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Consistent Fuzzy Preference Relation Method and Its Application to Criteria Assessment of Quality of Life Among Population of Setiu Wetlands

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ABSTRACT

Consistent Fuzzy Preference Relation (CFPR) method is widely used in solving multi-criteria group decision making problems. The main purpose of this study is to assess criteria that determine the quality of life (QoL) among the population in Setiu Wetlands by using CFPR method. QoL is an important aspect in the development of human society. It can be determined by eleven criteria. Personal communications with four experts were employed in the data collection. From the data obtained, seven-steps computations had been applied to evaluate the most important criterion. The advantages of using CFPR method are computationally efficient and simple. With the application of CFPR method, decision matrices can be reduced from n (n-1) /2 to (n-1) for a grouped of n-criteria. Results show that income aspect is the most important criteria. The contribution of this study lies in the result obtained which can help government to make major development in order to increase income factor among community to improve QoL among Setiu Wetlands's population.

Keywords: fuzzy preference relation, decision matrix, quality of life

INTRODUCTION

Group decision making is one of the most common activities in the real world. In the process of decision making, a group of experts will be asked to give their individual preferences. Preference relations or known as pairwise comparison matrices are the common tool used to help decision makers in expressing their preference information. During the past years, preference relations are receiving a lot of attention among researchers such as Menger (1951), Roubens and Vincke (1987), and Orlovsky (1978). Various types of preference relations have been developed including multiplicative preference relation, incomplete multiplicative preference relation, interval multiplicative preference relation, incomplete interval multiplicative preference relation, triangular fuzzy multiplicative preference relation, incomplete triangular fuzzy multiplicative preference relation, fuzzy preference relation, incomplete fuzzy preference relation, interval fuzzy preference relation, incomplete interval fuzzy preference relation, incomplete triangular fuzzy preference relation, linguistic preference relation, incomplete linguistic preference relation, consistent fuzzy preference relation and intuitionistic fuzzy preference relation.

In this study, the consistent fuzzy preference relation (CFPR) method is used to evaluate criteria to assess quality of life (QoL) among the population in Setiu Wetlands. This study uses CFPR method to find the priority weights of the criteria. CFPR method is computationally effective and simple while preserving the consistency of the pairwise comparisons. As reported in Economic Planning Unit (2011), QoL can be generally assessed by eleven criteria. It can be different based on the population. Development process in the population area can be done effectively with the evaluated criteria to assess quality of life. This paper constructs as follows: the related studies involving the CFPR method will be discussed in Section 2. The implementation of CFPR towards the evaluation of the criteria assessment and discussion of the results obtained will be explained in Section 3. Section 4 concludes this study.

CONSISTENT FUZZY PREFERENCE RELATION (CFPR) METHOD

Some judgments made in decision making problems are based on linguistic term. The idea of using values to demonstrate the preference degree between two elements in a given set comes from Menger (1951). Research about fuzzy relations is continuously evolving based on that idea. Orlovsky (1978) was the first researcher that defined fuzzy strict preference and indifference relations. The fuzzy preference relations are further investigated based on its comparability and consistency. Incomparability relations are first discussed simultaneously with fuzzy strict preference in 1987 (Roubens and Vincke, 1987). In 1991, the general functional form of fuzzy strict preference relations with the condition of min-asymmetry was discovered (Ovchinnikov and Roubens, 1991). Based on several studies, fuzzy preference relations can be generally defined as:

Definition 1. A fuzzy preference relation $R = (r_{ij})_{n\times n}$ is called an additive consistent fuzzy preference relation if it satisfies the additive transitivity (Tanino, 1984), (Herrea-Viedma et al., 2004):

$$r_{ij} = r_{ik} - r_{jk} + 0.5, \text{ for all } i, j, k = 1,2,...,n \tag{1}$$

Meanwhile, a fuzzy preference relation $R = (r_{ij})_{n\times n}$ is called multiplicative consistent fuzzy preference relation if it satisfies the multiplicative transitivity (Tanino, 1984), (Herrea-Viedma et al., 2004):

$$r_{ij} \cdot r_{jk} \cdot r_{ki} = r_{ik} \cdot r_{kj} \cdot r_{ji}, \text{ for all } i, j, k = 1,2,...,n \tag{2}$$

Over the years, fuzzy preference relations had been actively studied. Some properties of fuzzy preference relations also had been discussed (Herrea-Viedma et al., 2004). Fuzzy preference relations also involved the priority vector. Several methods had been proposed such as a convergent iterative algorithm (Xu, 2005), least square method and eigenvector methods (Xu, 2002), and optimization approach (Lipovetsky and Michael, 2002) to obtain the priority vector from fuzzy preference relations. Other than that, consistency of fuzzy preference relations also has been highlighted by several researchers. An analysis method to establish the inconsistency and weak transitivity of fuzzy preference relations and method to repair the problems has been discovered by Ma et al. (2006).

Besides, the development of various types of fuzzy preference relations has been made to improve the consistency and applications of fuzzy preference relations such as interval fuzzy preference relations (Xu, 2004), intuitionistic preference relations (Xu, 2007), interval intuitionistic preference relations (Herrera et al., 2005) and CFPR method (Herrea-Viedma et al., 2004). CFPR method had been applied to solve various types of decision making problems such as in the selection of a merger strategy for commercial banks (Wang and Lin 2009) and supplier selection (Chen and Chao, 2012). In this study, proposition of CFPR method from Herrera-Viedma et al., (2004) are adapted. There are three propositions used in the CFPR method.

Proposition 1. A group of alternatives, $A = \{a_1,...,a_n\}$ corresponded with a reciprocal multiplicative preference relation $S = (s_{ij})$ with $s_{ij} \in [1/9,9]$. Afterwards, the correlating reciprocal fuzzy preference relation, $R = (r_{ij})$ with $r_{ij} \in [0,1]$ associated with *S* is given as

$$R = g(S), i.e., r_{ij} = g(s_{ij}) = \frac{1}{2}(1 + \log_9 s_{ij}), \tag{3}$$

where *S* is a transformation function. $\log_9 s_{ij}$ is examined since $s_{ij} \in [1/9,9]$. If $s_{ij} \in [1/7,7]$, then $\log_7 s_{ij}$ is used. In general, $\log_n s_{ij}$ is used if $s_{ij} \in [1/n, n]$.

Proposition 2: For a reciprocal fuzzy preference relation, where, the following statements are matched.

$$r_{ij} + r_{jk} + r_{ki} = \frac{3}{2}, \forall i, j, k, \tag{4}$$

$$r_{ij} + r_{jk} + r_{ki} = \frac{3}{2}, \forall i < j < k, \tag{5}$$

Proposition 3: For a reciprocal fuzzy preference relation, $R = (r_{ij})$, the following expressions are equivalent.

$$r_{ij} + r_{jk} + r_{ki} = \frac{3}{2}, \forall i < j < k, \tag{6}$$

$$r_{i(i+1)} + r_{(i+1)(i+2)} + ... + r_{(i+k-1)(i+k)} + r_{(i+k)i} = \frac{k+1}{2}, \forall i, k \tag{7}$$

Respectively, each answered questionnaires are expressed in terms of consistent decision matrices. A decision matrix with entries in an interval $[-p, 1+p], p > 0$, not in the interval $[0,1]$ should be converted by using a transformation function that preserves reciprocity and additive consistency. Transformation function $f(x)$ is given as the following:

$$f : [-p, 1+p] \to [0,1], f(a) = \frac{a+p}{1+2p} \tag{8}$$

The fuzzy preference relation matrices, $R = (r_{ij})$ of pairwise comparisons are created and the weight of the criterion is calculated. The aggregation score, μ_i of each criterion can be evaluated by the following:

$$u_i = \frac{1}{n_f}\left(\sum_{j=1}^{n_f} r_{ij}\right) \tag{9}$$

where n_f is the number of criteria will be computed. r_{ij} is the value in the i^{th} row and j^{th} column of the preference relation matrix R. The weight, *w* of each criterion can be clarified as

$$w_i = \frac{u_i}{\sum_{i=1}^{n_f} u_i} \tag{10}$$

Based on the definition and propositions of CFPR method, it is relevant to implement this method to solve MCDM problem in assessing QoL among the population in Setiu Wetlands. This problem involved an evaluation of a number of criteria and information gathered from a group of experts. Therefore, CFPR method helps in providing clear fuzzy information as this method reduce the number of pairwise comparison needed to obtain the final result. Clear presentation of the collected data can be obtained by using CFPR method. Further explanation of the implementation process is discussed in the next section.

IMPLEMENTATION

Quality of life (QoL) is the common well-being of humanity and individuals. Quality of life can be classified into different fields such as health care, governments, and works. There are eleven criteria used to measure the QoL as a general as shown in Table 1 below.

Table 1:. Criteria to assess QoL among a population of Setiu Wetland

Notation	Criteria	Description
$\mathbf{a_1}$	Education	• The main approach to give knowledge and cultural exposure to the new generation • Act as a base to form and develop the innovation of new technologies
$\mathbf{a_2}$	Transport and Communication	• The important factor to rate the development of the community as it allows required source to be transferred • Provides job opportunities, education, and services
$\mathbf{a_3}$	Housing quality	• A basic social needs for a perfect living, safety securement, and protection to each family
$\mathbf{a_4}$	Culture and Entertainment	• An important component to determine the identities of the community • Can be described through human behaviors and act of thinking, conversations, social and religious practices of the community
$\mathbf{a_5}$	Income	• Enables an individual to cover self and family's expenses
$\mathbf{a_6}$	Public Safety	• To ensure peace and social stability
$\mathbf{a_7}$	Health	• Physical and mental health that increases the productivity and social participation
$\mathbf{a_8}$	Social participation	• A component that determines commitment and ability of the community to participate in social activities, politics, religion and community services
$\mathbf{a_9}$	Environment	• Effect the community's welfare • Forest preservation and pollutions act as the indicator to the QoL among the population
$\mathbf{a_{10}}$	Family living	• Family unit represents the social structure that provides social needs, economics and individual psychology

a_{11}	Working environment	• A safe and conducive working environment increases the productivity • High productivity ensures a high income and leads to a better QoL

In order to find the weight of the criteria, eight-steps of computation are required.

Computations & Results

Among the criteria, the evaluators have to make their preference on which criteria are more important comparing to another criteria respectively to assessing QoL. There are seven-steps involved in the computation process.

Step 1: Collect the questionnaires from the evaluators. The questionnaire is expressed as in linguistic variables.

Step 2: Convert the linguistic variables to the linguistic scale based on Saaty's scale as presented in Table 2.

Table 2: Linguistic scale

Relative importance	Definition	Explanation
1	Equally importance	Two factors contribute equally to the objective
3	Moderate importance	Experience and judgment slightly favor one factor over another
5	Strong importance	Experience and judgment strongly favor one factor over another
7	Very strong importance	A factor is strongly favoring and its dominance demonstrated in practice
9	Extreme importance	The evidence is favoring one factor over another is of the highest possible order of affirmation

Step 3: Compute the average score and initial fuzzy preference ratios for each pairwise comparison. The initial fuzzy preference ratios are converted from an average score according to the Equation (3) in Proposition 1. Table 3 shows the computed initial values of the criteria. The first row shows the converted average scores. Meanwhile, the second row of Table 3 lists the initial fuzzy preference ratio. The third row represents the corresponding denotation used in the decision matrix, M for each criterion.

Table 3:. The initial score of pairwise comparison in decision matrix, M

	a_1/a_2	a_2/a_3	a_3/a_4	a_4/a_5	a_5/a_6	a_6/a_7	a_7/a_8	a_8/a_9	a_9/a_{10}	a_{10}/a_{11}
Fuzzy preference ratio, *R*	0.9151	0.0212	0.1508	0.0769	0.8300	0.9788	0.6400	0.8266	0.8080	0.9428
Notation r_{ij}	r_{12}	r_{23}	r_{34}	r_{45}	r_{56}	r_{67}	r_{78}	r_{89}	r_{910}	r_{1011}

Step 4: Construct the initial decision matrix, M. Table 4 shows the initial value of some fuzzy preference ratios in the decision matrix. The values are computed according to Equation (4). The diagonal values are all 0.5 and the "r_{ij}" values will be computed in the next step.

Table 4: The initial decision matrix, M

	a1	**a2**	**a3**	**a4**	**a5**	**a6**	**a7**	**a8**	**a9**	**a10**	**a11**
a1	0.5000	0.9151	r_{13}	r_{14}	r_{15}	r_{16}	r_{17}	r_{18}	r_{19}	r_{110}	r_{111}
a2	r_{21}	0.5000	0.0212	r_{24}	r_{25}	r_{26}	r_{27}	r_{28}	r_{29}	r_{210}	r_{211}
a3	r_{31}	r_{32}	0.5000	0.1508	r_{35}	r_{36}	r_{37}	r_{38}	r_{39}	r_{310}	r_{311}
a4	r_{41}	r_{42}	r_{43}	0.5000	0.0769	r_{46}	r_{47}	r_{48}	r_{49}	r_{410}	r_{411}
a5	r_{51}	r_{52}	r_{53}	r_{54}	0.5000	0.8300	r_{57}	r_{58}	r_{59}	r_{510}	r_{511}
a6	r_{61}	r_{62}	r_{63}	r_{64}	r_{65}	0.5000	0.9788	r_{68}	r_{69}	r_{610}	r_{611}
a7	r_{71}	r_{72}	r_{73}	r_{74}	r_{75}	r_{76}	0.5000	0.6400	r_{23}	r_{710}	r_{711}
a8	r_{81}	r_{82}	r_{83}	r_{84}	r_{85}	r_{86}	r_{87}	0.5000	0.8266	r_{810}	r_{811}
a9	r_{91}	r_{92}	r_{93}	r_{94}	r_{95}	r_{96}	r_{97}	r_{98}	0.5000	0.8080	r_{911}
a10	r_{101}	r_{102}	r_{103}	r_{104}	r_{105}	r_{106}	r_{107}	r_{108}	r_{109}	0.5000	0.9428
a11	r_{111}	r_{112}	r_{113}	r_{114}	r_{115}	r_{116}	r_{117}	r_{118}	r_{119}	r_{1110}	0.5000

Step 5: Compute the rest of element "r" in the decision matrix using Equation (6) in Proposition 3. The detailed computation is demonstrated as follows.

Since

$i < k$, let $r_{21} = r_{(i+k)i}$

therefore,

$$2 = 1 + k$$
$$k = 2 - 1$$
$$k = 1$$

From Equation (7),

$$r_{i(i+1)} + r_{(i+1)(i+2)} + \ldots + r_{(i+k-1)(i+k)} + r_{(i+k)i} = \frac{k+1}{2}, \forall i, k$$
$$r_{(i+k-1)(i+k)} = r_{12} = r_{i(i+1)}$$

hence,

$$r_{12} + r_{21} = \frac{1+1}{2}$$
$$r_{21} = 1 - r_{12}$$
$$r_{21} = 1 - 0.9151 = 0.0850$$

All values of r were calculated by using the same process. The complete decision matrix, M is represented in Table 5.

Table 5: The complete decision matrix, M

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}
a_1	0.5000	0.9152	0.4364	0.0872	-0.3359	-0.0059	0.4729	0.6129	0.9395	1.2475	1.6903
a_2	0.0848	0.5000	0.0212	-0.3280	-0.7511	-0.4211	0.0577	0.1977	0.5243	0.8323	1.2751
a_3	0.5636	0.9788	0.5000	0.1508	-0.2723	0.0577	0.5365	0.6765	1.0031	1.3111	1.7539
a_4	0.9128	1.328	0.8492	0.5000	0.0769	0.4069	0.4069	1.0257	1.3523	1.6603	2.1031
a_5	1.3359	1.7511	1.2723	0.9231	0.5000	0.8300	1.3088	1.4488	1.7754	2.0834	2.5262
a_6	1.0059	1.4211	0.9423	0.5931	0.1700	0.500	0.9788	1.1188	1.4454	1.7534	2.1962
a_7	0.5271	0.9423	0.4635	0.1143	-0.3088	0.0212	0.5000	0.6400	0.9666	1.2746	1.7174
a_8	0.3871	0.8023	0.3235	-0.0257	-0.4488	-0.1188	0.3600	0.5000	0.8266	1.1346	1.5774
a_9	0.0605	0.4757	-0.0031	-0.3523	-0.7754	-0.4454	0.0334	0.1734	0.5000	0.8080	1.2508
a_{10}	-0.2475	0.1677	-0.3111	-0.6603	-1.0834	-0.7534	-0.2746	-0.1346	0.1920	0.5000	0.9428
a_{11}	-0.6903	-0.2751	-0.7539	-1.1031	-1.5262	-1.1962	-0.7174	-0.5774	-0.2508	0.0572	0.5000

Step 6: Normalize the data since the converted data not in the range of [0, 1] by using a transformation function as shown in Equation (8). Since the values are in the range of [-1.5262, 2.5262],

The complete altered data are shown in Table 6 below:

Table 6. The complete altered decision matrix, M

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}
a_1	0.5000	0.6024	0.4843	0.3981	0.2937	0.3751	0.4933	0.5278	0.6084	0.6844	0.7937
a_2	0.3975	0.5000	0.3818	0.2956	0.1912	0.2727	0.3908	0.4254	0.5059	0.5820	0.6912
a_3	0.5156	0.6181	0.5000	0.4138	0.3094	0.3908	0.50900	0.5435	0.6241	0.7001	0.80942
a_4	0.6018	0.7043	0.5861	0.5000	0.3955	0.4770	0.4770	0.6297	0.7103	0.7863	0.8955
a_5	0.7062	0.8087	0.6905	0.6044	0.5000	0.5814	0.6995	0.7341	0.8147	0.8907	1.0000
a_6	0.6248	0.7272	0.6091	0.5229	0.4185	0.5000	0.6181	0.6526	0.7332	0.8092	0.9185
a_7	0.5066	0.6091	0.4909	0.4048	0.3004	0.3818	0.5000	0.5345	0.6151	0.6911	0.8004
a_8	0.4721	0.5745	0.4564	0.3702	0.2658	0.3473	0.4654	0.5000	0.5805	0.6565	0.7658
a_9	0.3915	0.4940	0.3758	0.2896	0.1852	0.2667	0.3848	0.4194	0.5000	0.5760	0.6852
a_{10}	0.3155	0.4179	0.2998	0.2136	0.1092	0.1907	0.3088	0.3434	0.4239	0.5000	0.6092

a_{11}	0.2062	0.3087	0.1905	0.1044	0.0000	0.0814	0.1995	0.2341	0.3147	0.3907	0.5000

Step *7*: Evaluate the aggregation score and weight for each factor by utilizing Equation (9) and (10) respectively. Table 7 demonstrates the complete decision matrix, M with the score, weight and rank of each criterion. Based on Table 7, the most important criteria is income . This shows an income among Setiu Wetland population contributes a lot to the QoL of the community.

Table 7. The score, weight and rank of the criteria

Notations	Criteria	Aggregation score, u_i	Priority weight	Rank
a_1	Education	5.7615	0.0954	6
a_2	Transport and Communication	4.6345	0.0767	8
a_3	Housing	5.9342	0.0982	4
a_4	Culture and Entertainment	6.7639	0.1120	3
a_5	Income	8.0305	0.1329	1
a_6	Public Safety	7.1348	0.1181	2
a_7	Health	5.8351	0.0966	5
a_8	Social participation	5.4551	0.0903	7
a_9	Environment	4.5686	0.0756	9
a_{10}	Family living	3.7325	0.0618	10
a_{11}	Working environment	2.5305	0.0419	11

By using CFPR method to evaluate the result, the relative weights in each criterion can be easily measured. The CFPR method process helps to reduce pairwise comparison used in questionnaires. This helps expert to avoid confusion and make a proper judgment. Besides, the computation needed to be performed is simple and efficient. Additive transitivity used in CFPR method guarantees consistency of the results. The computation can be done by using Excel spreadsheets.

CONCLUSION

Fuzzy preference relations are used to solve decision making problems that counter linguistic term as the medium of judgment. This study presents CFPR method as a simple and efficient method to assess quality of life among the Setiu Wetlands population. By using CFPR method, number of pairwise comparison used in the questionnaires can be reduced from n(n-1)/2 to (n-1) for a grouped of n-factor. Other pairwise comparisons can be computed by using CFPR method. Therefore, the representation of preferences involved is clear. Besides, the procedure involved is simple and practical. Additive transitivity guarantees the consistency in creating decision matrices. Results show that income aspect with the weight of 0.1329 ranks as the most important criteria to assess QoL among the Setiu Wetlands population. Major and continuous development in the economy of Setiu Wetlands can be taken to ensure quality of life among the community.

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