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One Step Block Method for Solving Third Order Ordinary Differential Equations Directly

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ABSTRACT

The purpose of this research is to discuss a direct two-point one step block method for solving general third-order initial value problems (IVPs) of ordinary differential equations (ODEs) using constant step size. The proposed method will compute the approximation solutions directly without reducing to systems of first order ODEs at two-points in a block simultaneously. Lagrange polynomial has been used to derive the block method. The order, zero stability and consistency of the resulting method also will be discussed. In the numerical results, the method shown to be more accurate and less total function calls compare to the existing method when solving the general third order ODEs.

Keywords: One step method, two-point block, third order ODEs, higher order, direct method

INTRODUCTION

In this study, we considered the direct solving of the general third order initial value problems (IVPs) of ordinary differential equations (ODEs) and the form can be write as shown below:

$$y''' = f(x, y, y', y'') \tag{1}$$

with initial conditions

$$y(x_0) = Y_0 \;\; , y'(x_0) = Y_1, \;\; y''(x_0) = Y_2$$

is solved directly using constant step size. The suggested two-point one step block method has been deployed to solve the third-order problems of ODEs based on the extension method in Majid (2004). This study also considered the utilization of the direct block one step method in solving the IVPs of ODEs, as shown in Eqn.(1). The block method for solving ODEs has been investigated by various researchers such as Shampine and Watts (1969) and; Majid and Suleiman (2007). A direct solution to solve third order of ODEs using block method has been developed by several researchers such as Kuboyo and Omar (2015), Awoyemi (2003) and Obabode (2009). Awoyemi (2003) shown a P-stable linear multistep method for solving general third-order ODEs. An accurate block method for third-order ordinary differential equations has been proposed by Obabode (2009). In addition, Mehrkanoon (2011) has suggested a direct variable step block multistep method for solving third order ODEs without reducing the system to first-order equations. Then, Majid et al.(2012) has proposed a simple form of Adams Moulton method for solving directly the general third order ODEs using a two-point implicit block method using variable step size. Later, Waeleh and Majid (2016) has proposed a four-point block method for solving higher order ordinary differential equations directly.

DERIVATION OF THE FORMULAE

In this section, the interval [a,b] was divided into a series of blocks to evaluate the approximate solution in each block.

In Figure 1, each block has a step size of $2h$ in the interval of [a,b] for the approximation solutions of y_{n+1} and y_{n+2} simultaneously in the block.

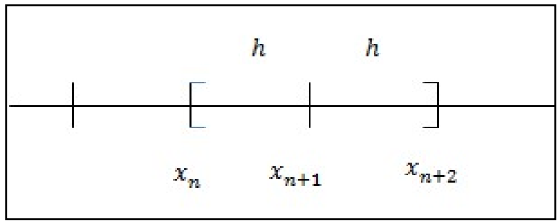

Figure 1: Two point one-step block method

The first corrector formula, y_{n+1} at x_{n+1} was derived by integrating Eqn. (1) once, twice, and thrice over the interval $[x_n, x_{n+1}]$ as follows:

Integrate Once:

$$\int_{x_n}^{x_{n+1}} y''' \, dx = \int_{x_n}^{x_{n+1}} f(x, y, y', y'')dx. \tag{2}$$

Integrate Twice:

$$\int_{x_n}^{x_{n+1}} \int_{x_n}^{x} y''' dxdx = \int_{x_n}^{x_{n+1}} \int_{x_n}^{x} f(x, y, y', y'')dxdx. \tag{3}$$

Integrate Thrice:

$$\int_{x_n}^{x_{n+1}} \int_{x_n}^{x} \int_{x_n}^{x} y''' dxdxdx = \int_{x_n}^{x_{n+1}} \int_{x_n}^{x} \int_{x_n}^{x} f(x, y, y', y'')dxdxdx. \tag{4}$$

The function $f(x, y, y', y'')$ in (2) – (4) will be substitute with the Lagrange interpolation polynomial defined as $P_2(x)$. Furthermore, the interpolation points involved were $(x_{n+2}, f_{n+2}), (x_{n+1}, f_{n+1})$ and (x_n, f_n) as follows:

$$P_2(x) = \frac{(x - x_n)(x - x_{n+1})}{(x_n - x_{n+1})(x_n - x_{n+2})} f_n + \frac{(x - x_n)(x - x_{n+2})}{(x_{n+1} - x_n)(x_{n+1} - x_{n+2})} f_{n+1}$$
$$+ \frac{(x - x_n)(x - x_{n+1})}{(x_{n+2} - x_n)(x_{n+2} - x_{n+1})} f_{n+2}. \tag{5}$$

Let $x = x_{n+1} + sh$ and $s = \dfrac{x - x_{n+1}}{h}$ and replacing $dx = hds$; then changing the limit of integration from -2 to -1 in Eqn. (2) – (4). We will obtained the first formula of the two-point one step block method as follows:

$$y''(x_{n+1}) = y''(x_n) + \frac{h}{12}(5f_n + 8f_{n+1} - f_{n+2})$$

$$y'(x_{n+1}) = y'(x_n) + hy''(x_n) + \frac{h^2}{24}(7f_n + 6f_{n+1} - f_{n+2})$$

$$y(x_{n+1}) = y(x_n) + hy'(x_n) + \frac{h^2}{2!}y''(x_n) + \frac{h^3}{240}(27f_n + 16f_{n+1} - 3f_{n+2}) \tag{6}$$

In order to obtain the second point, y_{n+2}, we need to integrating Eqn (1) once, twice, and thrice over the interval of $[x_{n+1}, x_{n+2}]$ to obtain the approximate solutions of the second point y''_{n+2}, y'_{n+2} and y_{n+2} as follows:

Integrate once:

$$\int_{x_{n+1}}^{x_{n+2}} y''' \, dx = \int_{x_{n+1}}^{x_{n+2}} f(x, y, y', y'') dx \tag{7}$$

Integrate twice:

$$\int_{x_{n+1}}^{x_{n+2}} \int_{x_{n+1}}^{x} y''' dxdx = \int_{x_{n+1}}^{x_{n+2}} \int_{x_{n+1}}^{x} f(x, y, y', y'') dxdx \tag{8}$$

Integrate thrice:

$$\int_{x_{n+1}}^{x_{n+2}} \int_{x_{n+1}}^{x} \int_{x_{n+1}}^{x} y''' dxdxdx = \int_{x_{n+1}}^{x_{n+2}} \int_{x_{n+1}}^{x} \int_{x_{n+1}}^{x} f(x, y, y', y'') dxdxdx \tag{9}$$

The function $f(x, y, y', y'')$ in Eqn. (7) – (9) will be substitute with the Lagrange interpolation polynomial as in (5). Furthermore, the interpolation points involved were $(x_{n+2}, f_{n+2}), (x_{n+1}, f_{n+1})$ and (x_n, f_n) as earlier described. The limit of integration was from -1 to 0 and the following formula were obtained:

$$\begin{aligned}
y''(x_{n+2}) &= y''(x_{n+1}) + \frac{h}{12}(-f_n + 8f_{n+1} + 5f_{n+2}) \\
y'(x_{n+2}) &= y'(x_{n+1}) + hy''(x_n + 1) + \frac{h^2}{24}(-f_n + 10f_{n+1} + 3f_{n+2}) \\
y(x_{n+2}) &= y(x_{n+1}) + hy'(x_{n+1}) + \frac{h^2}{2!}y''(x_{n+1}) + \frac{h^3}{240}(-3f_n + 36f_{n+1} + 7f_{n+2})
\end{aligned} \tag{10}$$

ORDER OF THE METHOD

The matrix form of the two-point one-step block formula is given in Eqn. (6) and (10) as follows:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} y_{n-3} \\ y_{n-2} \\ y_{n-1} \\ y_n \\ y_{n+1} \\ y_{n+2} \end{bmatrix} = h \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y'_{n-3} \\ y'_{n-2} \\ y'_{n-1} \\ y'_n \\ y'_{n+1} \\ y'_{n+2} \end{bmatrix}$$

$$+h^2\begin{bmatrix} 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \end{bmatrix}\begin{bmatrix} y''_{n-3} \\ y''_{n-2} \\ y''_{n-1} \\ y''_{n} \\ y''_{n+1} \\ y''_{n+2} \end{bmatrix} + h^3\begin{bmatrix} 0 & 0 & 0 & \frac{5}{12} & \frac{8}{12} & \frac{-1}{12} \\ 0 & 0 & 0 & \frac{7}{24} & \frac{6}{24} & \frac{-1}{24} \\ 0 & 0 & 0 & \frac{27}{240} & \frac{16}{240} & \frac{-3}{240} \\ 0 & 0 & 0 & \frac{-1}{12} & \frac{8}{12} & \frac{5}{12} \\ 0 & 0 & 0 & \frac{-1}{24} & \frac{10}{24} & \frac{3}{24} \\ 0 & 0 & 0 & \frac{-3}{240} & \frac{36}{240} & \frac{7}{240} \end{bmatrix}\begin{bmatrix} f_{n-3} \\ f_{n-2} \\ f_{n-1} \\ f_{n} \\ f_{n+1} \\ f_{n+2} \end{bmatrix}.$$

In this part, the order of the two-point one step block method derived in the previous section will be obtained. The approach to compute the order and error constants of the numerical method can be referred in Lambert (1991). The order of the two-point one step block method was investigated by extending the idea of Lambert (1991) and applying the technique to obtain the order and error constants for general third order ODEs. The linear difference operator, L, associated with the linear multistep method is defined by:

$$L[y(x);h]=\sum(\alpha_j y(x+jh)-h\beta_j y'(x+jh))-\ldots-h^d\sum(\alpha_j y^d(x+jh), \tag{11}$$

where $y(x)$ is an arbitrary function and continuously differentiable on $[a,b]$. Expanding the test function $y(x+jh)$ and its derivative $y'(x+jh)$ as Taylor series about x, and collecting terms in will give:

$$L[y(x);h]=C_0 y(x)+C_1 h y^1(x)+\ldots+C_q h^q y^q(x)+\ldots, \tag{12}$$

where C_q are constants. The difference operator in (6) and (10); and the associated multistep method are said to be order p if $C_0 = C_1 = \ldots = C_p = 0$ and $C_{p+3} \neq 0$.

$$C_0 = \sum_{j=0}^{k}\alpha_j,$$

$$C_1 = \sum_{j=0}^{k}(j\alpha_j - \beta_j),$$

$$C_2 = \sum_{j=0}^{k}(\frac{j^2}{2!}\alpha_j - j\beta_j - \gamma_j),$$

$$C_3 = \sum_{j=0}^{k}(\frac{j^3}{3!}\alpha_j - \frac{j^2}{2!}\beta_j - j\gamma_j - \delta_j),$$

$$\vdots$$

$$C_q = \sum_{j=0}^{k}(\frac{j^q}{q!}\alpha_j - \frac{j^{q-1}}{(q-1)!}\beta_j - \frac{j^{q-2}}{(q-2)!}\gamma_j - \frac{j^{q-3}}{(q-3)!}\delta_j), \tag{13}$$

Therefore, the order and error constant of the two-point one-step block method can be obtained by using Eqn. (13) and obtained as follows,

$C_0 = [0\ 0\ 0\ 0\ 0\ 0\ 0]^T$.

$C_1 = [0\ 0\ 0\ 0\ 0\ 0\ 0]^T$.

$\vdots$

$\vdots$

$$C_6 = \sum_{j=0}^{5}\left(\frac{1}{6!}j^6\alpha_j - \frac{1}{5!}j^5\beta_j - \frac{1}{4!}j^4\gamma_j - \frac{1}{3!}j^3\delta_j\right) = \left[\frac{1}{24}\ \frac{1}{45}\ \frac{1}{144}\ \frac{-1}{24}\ \frac{-7}{360}\ \frac{-1}{180}\right]^T.$$

According to Lambert (1993), a method is order p, if $C_0 = C_1 = \ldots = C_{p+2} = 0$, and $C_{p+3} \neq 0$ is the error constant. Hence,

$C_6 = C_{p+3} = \left[\frac{1}{24}, \frac{1}{45}, \frac{1}{144}, \frac{-1}{24}, \frac{-7}{360}, \frac{-1}{180}\right]$. The proposed method is of order 3.

STABILITY ANALYSIS

Zero stability

In this part, the zero stability of the two-point one step block method will be discussed. The corrector formulae in (6) and (10) were considered. The method is said to have a zero stability if the first characteristic polynomial $\rho(R)$ is defined as:

$$\rho(R) = \det\left[RA^0 - RA^1\right] = 0$$

where

$$A^0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, A^1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\rho(R) = \det\begin{bmatrix} R-1 & 0 & 0 & 0 & 0 & 0 \\ 0 & R-1 & 0 & 0 & 0 & 0 \\ 0 & 0 & R-1 & 0 & 0 & 0 \\ -1 & 0 & 0 & R & 0 & 0 \\ 0 & -1 & 0 & 0 & R & 0 \\ 0 & 0 & -1 & 0 & 0 & R \end{bmatrix}$$

$$= R^3.(R-1)^3 = 0,0,0,1,1,1; \quad |R| \leq 1.$$

Hence, the two-point one step block method is zero stable.

Consistency

The method in (6) and (10) is said to be consistent if the method is order p where $C_0 = C_1 = \ldots = C_{p+2} = 0$ and $C_{p+3} \neq .0$

Hence, the proposed method is consistent as it has order 3.

Convergence

Theorem 1 : Zero stability and consistency are sufficient conditions for a linear multistep method to be convergent.

Hence, the proposed two-point one step block is zero stable and consistent, as a result, the proposed block method can be said to be convergent.

NUMERICAL RESULTS

The following test problems were numerically solved to illustrate the efficiency of the two-point one step block method

Problem 1: $y''' = -y'$
$y(0) = 0,\ y'(0) = 1,\ y''(0) = 2,\ [0,1]$
Exact solution: $y(x) = 2(1 - \cos x) + \sin x.$

Problem 2: $y''' = -2y'' + 9y' + 18y - 18x^2 - 18x + 22$
$y(0) = -2, y'(0) = -8, y''(0) = -12\ ,[0,1]$
Exact solution: $y(x) = -2e^{3x} + e^{-2x} + x^2 - 1.$

Problem 3: $y''' = 8y' - 3y - 4e^x$
$y(0) = 2,\ y'(0) = -2,\ y''(0) = 10,\ [0,1]$
Exact solution: $y(x) = e^x + e^{-3x}.$

Problem 4: $y''' = 3\sin x$
$y(0) = 1,\ y'(0) = 0,\ y''(0) = -2\ ,[0,1]$
Exact solution $y(x) = 3\cos x + \frac{x^2}{2} - 2.$

Problem 5: $y''' = -4y'' + x$
$y(0) = 0,\ y'(0) = 0,\ y''(0) = 1,\ [0,1]$
Exact solution: $y(x) = \left(\frac{3}{16}\right)(1 - \cos 2x) + \left(\frac{1}{8}\right)x^2.$

The following notations were used in the tables presented in this section.

h	Step size
TS	Total of steps
TFC	Total function calls
TIME	Timing in seconds
RK3	Runge-Kutta method of order three
2PD	Two-point one step block method of order three proposed in this research
1.19 e-08	1.19×10^{-8}

Table 1: Comparison of the errors for 2PD and RK3 when solving problem 1 at different values of *h*

x	h=0.1		h=0.05		h=0.01	
	2PD	RK3	2PD	RK3	2PD	RK3
0.1	1.19 e-08	8.24 e-06	1.22 e-09	5.18 e-07	1.97 e-02	7.91 e-09
0.2	6.84 e-08	1.55 e-05	4.32 e-09	1.46 e-06	1.97 e-12	7.91 e-09
0.3	1.30 e-07	2.16 e-05	8.51 e-09	2.27 e-06	1.36 e-11	2.03 e-08
0.4	2.09 e-07	2.63 e-05	1.32 e-08	2.91 e-06	2.12 e-11	2.81 e-08
0.5	2.82 e-07	3.60 e-05	1.79 e-08	4.01 e-06	2.90 e-11	3.80 e-08
0.6	3.57 e-07	4.65 e-05	2.26 e-08	5.29 e-06	3.65 e-11	4.86 e-08
0.7	4.22 e-07	5.76 e-05	2.26 e-08	5.29 e-06	4.34 e-11	5.96 e-08
0.8	4.83 e-07	6.90 e-05	4.83 e-08	8.05 e-06	4.97 e-11	7.08 e-08
0.9	5.33 e-07	8.04 e-05	3.40 e-08	9.46 e-06	5.52 e-11	8.19 e-08
1.0	5.77 e-07	1.01 e-05	3.68 e-08	1.08 e-05	5.99 e-11	9.15 e-08

Table 2: Comparison of the errors for 2PD and RK3 when solving problem 2 at different values of *h*

x	h=0.1		h=0.05		h=0.01	
	2PD	RK3	2PD	RK3	2PD	RK3
0.1	8.18 e-08	6.53 e-04	8.24 e-09	9.33 e-05	1.31 e-11	8.29 e-07
0.2	5.10 e-07	1.82 e-03	3.16 e-08	2.58 e-04	5.09 e-11	2.29 e-06
0.3	1.03 e-06	3.76 e-03	6.70 e-08	5.33 e-04	1.08 e-10	4.71 e-06
0.4	1.74 e-06	6.86 e-03	1.09 e-07	9.70 e-04	2.54 e-10	1.45 e-05
0.5	2.40 e-06	1.16 e-02	1.54 e-07	1.64 e-03	2.54 e-10	1.45 e-05
0.6	3.06 e-06	1.90 e-02	1.97 e-07	2.68 e-03	3.65 e-11	4.86 e-08
0.7	3.57 e-06	3.00 e-02	2.34 e-07	4.23 e-03	3.92 e-10	3.73 e-05
0.8	1.93 e-06	4.64 e-02	2.61 e-07	6.54 e-03	4.42 e-10	5.76 e-05
0.9	4.09 e-06	7.06 e-02	2.76 e-07	9.95 e-03	4.74 e-10	8.76 e-05
1.0	4.02 e-06	1.05 e-01	2.79 e-07	1.49 e-02	4.86 e-10	1.31 e-04

Table 3: Comparison of the errors for 2PD and RK3 when solving problem 3 at different values of h

x	h=0.1		h=0.05		h=0.01	
	2PD	RK3	2PD	RK3	2PD	RK3
0.1	2.09 e-08	3.22 e-04	2.10 e-09	3.58 e-05	1.36 e-12	2.61 e-07
0.2	1.38 e-07	4.81 e-04	8.65 e-09	5.36 e-05	1.38 e-11	3.92 e-07
0.3	3.13 e-07	5.43 e-04	2.04 e-08	6.07 e-05	3.27 e-11	4.44 e-07
0.4	6.25 e-07	5.49 e-04	3.91 e-08	6.16 e-05	6.26 e-11	4.54 e-07
0.5	1.06 e-06	5.27 e-04	6.76 e-08	5.97 e-05	1.08 e-10	4.42 e-07
0.6	1.78 e-06	4.96 e-04	1.11 e-07	5.68 e-05	1.78 e-10	4.25 e-07
0.7	2.91 e-06	4.68 e-04	1.83 e-07	5.44 e-05	2.93 e-10	4.13 e-07
0.8	4.99 e-06	4.51 e-04	3.12 e-07	5.35 e-05	4.99 e-10	4.13 e-07
0.9	9.47 e-06	4.50 e-04	5.94 e-07	5.48 e-05	9.51 e-10	4.31 e-07
1.0	2.58 e-05	4.72 e-04	1.61 e-06	5.89 e-05	2.59 e-09	4.73 e-07

Table 4: Comparison of the errors for 2PD and RK3 when solving problem 4 at different values of h

x	h=0.1		h=0.05		h=0.01	
	2PD	RK3	2PD	RK3	2PD	RK3
0.1	4.15 e-07	1.24 e-05	3.95 e-08	1.56 e-06	6.21 e-11	1.24 e-08
0.2	1.20 e-06	2.48 e-05	7.31 e-08	3.10 e-06	1.15 e-10	2.47 e-08
0.3	1.57 e-06	3.69 e-05	1.02 e-07	4.60 e-06	1.62 e-10	3.67 e-08
0.4	2.06 e-06	4.86 e-05	1.27 e-07	6.04 e-06	2.02 e-10	4.81 e-08
0.5	2.34 e-06	5.96 e-05	1.48 e-07	7.40 e-06	2.36 e-10	5.88 e-08
0.6	2.69 e-06	6.97 e-05	1.67 e-07	8.65 e-06	2.66 e-10	6.87 e-08
0.7	2.90 e-06	7.90 e-05	1.82 e-07	9.78 e-06	2.91 e-10	7.75 e-08
0.8	3.16 e-06	8.71 e-05	1.96 e-07	1.07 e-05	3.13 e-10	8.52 e-08
0.9	3.31 e-06	9.40 e-05	2.07 e-07	1.15 e-05	3.31 e-10	9.16 e-08
1.0	3.50 e-06	9.95 e-05	2.17 e-07	1.22 e-05	3.47 e-10	9.66 e-08

Table 5: Comparison of the errors for 2PD and RK3 when solving problem 5 at different values of *h*

x	h=0.1		h=0.05		h=0.01	
	2PD	RK3	2PD	RK3	2PD	RK3
0.1	1.63 e-05	1.24 e-05	8.24 e-09	1.56 e-06	1.31 e-11	1.22 e-08
0.2	2.61 e-05	2.42 e-05	3.16 e-08	3.10 e-06	5.09 e-11	2.31 e-08
0.3	2.47 e-05	3.39 e-05	6.70 e-08	4.60 e-06	1.08 e-10	3.12 e-08
0.4	2.50 e-05	4.00 e-05	1.09 e-07	6.04 e-06	1.78 e-10	3.54 e-08
0.5	2.40 e-05	4.16 e-05	1.54 e-07	7.40 e-06	2.54 e-10	3.46e-08
0.6	2.33 e-05	3.79 e-05	1.97 e-07	8.65 e-06	3.27 e-10	2.82 e-08
0.7	2.21 e-05	2.83 e-05	2.34 e-07	9.78 e-06	3.92 e-10	1.62 e-08
0.8	2.08 e-05	1.30 e-05	2.61 e-07	1.07 e-05	4.42 e-10	1.32 e-09
0.9	1.93 e-05	7.77 e-05	2.76 e-07	1.15 e-05	4.74 e-10	2.38 e-08
1.0	1.76 e-05	3.32 e-05	2.79 e-07	1.22 e-05	4.86 e-10	5.01 e-08

Table 6: Comparison between 2PD and RK3 when solving problems 1-5

	h=0.1			h=0.05			h=0.01		
Problem 1 - 5	**TS**	**TFC**		**TS**	**TFC**		**TS**	**TFC**	
2PD	5	40		10	80		50	180	
RK3	10	99		20	180		100	900	

DISCUSSION

The performance of the proposed two-point one step block method is shown by solving different test problems and the numerical results are compared with the RK3 where RK3 need to reduce the same problems to a system of first order ODEs. The 2PD method requires less number of function calls as compared with the existing method RK3. This is because of the fact that when the problems are converted to a system of the first order ODEs, the number of equations increased three times. In Tables 1 - 5, the results presented at the step sizes of 0.1, 0.05, and 0.01 for all the test problems. In Table 6, the total step size and function calls in the proposed method was calculated for all the test problems and the number of total steps taken by the proposed method is lesser than RK3 at all test problems. This is expected because of the proposed methods are block method which computed the approximation at two points. It can be concluded from the computational results that the proposed methods performed better in terms of accuracy than the RK3 when computing the same test problems. As a result, the proposed methods are suitable for the direct solutions of the third order ODEs.

CONCLUSION

In the present paper, we have proposed a direct two-point one step block method to solve the general third-order ODEs directly using constant step size. Numerical results show that the proposed method performed better in terms of accuracy, total steps and total function calls compared to RK3. Hence, the proposed method is suitable for solving the general third order ODEs directly.

REFERENCES

Awoyemi, D. O. (2003). A P-stable linear multistep method for solving general third order ordinary differential equations. International Journal of Computer Mathematics, 80(8), 985-991.

Kuboye, J. O. and Omar, Z. (2015). Numerical solution of third order ordinary differential equations using a seven-step block method. International Journal of Mathematical Analysis, 9(15), 743-745.

Lambert, J. D. (1991). Numerical methods for ordinary differential systems: the initial value problem, New York: John Wiley & Sons, Inc.

Majid, Z. A. (2004). Parallel block methods for solving ordinary differential equations. Doctoral dissertation, Universiti Putra Malaysia, Malaysia.

Majid, Z. A. and Suleiman, M. B. (2007). Implementation of four-point fully implicit block method for solving ordinary differential equations. Applied Mathematics and Computation, 184(2), 514-522.

Majid, Z. A., Azmi, N. A., Suleiman, M. and Ibrahaim, Z. B. (2012). Solving directly general third order ordinary differential equations using two-point four step block method. Sains Malaysiana, 41(5), 623-632.

Mehrkanoon, S. (2011). A direct variable step block multistep method for solving general third-order ODEs. Numerical Algorithms, 57(1), 53-66.

Olabode, B. T. (2009). An accurate scheme by block method for the third order ordinary differential equation. Pacific journal of science and technology, 10(1), 136-142.

Shampine, L. F., Watts, H. A. (1969). Block implicit one-step methods. Mathematics of Computation, 23(108), 731-740.

Waeleh, N. and Abdul Majid, Z. (2016). A 4-point block method for solving higher order ordinary differential equations directly. International Journal of Mathematics and Mathematical Sciences.