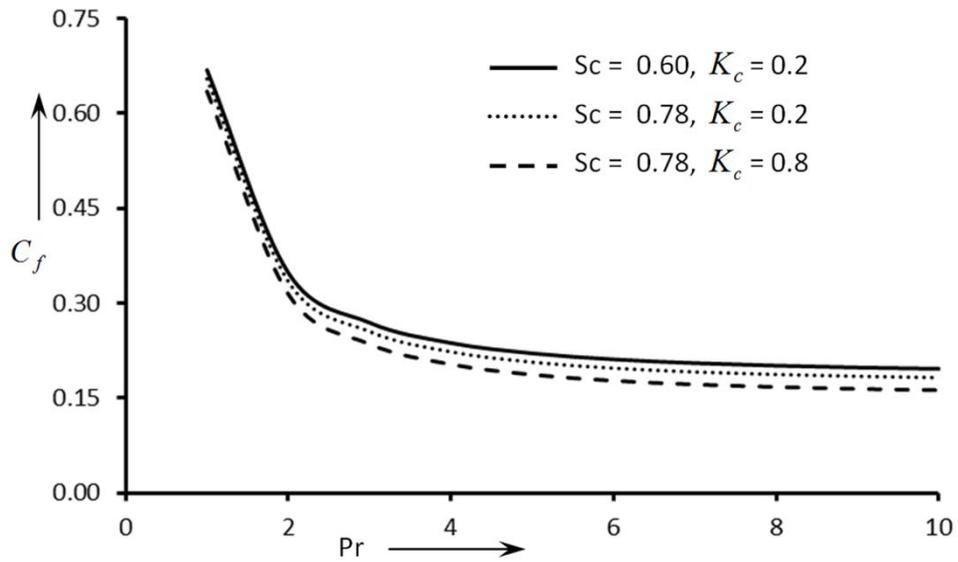
Fig.8: Skin friction coefficient against Pr for different values of Sc and K_c (Starting from Pr = 1.0 to present clear effects) when S = 0.1, $\omega = 1.13$, M = 2.0, $Gr_T = 1.0$, $\phi = 30^\circ$ and $Gr_C = 0.5$.
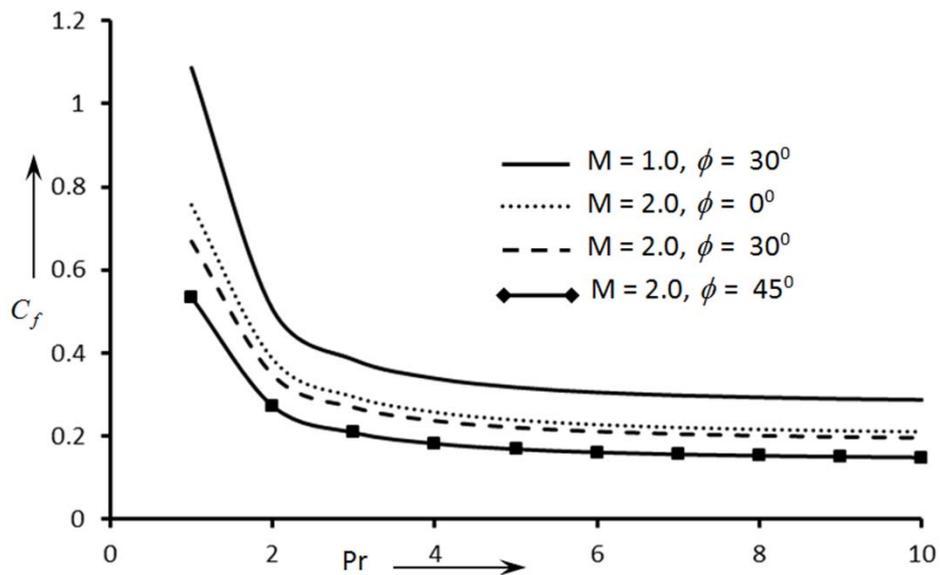

Fig.9: Skin friction coefficient against Pr for different values of M and ϕ (Starting from Pr = 1.0 to present clear effects) when Sc = 0.78, $K_c = 0.2$, S = 0.1, $\omega = 1.13$, $Gr_T = 1.0$ and $Gr_C = 0.5$.
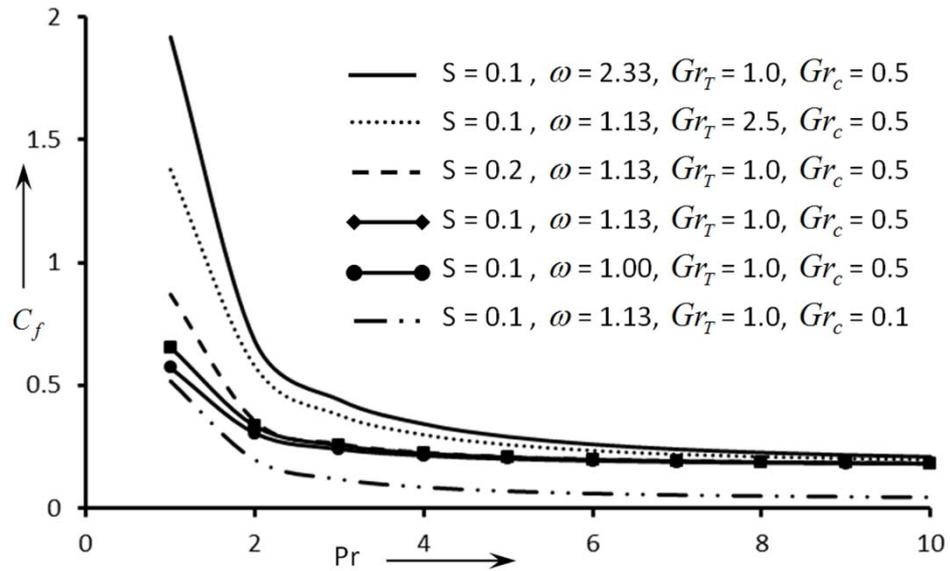
Fig.10: Skin friction coefficient against Pr for different values of S, ω, Gr_T and Gr_c (Starting from Pr = 1.0 to present clear effects) when Sc = 0.78, K_c = 0.2, M = 2.0 and $\phi = 30^\circ$.
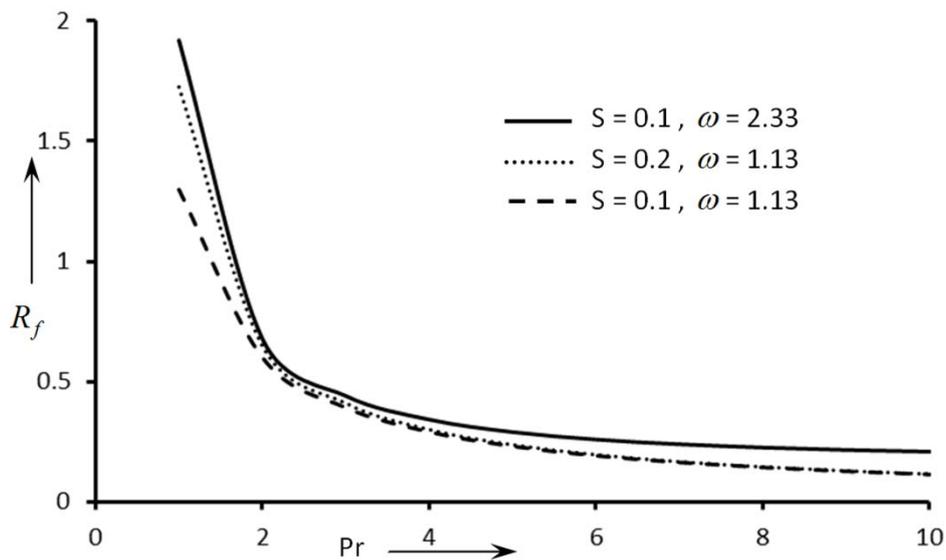
Fig.11: Recovery factor against Pr for different values of S and ω (Starting from Pr = 1.0 to present clear effects).
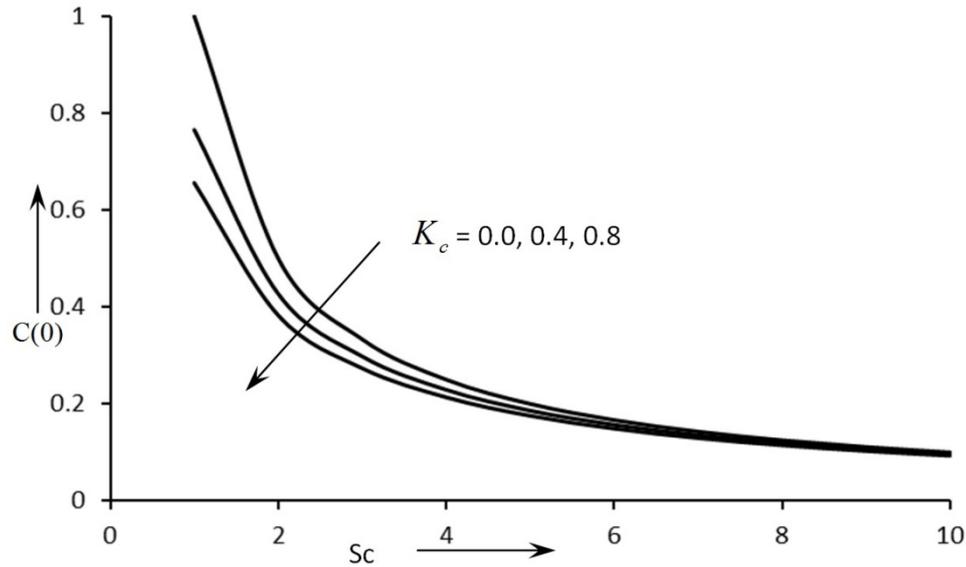
Fig.12: Sherwood number against Sc for different values of K_c (Starting from Sc = 1.0 to present clear effects).

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REFERENCES

Anghel, M., Hossain, M. A., Zeb, S. (2001). Combined heat and mass transfer by free convection past an inclined flat plate. *Int. J. Appl. Mech. and Engg.,* 2, 473–497

Bhuvaneswari, M., Sivasankaran, S., Kim, Y. J. (2010). Exact analysis of radiation convective flow heat and mass transfer over an inclined plate in a porous medium. *World Applied Journal*, 10, 774–778.

Brewster, M. Q. (1992). *Thermal Radiative Transfer and Properties*. John Wiley and sons, New York.

Cebeci, T., Bradshaw, P. (1977). *Momentum transfer in boundary layers*. Hemisphere, New York.

Cebeci, T., Bradshaw, P. (1988). *Physical and computational aspects of convective heat transfer*. Springer-Verlag, New York

Chen, C. H. (2004). Heat and mass transfer in MHD flow with variable wall temperature and concentration. *Acta Mechanica*, 172, 219–235

Ganesan, P., Palani, G. (2003). Natural convection effects on impulsively started inclined plate with heat and mass transfer. *Heat and Mass Transfer*, 39, 277–283.

Ganesan, P., Palani, G. (2004). Finite difference analysis of unsteady natural convection MHD flow past an inclined plate with variable surface heat and mass flux. *Int. J. Heat Mass Transfer*, 47, 4449–4457.

Ganesan, P., Loganathan, P., Soundalgekar, V. M. (2001). Radiation effects on Flow Past an Impulsively Started Infinite Vertical Plate. *Int. J. of Applied Mechanics and Engineering,* 6, 719–730.

Hossain, M. A., Pop, I., Ahmad, M. (1996). MHD free convection flow from an isothermal plate. *J. Theo. and Appl. Mech.*, 1, 194–207

Kandasamy, R., Muhaimin, I., Khamis, A. B. (2009). Thermophoresis and variable viscosity effects on MHD mixed convective heat and mass transfer past a porous wedge in the presence of chemical reaction. *Heat Mass Transfer*, 45, 703–712.

Kandasamy, R., Devi, S. P. A. (2004). Effects of chemical reaction, heat and mass transfer on non-linear laminar boundary-layer flow over a wedge with suction or injection. *Journal of Computational and Applied Mechanics*, 5, 21–31.

Kumar, H. (2009). Radiative heat transfer with hydromagnetic flow and viscous dissipation over a stretching surface in the presence of variable heat flux. *Thermal Science*, 13, 163–169

Kumar, H. (2011). Heat transfer over a stretching porous sheet subjected to power law heat flux in presence of heat source. *Thermal Science*, 15 (Suppl.2), S187–S194.

Modest, F. (2003). *Radiative Heat Transfer (2nd edition)*. Academic Press, New York

Shit, G. C., Haldar, R. (2011). Thermal radiation and hall effect on MHD flow, heat and mass transfer over an inclined permeable stretching sheet. *Thermal Science*, 15 (Suppl.2), S195–S204.

Sparrow, E. M., Husar, R. B. (1969). Longitudinal vortices in natural convection flow on inclined plates. *J. Fluid. Mech.*, 37, 251–255.

Siegel, R., Howell, J. R. (1992). *Thermal Radiation Heat Transfer (3rd edition)*. Hemisphere, New York

Sivasankaran, S., Bhuvaneswari, M., Kandaswamy, P., Ramasami, E. K. (2006). Lie Group Analysis of Natural Convection Heat and Mass Transfer in an Inclined Surface. *Nonlinear Analysis: Modelling and Control*, 11, 201–212.

Vajravelu, K., Hadjinicolaou, A. (1993). Heat transfer in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation. *Int. Comm. Heat Mass Transfer*, 20, 417–430.