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Dufour and Soret Effects on Magnetohydrodynamic Boundary Layer Slips Flow in Nanofluids with Microorganisms over a Heated Stretching Sheet with Temperature-Dependent Viscosity

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ABSTRACT

In this paper, steady two-dimensional magnetohydrodynamics boundary layer slip flow in a nanofluid with microorganism over a heated stretching sheet with temperature-dependent viscosity is studied using a numerical method. The diffusion-thermo (Dufour) as well as thermal-diffusion (Soret) effects are taken into consideration. The fluid viscosity is assumed to vary linearly with temperature. A scaling group of transformation is applied to the governing equations of continuity, momentum, energy, concentration and microorganism to obtain a set of non-linear ordinary differential equations (ODEs). These ODEs along with its associated supplementary boundary conditions are then solved numerically by using Runge-Kutta-Fehlberg fourth-fifth (RKF45) order method. The important physical parameters such as magnetic field M, temperature-dependent fluid viscosity β, Dufour Du, Soret Sr, Schmidt number Sc, Péclet number Pe, thermophoresis Nt, Brownian motion Nb, bioconvection Lewis number Lb, thermal slip p, mass slip q , and microorganisms slip r give effects on skin friction coefficient, Nusselt number, Sherwood number and density number of motile microorganism. Result shows that the skin friction increase with increasing of parameter Du, β and M. Nusselt number, Sherwood number and density number of motile microorganism tend to decrease with increasing of parameter Sr, Sc, Pe, Nt, Nb, Lb, p, q and r. All results are discussed and displayed graphically. The present study is compared with the previous published study and a good agreement was found.

Keywords: Nanofluid, Bioconvection, Magnetohydrodynamic

INTRODUCTION

Nanofluid refers to the fluid in which nanoparticles are suspended in base fluid. Nanoparticles are particles that sized between 1 and 100 nanometers. Nanoparticles are usually metal (Cu, Al, Ni), metal oxides (Al_2O_3, CuO), nitrides (SiN,AlN) or carbides (SiC). Several ordinary heat transfer fluids or base fluid such as water, toluene, polymer solutions, and ethylene glycol have rather low thermal conductivity in heat transfer process compared to nanofluid. Nanofluids have the ability to enhance the thermophysical properties of the base fluids (Yu and Xie, 2012) The study of heat and mass transfer of nanofluid has been widely reported by various researchers. Kandasamy et al. (2011) investigated the MHD boundary layer flow of a nanofluid over a vertical stretching surface in the existence of suction/injection using scaling group transformation. The model used for the nanofluid is taken into consideration effects of Brownian motion and effects of thermophoresis. Uddin et al. (2012) have investigated two-dimensional free convection boundary layer flow of a nanofluid from a convectively heated vertical plate with linear momentum slip boundary condition. By using the Buongiorno (2006) model, Kuznetsov & Nield (2014) investigated the natural convective boundary layer flow of a nanofluid past a vertical plate. The Buongiorno model was also used by Khan and Pop (2010) to investigate the boundary layer flow of a nanofluid past a plate. RamReddy et al. (2013)

analyzed the Soret effect on mixed convection flow in a nanofluid with convective boundary condition.

Bioconvection is occur due to upward swimming of microorganisms which are a slightly denser than water and this process leading to the increase of the density of the base fluid. Bioconvection has the ability to improve the stability of nanofluids instead of enhancing inducing mixing and mass transport. Mutuku and Makinde (2014) investigated the hydromagnetic flow of nanofluid with gyrotactic microorganisms flowing past a permeable vertical moving surface.

Magnetohydrodynamics (MHD) refers to the phenomena of the complex interaction between the magnetic field and electrically conducting fluid (Khan and Makinde (2014)). MHD is an interesting branch of research and for that reason many researchers choose to study and investigate it. MHD nanofluid bioconvection which is due to gyrotactic microorganisms over a convectively heat stretching sheet are investigated by Khan and Makinde (2014). Recently, Hassan et al. (2015) studied MHD boundary layer flow with the incorporation of variable viscosity over a heated stretching sheet and result shows that the dimensionless velocity increases by decreasing the magnetic field parameter.

The heat transfer due to the concentration gradient is termed as Dufour or diffusion-thermo effect while the mass transfer created by the temperature gradient is termed Soret or thermal-diffusion effect. Omowaye et al. (2015) studied and analyzed the Dufour and Soret effects on steady MHD convective flow of a fluid in a porous medium with temperature dependent-viscosity. Next, Moorthy and Senthilvadivu (2012) have analyzed the Soret and Dufour effects on natural convection flow past a vertical surface with variable viscosity and the result shows that the thermal and species concentration boundary layer thickness increases for gases but decreases for liquids. Dufor and Soret effects also gives impact on mixed convection boundary layer flow in a porous medium filled with a viscoelastic fluid (Hayat et al. (2010). Arasu et al. (2011) have explained about thermophoresis particle deposition which is gives effects on thermal-diffusion and diffusion-thermo. Uwanta et al. (2012) have investigated the MHD fluid flow over a vertical plate in the presence of Dufour and Soret effects. Mahdy (2010) investigated the Soret and Dufour effect on double diffusion mixed convection from a vertical surface in a porous medium which is saturated with a non-Newtonian fluid.

In most of the studies, the viscosity of the fluid has always been assumed to be constant but it is well known that physical properties of a fluid can change significantly with temperature and other factors. Compared to other constant problem, the flow characteristic are significantly changed when the effect and influence of variable viscosity is taken into consideration in a given flow model. Based on paper Mukhopadhyay et al. (2005), the fluid viscosity in MHD boundary layer flow is assumed to be a linear function of temperature. Muhaimin et al. (2010) have introduced the presence of thermophoresis and chemical reaction on MHD boundary layer flow with effect of temperature-dependent fluid viscosity. The temperature-dependent fluid viscosity is known to play an important role in shifting the fluid away from the wall.

The objective of this paper is to study the Dufour and Soret effects on MHD boundary layer slips flow of bionanofluid over a heated stretching sheet with temperature-dependent viscosity. The governing of the problem are nondimensionalized and transform into ODEs equations using similarity transformation developed by Lie group analysis. ODEs equations are then solved by Runge-Kutta-Fehlberg fourth-fifth (RKF45) order numerical procedure.

MATHEMATICAL FORMULATION

Consider a steady two-dimensional MHD nanofluids with microorganisms over a heated stretching sheet (in the region $\bar{y} > 0$) with temperature-dependent viscosity in the presence of thermal slip, mass slip, microorganisms slip together with the Dufour and Soret effects in Figure 1. The temperature T, the nanoparticle volume fraction C and the density of motile microorganisms N at the plate surface is denoted as T_w, C_w and N_w respectively. Meanwhile the corresponding ambient values are denoted respectively by T_∞, C_∞ and N_∞.

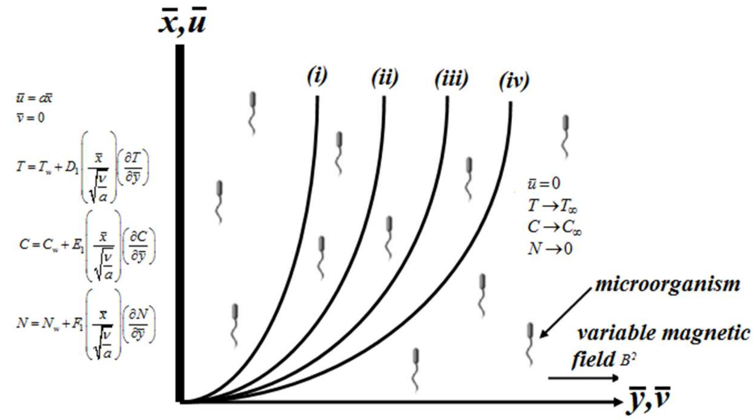


Figure 1: Physical model and coordinate system (Hassan et al., 2015)

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \frac{1}{\rho}\frac{\partial \mu_1}{\partial T}\frac{\partial T}{\partial \bar{y}}\frac{\partial \bar{u}}{\partial \bar{y}} + \frac{\mu_1}{\rho}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma B^2}{\rho}\bar{u}, \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha\frac{\partial^2 T}{\partial \bar{y}^2} + \frac{D_m K_T}{c_s c_p}\frac{\partial^2 C}{\partial \bar{y}^2} + \tau\left[D_B\frac{\partial T}{\partial \bar{y}}\frac{\partial C}{\partial \bar{y}} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial \bar{y}}\right)^2\right], \tag{3}$$

$$\bar{u}\frac{\partial C}{\partial \bar{x}} + \bar{v}\frac{\partial C}{\partial \bar{y}} = D_m\frac{\partial^2 C}{\partial \bar{y}^2} + \frac{D_m K_T}{T_m}\frac{\partial^2 T}{\partial \bar{y}^2}, \tag{4}$$

$$\bar{u}\frac{\partial N}{\partial \bar{x}} + \bar{v}\frac{\partial N}{\partial \bar{y}} + \frac{\tilde{b}W_c}{C_w - C_\infty}\left[\frac{\partial}{\partial \bar{y}}\left(N\frac{\partial C}{\partial \bar{y}}\right)\right] = D_n\left(\frac{\partial^2 N}{\partial \bar{y}^2}\right), \tag{5}$$

where the temperature-dependent fluid viscosity is $\mu_1 = \mu\left[a_1 + b_1\left(T_w - T\right)\right]$ (Batchelor, 1967), $\bar{u}$ and $\bar{v}$ are the velocity components respectively in $\bar{x}$ and $\bar{y}$ directions, μ_1 is the coefficient of fluid viscosity, ρ is the fluid density, σ is the electrical conductivity, B^2 is the strength of uniform magnetic field, μ is the dynamic viscosity, a_1, b_1 are positive constants, α is the thermal diffusivity, D_m is the mass diffusion coefficient, K_T is the thermal-diffusion ratio, c_s is the concentration susceptibility, c_p is the specific heat at constant pressure, τ is the ratio of nanoparticle heat capacity with the base fluid heat capacity, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, T_m is the mean fluid temperature, $\tilde{b}$ is the chemotaxis constant, W_c is the maximum cell swimming speed, D_n is the microorganisms diffusion coefficient. The corresponding boundary conditions are (Mukhopadhyay et al., 2005) and (Uddin et al., 2015)

$$\bar{u} = a\bar{x},\ \bar{v} = 0,\ T = T_w + D_1\left(\frac{\bar{x}}{\sqrt{(\upsilon / a)}}\right)\left(\frac{\partial T}{\partial \bar{y}}\right),\ C = C_w + E_1\left(\frac{\bar{x}}{\sqrt{(\upsilon / a)}}\right)\left(\frac{\partial C}{\partial \bar{y}}\right),$$

$$N = N_w + F_1\left(\frac{\bar{x}}{\sqrt{(\upsilon / a)}}\right)\left(\frac{\partial N}{\partial \bar{y}}\right) \text{ at } \bar{y} = 0, \tag{6}$$

$$\bar{u} = 0,\ T \to T_\infty,\ C \to C_\infty,\ N \to 0 \text{ as } \bar{y} \to \infty,$$

where a is a positive constant, $D_1\left(\frac{\bar{x}}{\sqrt{(\upsilon / a)}}\right)\left(\frac{\partial T}{\partial \bar{y}}\right)$ is the thermal slip, $E_1\left(\frac{\bar{x}}{\sqrt{(\upsilon / a)}}\right)\left(\frac{\partial C}{\partial \bar{y}}\right)$ is the mass slip and $F_1\left(\frac{\bar{x}}{\sqrt{(\upsilon / a)}}\right)\left(\frac{\partial N}{\partial \bar{y}}\right)$ is the microorganism slip.

NONDIMENSIONALIZATION OF THE GOVERNING EQUATIONS

Introducing the following dimensionless variables

$$x=\frac{\bar{x}}{\sqrt{(\upsilon/a)}},\ y=\frac{\bar{y}}{\sqrt{(\upsilon/a)}},\ u=\frac{\bar{u}}{\sqrt{\upsilon a}},\ v=\frac{\bar{v}}{\sqrt{\upsilon a}},\ \theta=\frac{T-T_\infty}{T_w-T_\infty},\ \phi=\frac{C-C_\infty}{C_w-C_\infty},\ \chi=\frac{N}{N_w} \tag{7}$$

By using scaling group transformations, we achieve the similarity transformations as follows,

$$\eta=y,\ \ \psi=xf(\eta),\ \ \theta=\theta(\eta),\ \ \phi=\phi(\eta),\ \ \chi=\chi(\eta),\ \ D_1=(D_1)_\circ,\ \ E_1=(E_1)_\circ,\ \ F_1=(F_1)_\circ, \tag{8}$$

where η is the similarity variable and ψ is the stream function defined as

$$u=\frac{\partial\psi}{\partial y}\text{ and } v=-\frac{\partial\psi}{\partial x} \tag{9}$$

The equation of continuity is satisfied identically. Now, by substituting Eq. (7), Eq. (8) and Eq. (9) into Eqs. (1) - (6), we obtained the following ordinary differential equations (ODEs):

$$f'''-\frac{\beta}{(a_1+\beta)}\theta f'''-\frac{\beta}{(a_1+\beta)}\theta' f''+\frac{1}{(a_1+\beta)}ff''-\frac{1}{(a_1+\beta)}f'^2-\frac{1}{(a_1+\beta)}M\,f'=0, \tag{10}$$

$$\theta''+Pr\,f\theta'+Nb\,\theta'\phi'+Nt\,\theta'^2+Du\,\phi''=0, \tag{11}$$

$$\phi''+Sc\,f\phi'+Sc\,Sr\,\theta''=0, \tag{12}$$

$$\chi''+Lb\,f\chi'-Pe\left[\chi\phi''+\phi'\chi'\right]=0, \tag{13}$$

subject to the boundary conditions:

$$\begin{gathered} f(0)=0,\ f'(0)=1,\ \theta(0)=1+p\ \theta'(0),\ \phi(0)=1+q\,\phi'(0),\ \chi(0)=1+r\,\chi'(0),\\ f'(\infty)=0,\ \theta(\infty)=0,\phi(\infty)=0,\ \chi(\infty)=0, \end{gathered} \tag{14}$$

where $\beta=b_1(T_w-T_\infty)$ is the temperature-dependent fluid viscosity, a_1 is a positive constant, $M=\frac{\sigma B^2}{a\rho}$ is the magnetic field parameter, $Pr=\frac{\upsilon}{\alpha}$ is the Prandtl number, $Du=\frac{D_mK_T}{c_sc_p}\frac{(C_w-C_\infty)}{\alpha(T_w-T_\infty)}$ is the Dufour number, $Nb=\frac{\tau D_B(C_w-C_\infty)}{\alpha}$ is the Brownian motion parameter, $Nt=\frac{\tau D_T(T_w-T_\infty)}{\alpha T_\infty}$ is the thermophoresis parameter, $Sc=\frac{\upsilon}{D_m}$ is the Schmidt number, $Sr=\frac{D_mK_T(T_w-T_\infty)}{\upsilon T_m(C_w-C_\infty)}$ is the Soret number, $Pe=\frac{\tilde{b}W_c}{D_n}$ is the Péclet number, $Lb=\frac{\upsilon}{D_n}$ is the bioconvection Lewis number, $p=\frac{a}{\sqrt{\upsilon a}}(D_1)_0$ represents the thermal slip parameter, $q=\frac{a}{\sqrt{\upsilon a}}(E_1)_0$ represents the mass slip parameter and $r=\frac{a}{\sqrt{\upsilon a}}(F_1)_0$ represents the microorganisms slip parameter.

QUANTITIES OF PRACTICAL INTEREST

The quantities of interest relevant to this investigation are the skin friction coefficient $C_{f_{\bar{x}}}$, the local Nusselt number $Nu_{\bar{x}}$, local Sherwood number $Sh_{\bar{x}}$ and local density number of motile microorganisms $Nn_{\bar{x}}$ which are defined as:

$$C_{f_{\bar{x}}} = \frac{\tau_w}{\rho \bar{u}_w^2}, \quad Nu_{\bar{x}} = \frac{\bar{x} q_w}{k(T_w - T_\infty)}, \quad Sh_{\bar{x}} = \frac{\bar{x} q_m}{D_B(C_w - C_\infty)}, \quad Nn_{\bar{x}} = \frac{\bar{x} q_n}{D_n(N_w - N_\infty)}, \tag{15}$$

where τ_w is the skin friction, q_w is the heat flux, q_m is the mass flux and q_n is the motile microorganism flux and defined by:

$$\tau_w = \mu \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right)_{\bar{y}=0}, q_w = -k \left(\frac{\partial T}{\partial \bar{y}} \right)_{\bar{y}=0}, q_m = -D_B \left(\frac{\partial C}{\partial \bar{y}} \right)_{\bar{y}=0}, q_n = -D_n \left(\frac{\partial N}{\partial \bar{y}} \right)_{\bar{y}=0}. \tag{16}$$

Substitute Eq. (16) and Eq. (8) into Eq. (15), we obtain,

$$C_{f_{\bar{x}}} Re_{\bar{x}}^{-\frac{1}{2}} = f''(0), \quad Nu_{\bar{x}} Re_{\bar{x}}^{\frac{1}{2}} = -\theta'(0), \quad Sh_{\bar{x}} Re_{\bar{x}}^{\frac{1}{2}} = -\phi'(0), \quad Nn_{\bar{x}} Re_{\bar{x}}^{\frac{1}{2}} = -\chi'(0), \tag{17}$$

NUMERICAL SOLUTION

Eqs. (10) - (13) subjected to the boundary conditions in Eq. (14) were solved using the Runge-Kutta-Fehlberg fourth-fifth (RKF45) order numerical scheme. In order to validate our method, we have compared the values of the local Nusselt number with various values of Prandtl number in Table 1 with previous published paper which are Pantokratoras (2008) and Hassan et al. (2015). The result for our present study is found to be in very good agreement.

Table 1: Comparison values of the local Nusselt number, $-\theta'(\eta)$ with various value of Prandtl number, Pr when $M = \beta = Nb = Nt = Du = Sc = Sr = Pe = Lb = p = q = r = 0$, $a_1 = 1$.

Pr	$-\theta'(0)$		
	Pantokratoras (2008)	Hassan et al. (2015)	Present (2016)
0.1	0.0925	0.0924952	0.1004902
0.7	0.4543	0.4539453	0.4539162
1.0	0.5820	0.5819919	0.5819767
10.0	2.3080	2.3080132	2.3080039

RESULTS AND DISCUSSION

Fig. 2 (i) shows that the dimensionless velocity decreasing with the increase of magnetic field parameter, *M*. This is because of the fact that application of transverse magnetic field to a fluid which conducts electricity produces a drag-like force termed the Lorentz force. The Lorentz force retards the force on the velocity field and hence the dimensionless velocity is decreases. Meanwhile, in Fig. 2 (ii), the dimensionless temperature increases as *M* increase and this happen due to the Lorentz force tendency to reduce the fluid velocity within the boundary layer which results in increases of dimensionless temperature. Fig. 2 (iii) and (iv) displayed that both dimensionless nanoparticle volume fraction as well as dimensionless density of motile microorganisms increase with the increases of M respectively.

Fig. 3 (i) express that the dimensionless velocity is increasing as the temperature-dependent fluid viscosity parameter β increasing. As β is increasing, the fluid viscosity decreases which resulting in the increment of the velocity boundary layer thickness. The dimensionless temperature decreases as β increase as shown in Fig. 3 (ii). The increase of β causes the decrease of the thermal boundary layer thickness and hence the dimensionless temperature decrease. The dimensionless nanoparticle volume fraction and dimensionless density of motile microorganisms are decreasing with the increase of β as shown in Fig. 3 (iii) and (iv) respectively.

Fig. 4 shows that as the values of Dufour parameter, Du increasing, the dimensionless temperature is increasing. Thermal boundary layer thickness increases as Du increases.

The effect of Schmidt parameter, Sc on the dimensionless nanoparticle volume fraction is displayed in Fig. 5. As the value of Sc increases, the mass transfer rate also increases and results in the decreasing of dimensionless nanoparticle volume fraction.

Fig. 6 shows the dimensionless nanoparticle volume fraction is increasing when the Soret parameter, Sr is increasing. Increasing Sr results in increases of boundary layer thickness of concentration.

Fig. 7 (i) shows that the dimensionless velocity is increasing with the increase of thermal slip parameter which leads to an increase of the thickness of the thermal boundary layer. On the other hand, in Fig. 7 (ii)-(iv), the dimensionless temperature, dimensionless nanoparticle volume fraction and also dimensionless density of motile microorganisms are found to decrease with the increase of value of thermal slip parameter respectively.

Fig. 8 (i)-(iii) describes the effect of mass slip parameter, q on the dimensionless velocity, temperature, nanoparticle volume fraction and density of motile microorganisms respectively. Both temperature and nanoparticle volume fraction graph are decreasing with the increase of mass slip parameter. This can be seen in Fig. 8 (ii) and (iii). Meanwhile in the Fig. 8 (i) and (iv), the dimensionless velocity and density of motile microorganisms increases as the mass slip parameter increase.

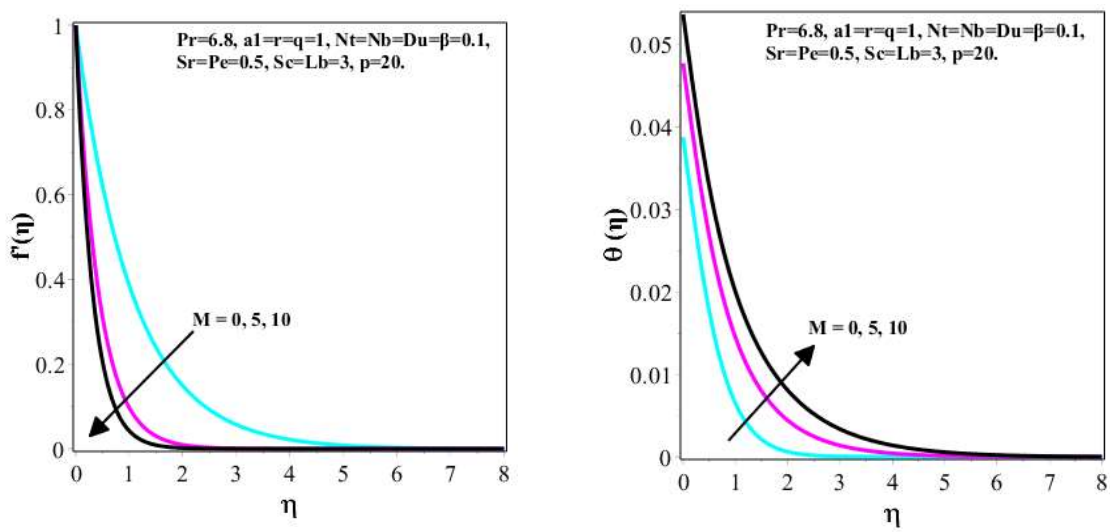
The effect of microorganisms slip parameter, r on the dimensionless density of motile microorganisms is depicted in Fig. 9. As depicted in the Fig. 9, the dimensionless density of motile microorganisms is decreasing with the increase of microorganisms slip parameter.

Fig. 10 displayed the variation of the local skin friction coefficient, $f''(0)$ with Dufour parameter, Du and temperature-dependent fluid viscosity parameter, β along with various values of magnetic field strength parameter, M. It shows that the local skin friction factor is increasing with the increase of Dufour parameter, temperature-dependent fluid viscosity parameter as well as magnetic field strength parameter.

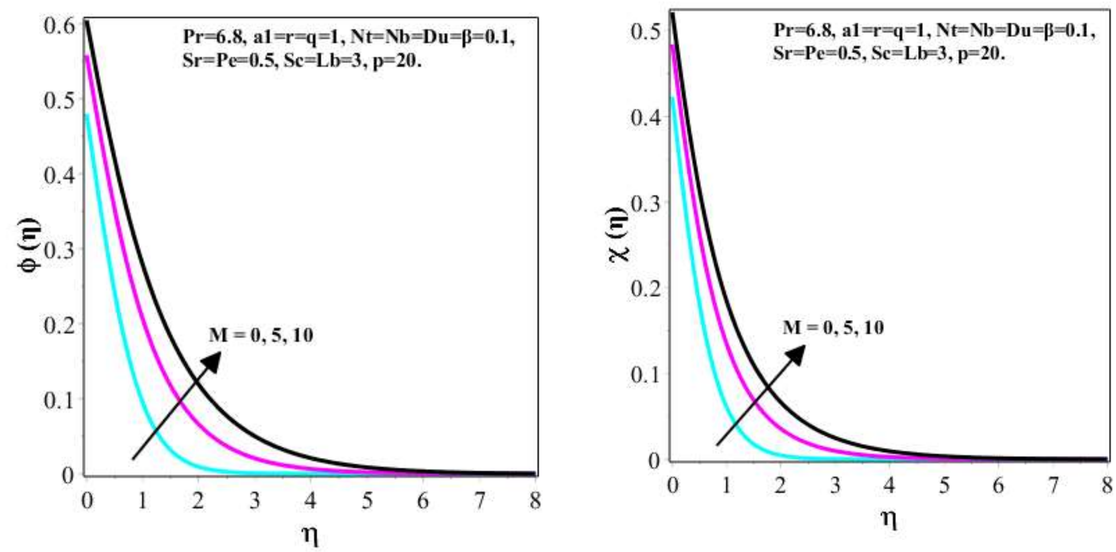
Fig. 11 demonstrates the variation of the local Nusselt number, $-\theta'(0)$ versus Brownian motion parameter, Nb and thermal slip parameter, p with different values of thermophoresis parameter, Nt. From Fig.11, it can be seen that the local Nusselt number decreases as the Brownian motion parameter, thermal slip parameter and thermophoresis parameter increase.

Fig. 12 illustrates graphically the variation of the local Sherwood number, $-\phi'(0)$ with Soret parameter, *Sr* and mass slip parameter, *q* with different values of Schmidt parameter, *Sc*. As shown in the Fig. 12, the local Sherwood number is decreasing with the increase of Soret parameter, Schmidt parameter and mass slip parameter.

Fig. 13 represents the variation of the local density number of motile microorganisms, $-\chi'(0)$ versus bioconvection Lewis parameter, *Lb* and microorganism slip parameter, *r* along with various values of Péclet number, *Pe*. The local density of motile microorganisms is decreases as the bioconvection Lewis parameter, microorganism slip parameter and Péclet parameter increase.

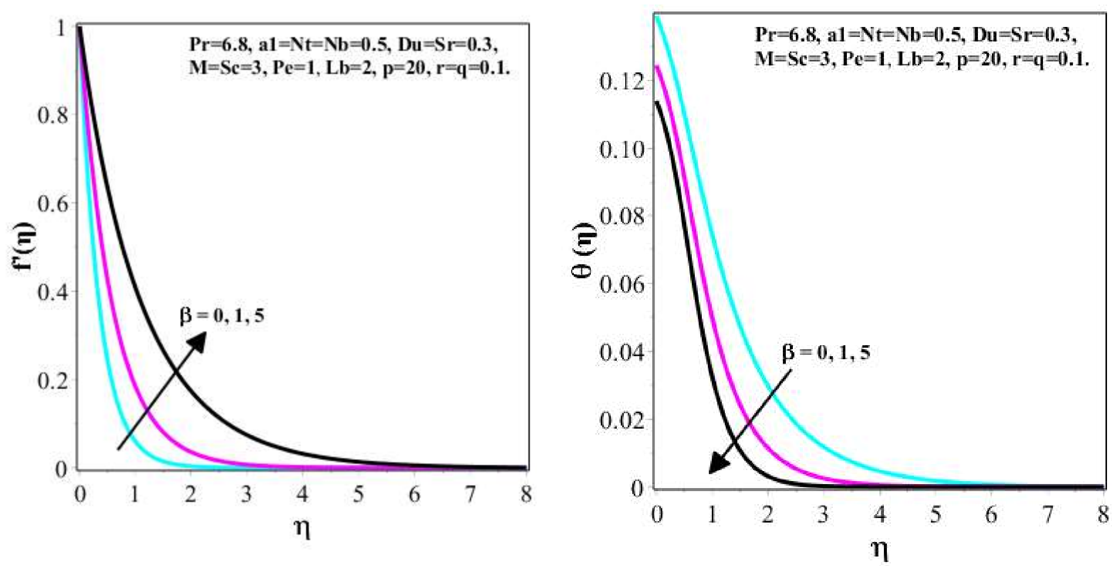


(i) (ii)



(iii) (iv)

Figure 2: Effect of magnetic field strength on the dimensionless (i) velocity, (ii) temperature, (iii) nanoparticle volume fraction and (iv) density of motile microorganism.



(i) (ii)

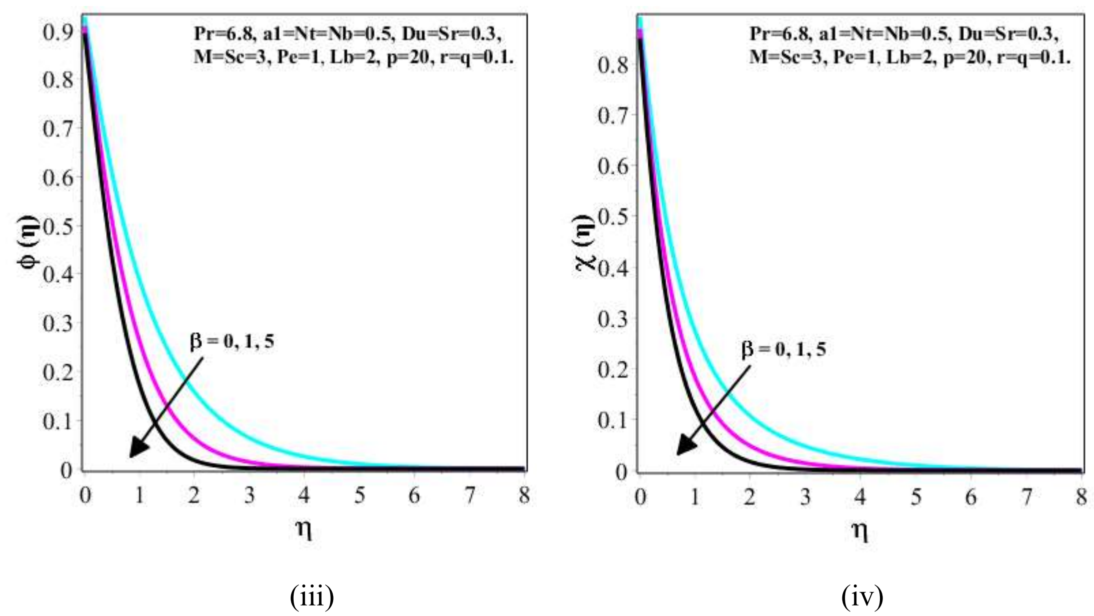


Figure 3: Effect of temperature-dependent fluid viscosity parameter on the dimensionless (i) velocity, (ii) temperature, (iii) nanoparticle volume fraction and (iv) density of motile microorganism.

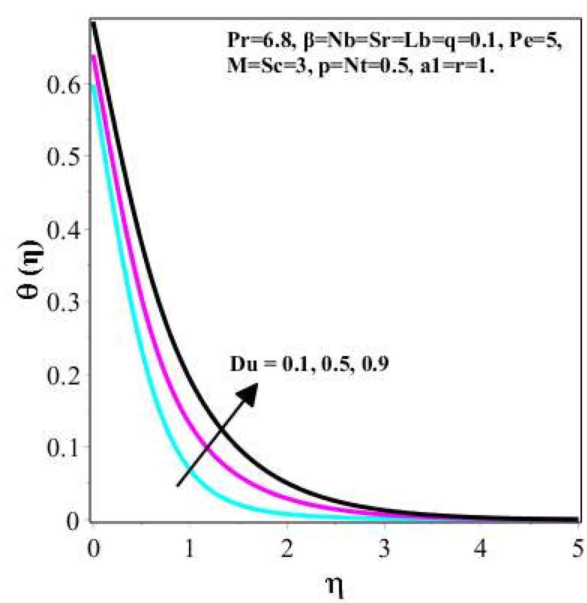


Figure 4: Effect of Dufour parameter on the dimensionless temperature.

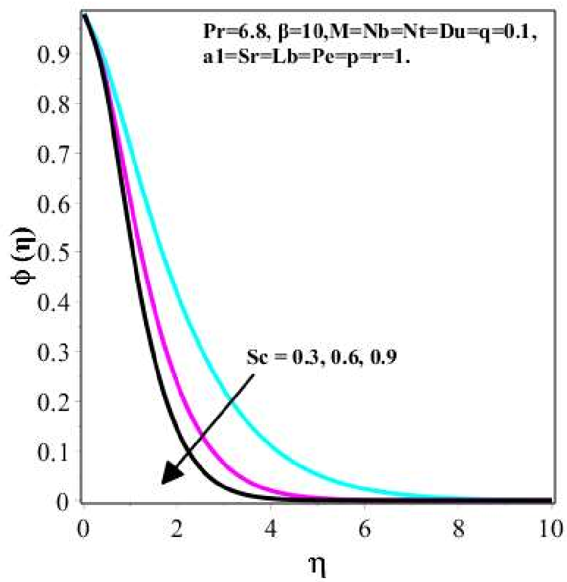


Figure 5: Effect of Schmidt parameter on the dimensionless nanoparticle volume fraction.

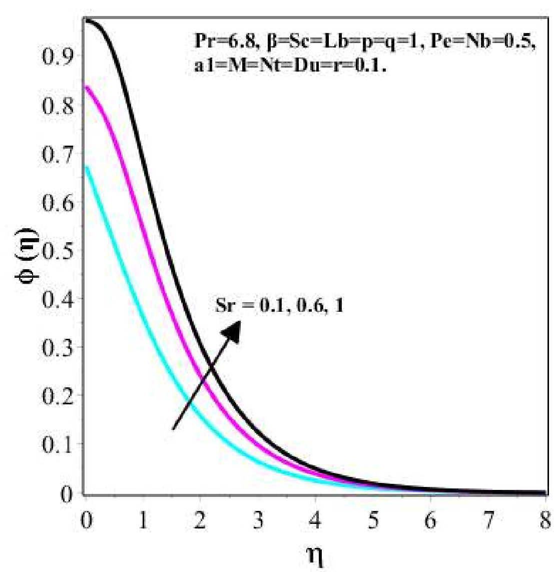


Figure 6: Effect of Soret parameter on the dimensionless nanoparticle volume fraction.

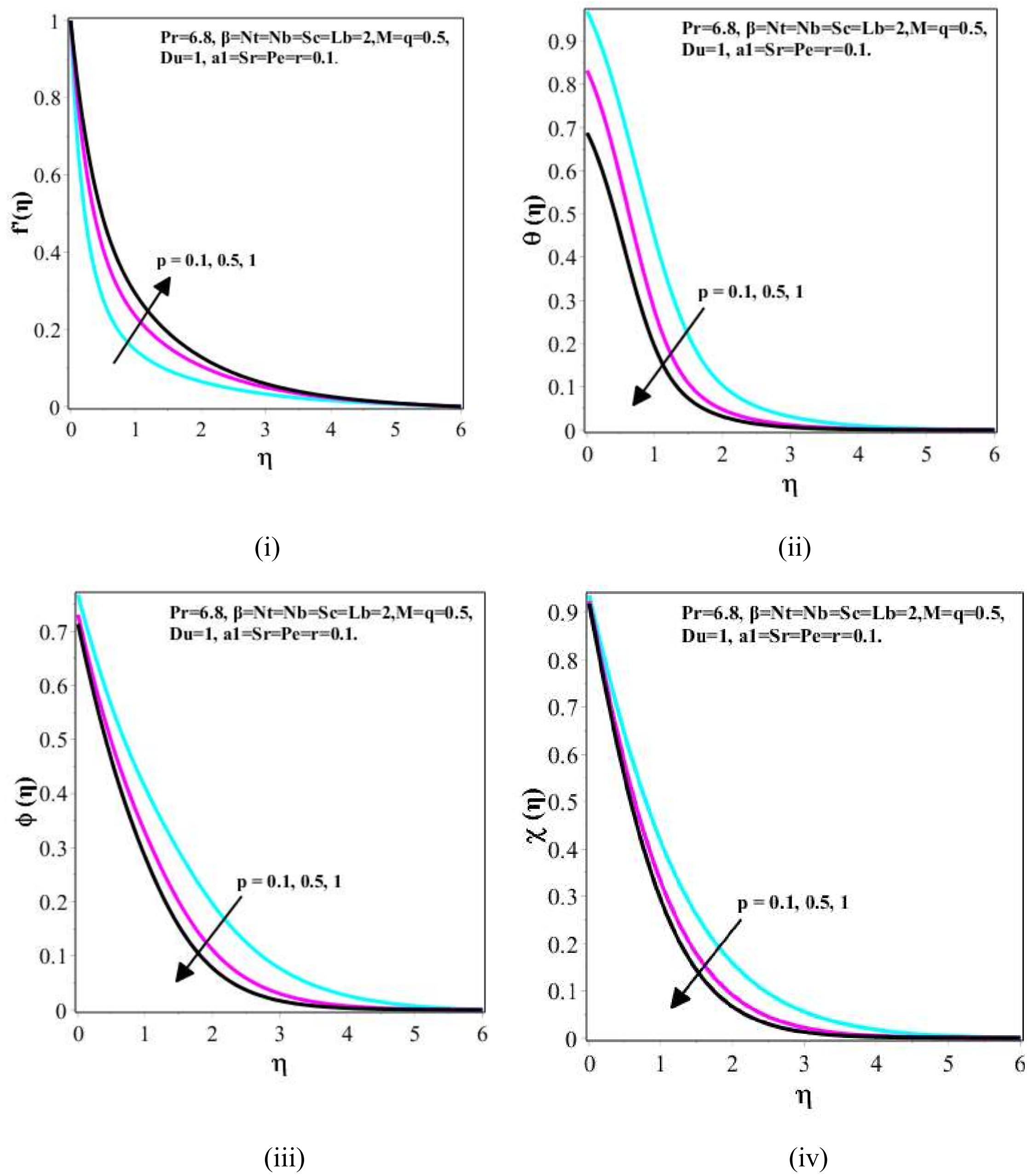


Figure 7: Effect of thermal slip parameter on the dimensionless (i) velocity, (ii) temperature, (iii) nanoparticle volume fraction and (iv) density of motile microorganism.

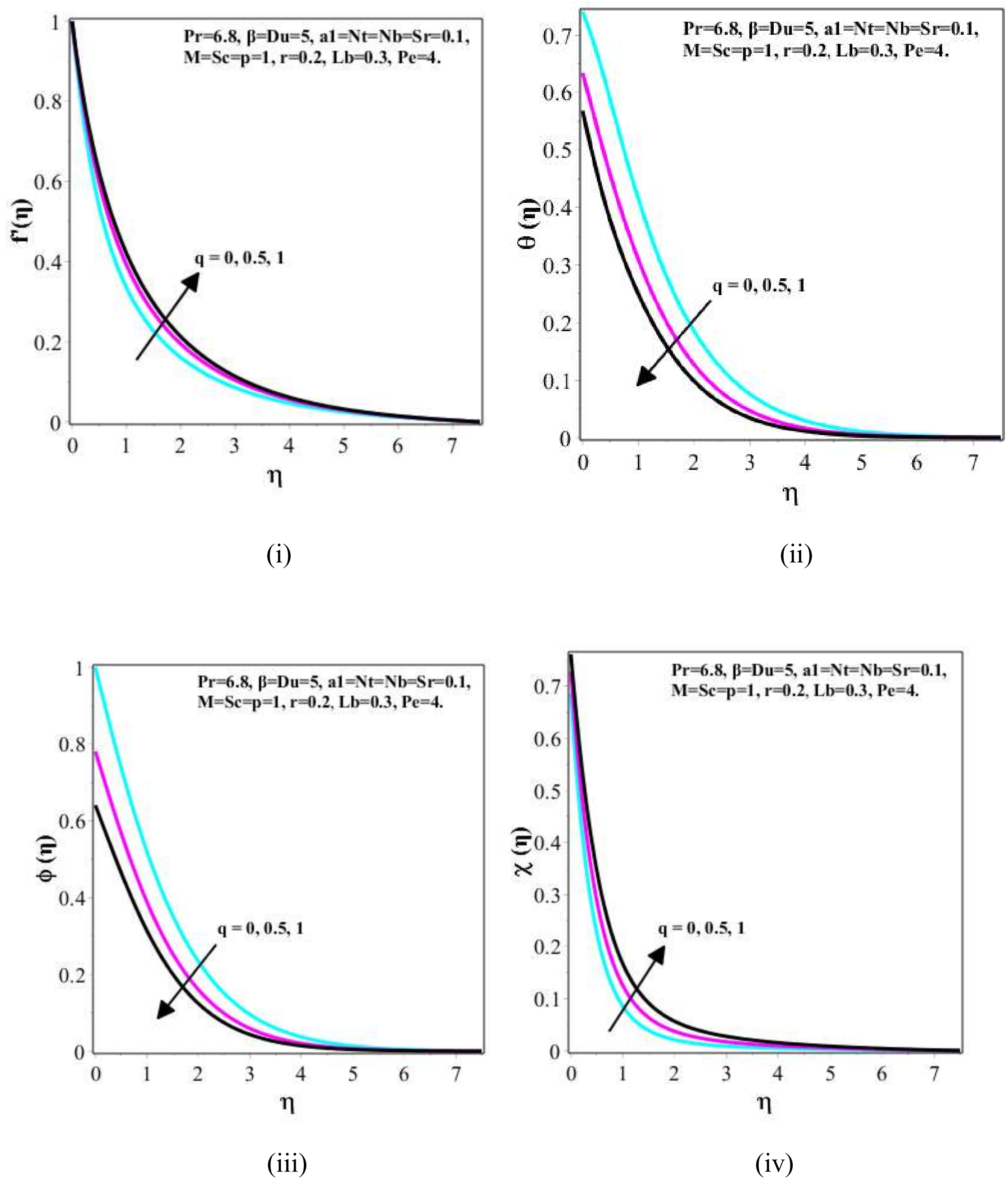


Figure 8: Effect of mass slip parameter on the dimensionless (i) velocity, (ii) temperature, (iii) nanoparticle volume fraction and (iv) density of motile microorganism.

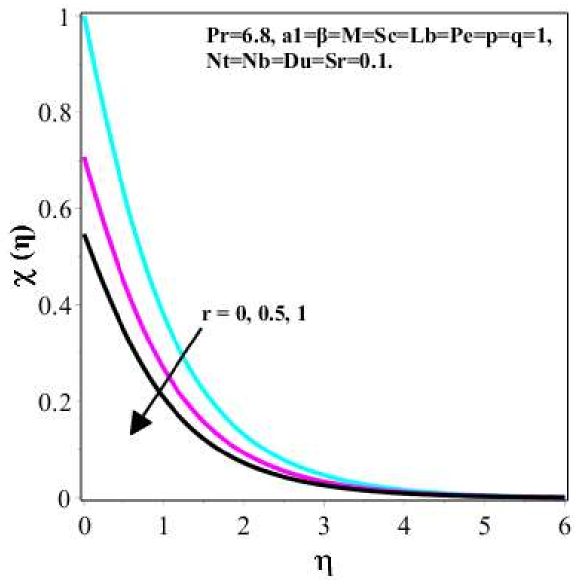


Figure 9: Effect of microorganism slip parameter on the dimensionless density of motile microorganisms.

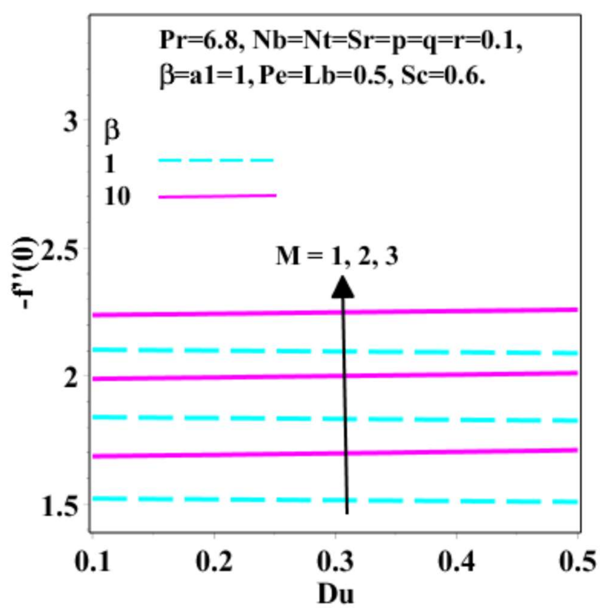


Figure 10: Variation of the local skin friction coefficient with parameter of Dufour and temperature-dependent fluid viscosity along with various values of magnetic field strength.

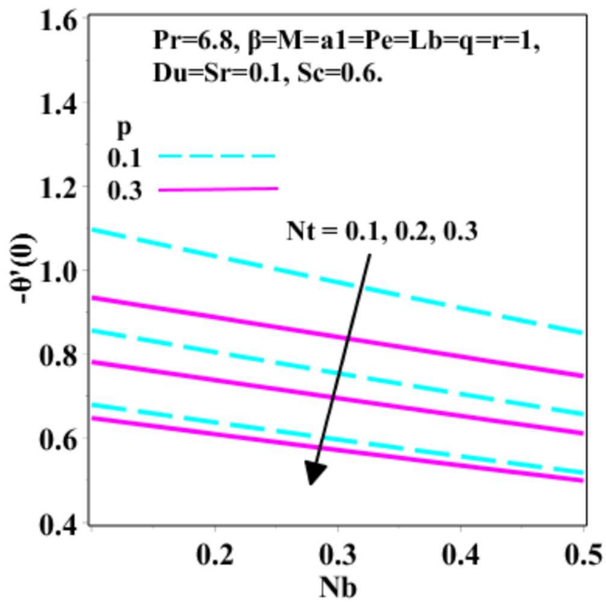


Figure 11: Variation of the local Nusselt number with Brownian motion parameter and thermal slip parameter along with various values of thermophoresis parameter.

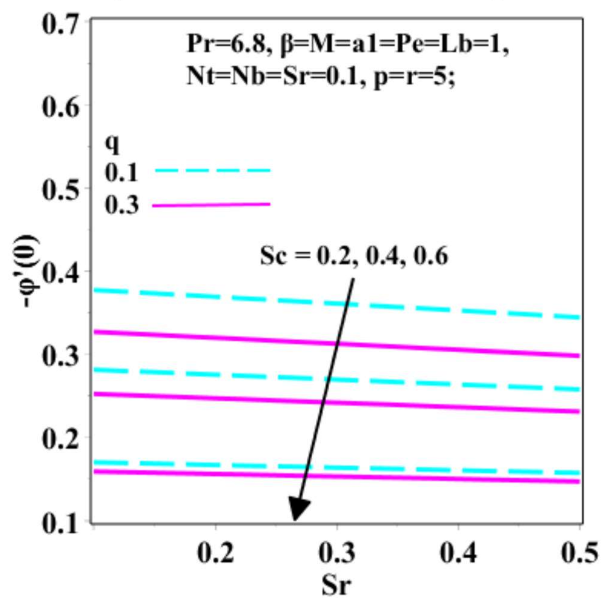


Figure 12: Variation of the local Sherwood number with Soret parameter and mass slip parameter along with various values of Schmidt parameter.

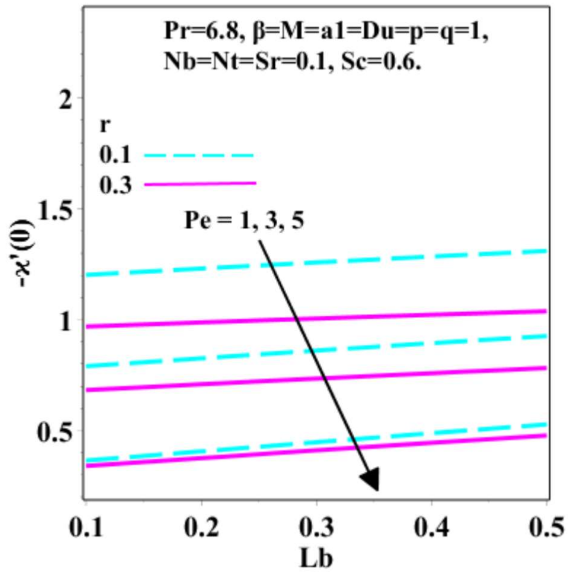


Figure 13: Variation of the local density number of motile microorganism with bioconvection Lewis parameter and microorganism slip parameter along with various values of Péclet parameter.

CONCLUSION

In this paper, a steady two-dimensional incompressible boundary layer slips flow of bionanofluid over a heated stretching sheet with temperature-dependent viscosity in which the effects of Dufour and Soret effects are taken into account has been investigated. The system of non-linear governing partial differential equations is reduced by using similarity transformation. The resulting ODEs are solved by Runge-Kutta Fehlberg fourth-fifth (RKF45) order method. The results of this study showed that the temperature-dependent fluid viscosity plays an important role in shifting the fluid away from the wall. The influences of various governing parameters on dimensionless velocity, dimensionless temperature, dimensionless nanoparticle volume fraction and dimensionless density of motile microorganisms along with local skin friction coefficient, local Nusselt number, local Sherwood number and local density number of motile microorganisms were examined.

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