NETWORK TOPOLOGY AND ULTRAMETRICITY IN FINANCIAL MARKETS

Abstract

Minimal spanning tree (MST) and Sub-dominant ultrametric (SDU) are one of the principal mainstreams in financial markets analysis. We show that the use of MST to filter important information in a complex system of financial market's commodities is not robust except when the system consists of a unique MST. In this talk we propose to use the forest of all MSTs as a robust filter. For that purpose, an algorithm to construct that forest will be provided. Our approach is based on the notion of min-max transitive closure of a fuzzy relation to find the SDU. The result is in the form of five theorems. To illustrate the advantages of the forest, the analysis results of NYSE 100 stocks will be presented.

Introduction

- Financial market’s commodities
- UVTS or MVTS representation of each commodity
- Similarities among commodities
- Complex system analysis of similarities

Introduction

- Similarity depends on the nature of TS: Mathematical law versus support

- In general

		SUPPORT	
		Same	Different
LAW	GBM	PCC	*
	Non-GBM	*	*

* : other similarity measures e.g. DTW and Detrended similarity

Complex system analysis

THE TWO PRINCIPAL MAINSTREAMS

- RMT Approach (Wigner, 1951)
  This approach focus on PCC

- MST and SDU Approach (Mantegna, 1991)
  This approach is not limited to PCC but also other similarity measures

Complex system analysis

MST AND SDU APPROACH

- Analysis of complex system of dissimilarities
- MST is to filter the economic information in terms of topological properties of commodities
- SDU is to cluster similar commodities in the form of hierarchical tree

Problem

- MST is not robust except when the complex system consists of unique MST
- Therefore, the economic interpretation could be misleading
- How to find ROBUST FILTER?

Proposal

USE THE FOREST OF ALL POSSIBLE MSTs
AS A ROBUST FILTER

Results

Let D be a complex system representing dissimilarities among n commodities. It is thus a matrix where its i-th row and j-th column is the dissimilarity $d(i,j)$ for all $i, j = 1, 2, \ldots, n$. From fuzzy relation viewpoint, D is a symmetric and anti-reflexive fuzzy relation with d as the membership function [1].

We have proved the following theorems [2,3].

Results

Theorem 1. Consider k times min-max transitive operation '*' on D and itself,

$$D^{*k} = D * D^{*(k-1)} \text{ for all } k = 2, 3, \ldots$$

where

(i) $D^{*1} = D$,

(ii) the membership function d^{*k} in D^{*k} is

$$d^{*k}(i,j) = \bigwedge_{m=1}^{n} \left\{ d(i,m) \vee d^{*(k-1)}(m,j) \right\},$$

(iii) $a \wedge b = \min\{a, b\}$ and $a \vee b = \max\{a, b\}$ for all real numbers a and b,

Then, the sequence D, D^{*2}, D^{*3}, …, D^{*k}, … is monotone decreasing, i.e.,

$$\ldots\ldots \subseteq D^{*k} \subseteq \ldots\ldots \subseteq D^{*3} \subseteq D^{*2} \subseteq D.$$

Results

Corollary.

The min-max transitive closure D^+ of D, defined in [1] as

$$D^+ = D \cap D^{*2} \cap D^{*3} \cap \ldots\ldots \cap D^{*k},$$

is simply $D^+ = D^{*k}$.

Results

Theorem 2. Let E be the set of all commodities and d^+ be the membership function of D^+. Then, $d^+(i, j) = \bigwedge_{\gamma \text{ in } \Gamma} L(\gamma)$ for all pairs (i, j) in E × E, where

(i) $\Gamma = \{\gamma \mid \gamma = (i = i_1, i_2, \ldots\ldots, i_p = j) \text{ is a chain from } i \text{ to } j\}$,

(ii) $L(\gamma) = \bigvee_{k=1}^{p-1} d(i_k, i_{k+1})$.

Theorem 3. D^+ is the SDU of D.

Results

Theorem 4. Let M be a MST of D and $(i = i_1, i_2, \ldots, i_p = j)$ be the chain from i to j in M and

$$\hat{d}(i,j) = \bigwedge_{\gamma \text{ in } \Gamma} \bigvee_{k=1}^{p-1} d(i_k, i_{k+1})$$

If Δ is a fuzzy relation where its membership function

$$\delta(i,j) = \begin{cases} 1; \, d(i,j) - d^{+}(i,j) = 0 \text{ and } i \neq j \\ 0; \, d(i,j) - d^{+}(i,j) \neq 0 \text{ or } i = j \end{cases},$$

then, Δ is the adjacency matrix that corresponds to the forest of all MSTs.

Results

Theorem 5. Let N be the number of pairs (i, j) where $i > j$ and $\delta(i, j) = 1$. Then, MST in D is unique if and only if $N = (n-1)$.

Results

NOTE:

Instead of constructing the sequence D^{*2}, D^{*3}, D^{*4}, … , we construct the sequence D^{*2}, D^{*4}, D^{*8}, … . This computation process is stopped at the k-th iteration if $D^{*2^k} = D^{*2^{(k-1)}}$. This is the SDU of D. In this case, the number of iterations needed is k satisfying $2^k \leq n$, i.e., $k \leq \dfrac{\ln(n)}{\ln(2)}$.

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Example

- NYSE 100 traded stocks is analyzed using MST-based filter as well as Forest-based filter.
- It consists of 10 sectors (colored circles)
- We will see the significant different economic interpretation given by the two Filters in terms of centrality measures. See Figures 1 and 2.

Consumer Goods
Financials
Oil & Gas
Health Care
Telecommunications
Consumer Services
Technologies
Industrials
Utilities
Basic materials

Example

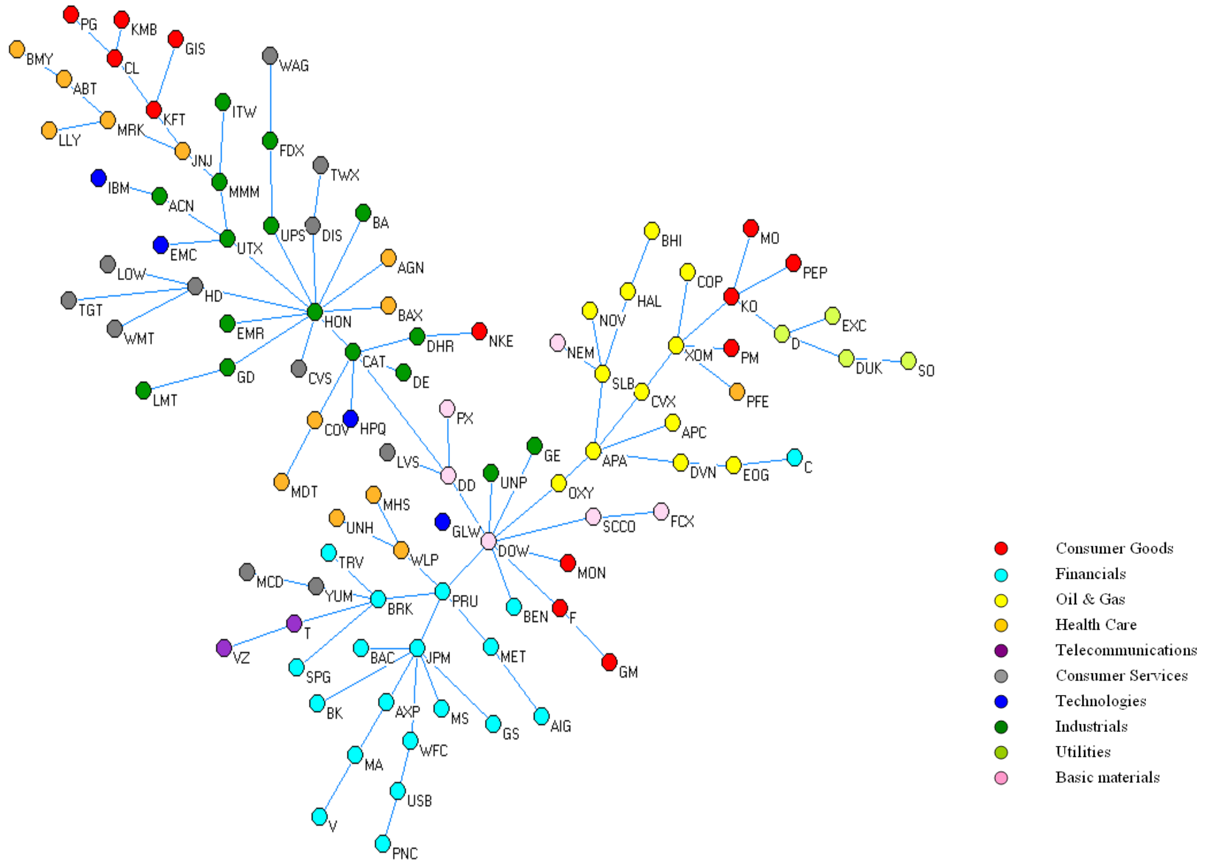

Figure 1. An MST given by Kruskal's algorithm

Example

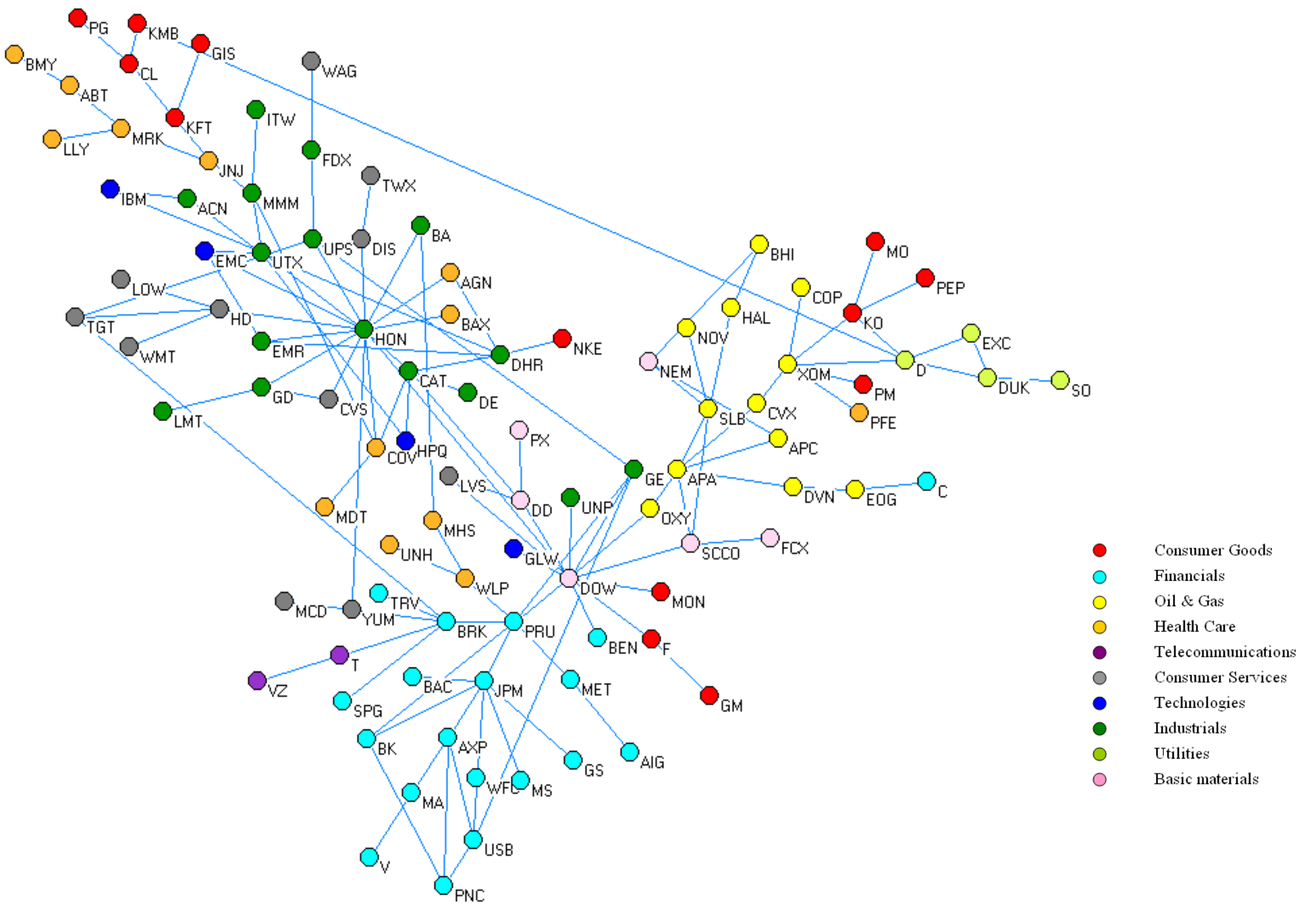

Figure 2. The forest of all possible MSTs

Concluding remarks

The topological properties of stock networks created by a MST such as, for an example, discussed in Tabak *et al*. [4] are different from those issued by the forest of all MSTs except when D contains one unique MST.

- With respect to any MST, centrality measures such as degree centrality, betweenness centrality, closeness centrality, and eigenvalue centrality might not be robust. This is not the case if one works based on the forest.

Concluding remarks

- According to the degree centrality measure [5,6], for example, the top ten stocks traded at NYSE ranked by the proposed filter is different from that given by MST. The forest-based filter puts UTX (United Technologies Corp.) and DHR (Danaher Corp.) among the top ten whereas, according to the MST-based filter, they are respectively at ranks 12 and 34. On the other hand, DD (E.I. DuPont de Nemours & Co.) and KO (Coca Cola Co.) are among the top ten when we use the MST-based filter but they are respectively at ranks 13 and 15 according to the forest.

Concluding remarks

- The same situation will also appear if we use other centrality measures. Whatever the centrality measure that we use, the list of stocks ordered from the smallest until the largest ranks given by the proposed filter is different from that given by the MST-based filter. All the above observations illustrate the advantages of the proposed filter.

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THANK YOU

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