Expository Quantum Lecture Series 2013
(EQualS 2013) at University Putra Malaysia
22-24, November 2013

Geometry-Property Relationship in Condensed Matter Physics

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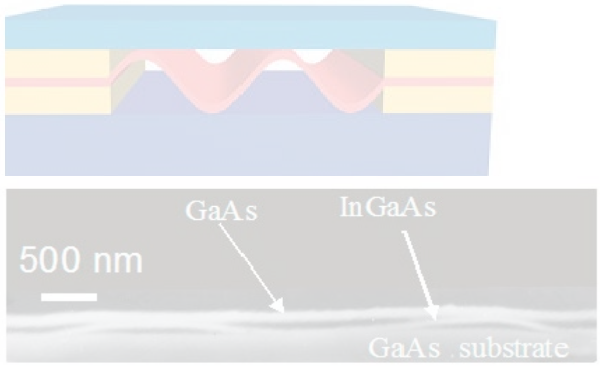
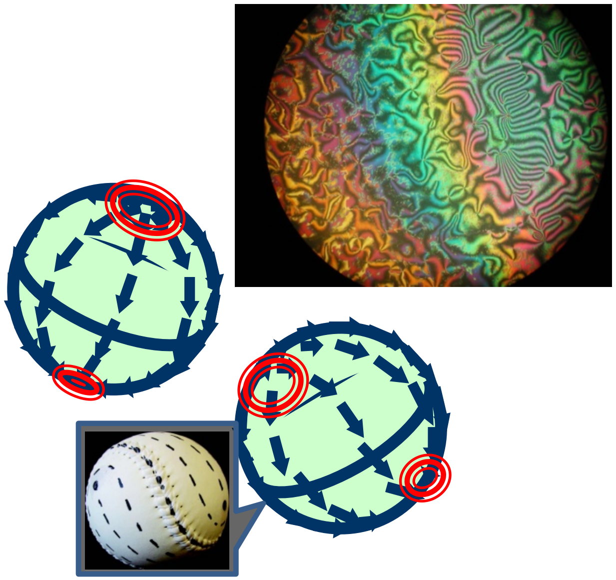
Quantum mechanics on Curved surfaces



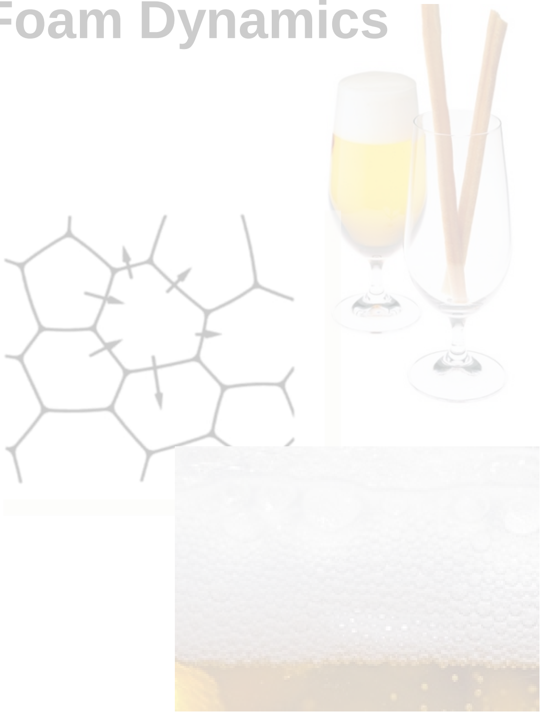
EPL **96** 27011 (2011)
J. Phys. Cond. Mat. **22**, 075301 (2010)
Phys. Rev. B **79,** 201401(R) (2009)
Phys. Rev. B **79**, 235407 (2009)

Liquid crystal on Curved substrates



J. Phys. Soc. Jpn. **79**, 074607 (2010).

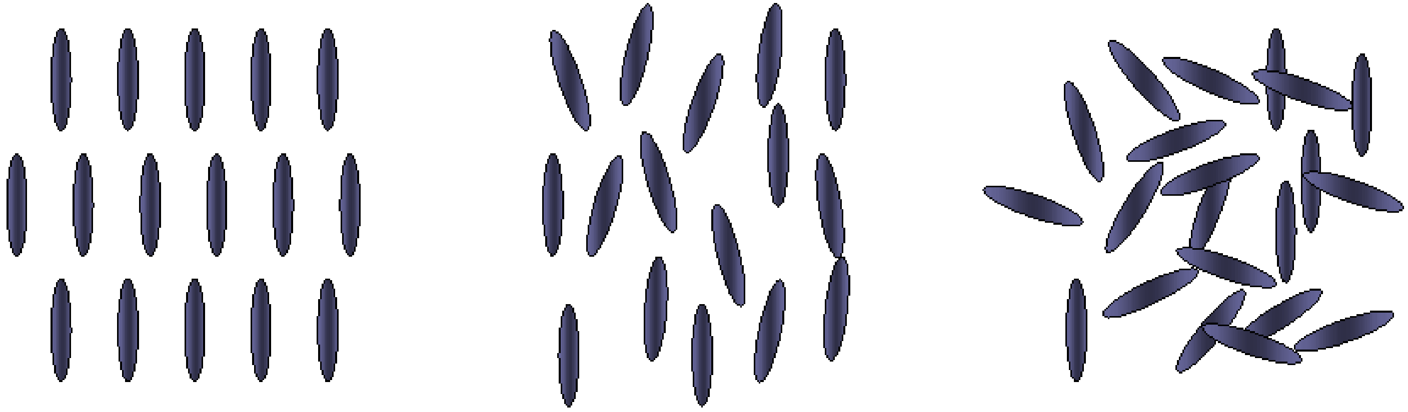
Foam Dynamics



J. Phys. Soc. Jpn. **79**, 074601 (2010).

1. Liquid crystal

= An intermediate state between a crystal and liquid



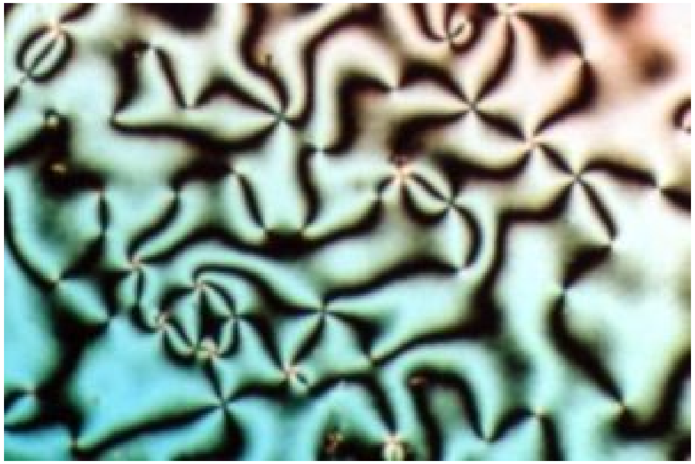
Crystal

Liquid Crustal (LC)

Liquid

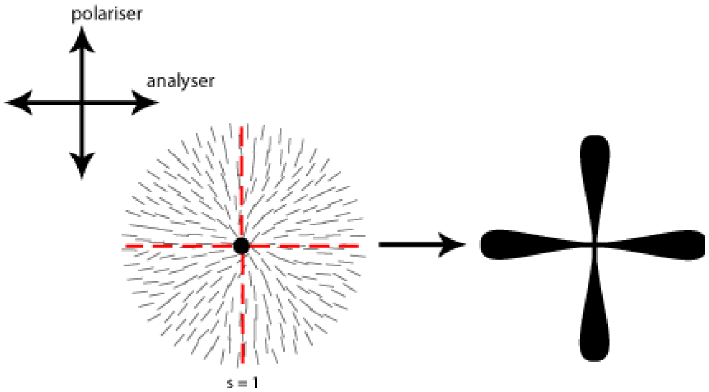
order

fluidity



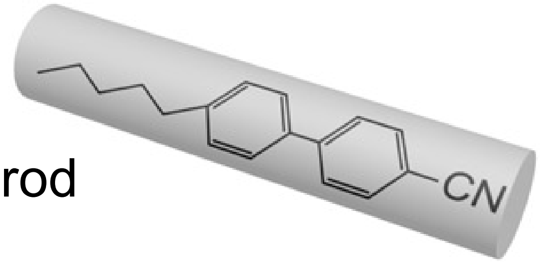
Schlieren texture:

… observed in (nematic) LC films sandwiched between crossed polarizers in a polarizing microscope.

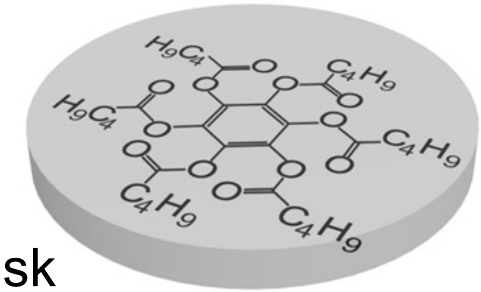


1. Liquid crystal

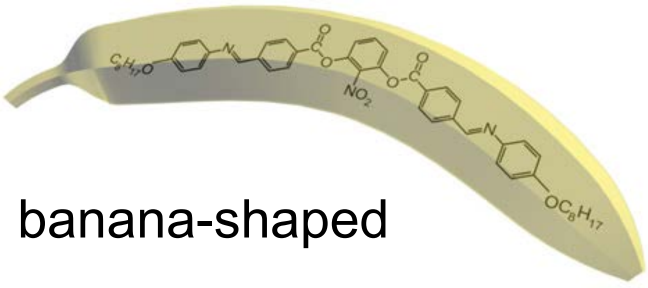
Classes of LC molecules:



rod

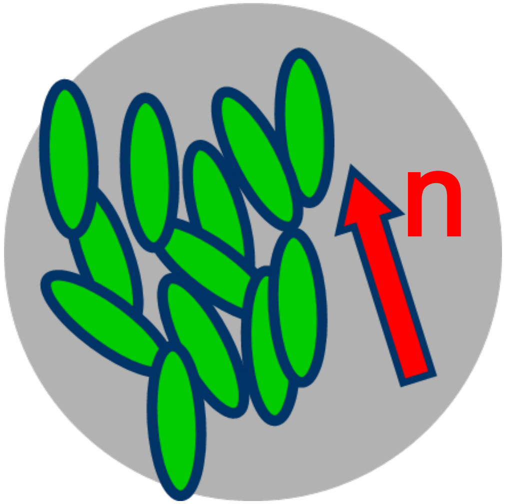


disk



banana-shaped

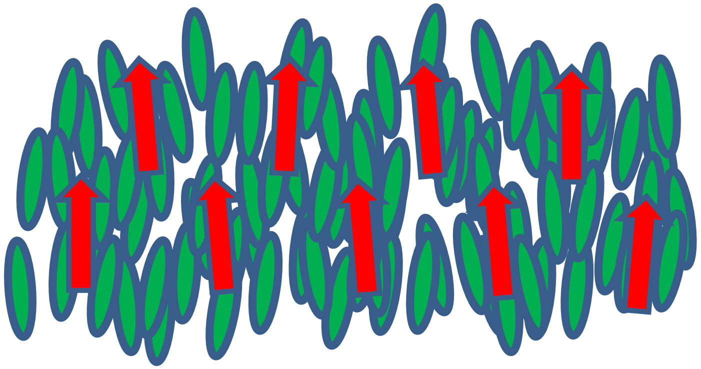
http://dept.kent.edu/spie/liquidcrystals/maintypes.html



Director **n**

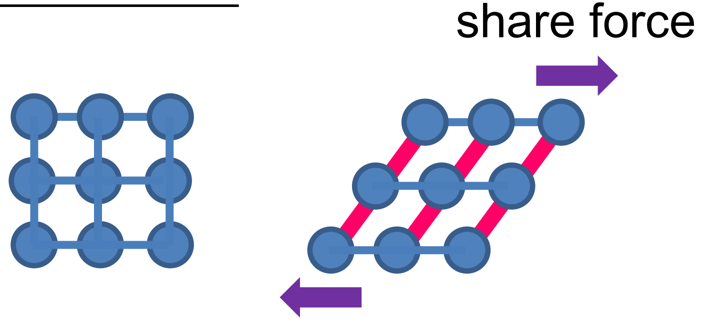
=The direction of the preferred orientation of LC molecules.

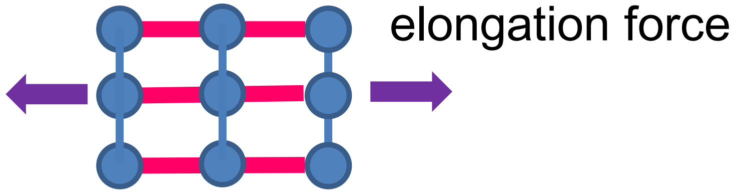
Ensemble of LC molecules is mapped onto the **director field** (continuum).



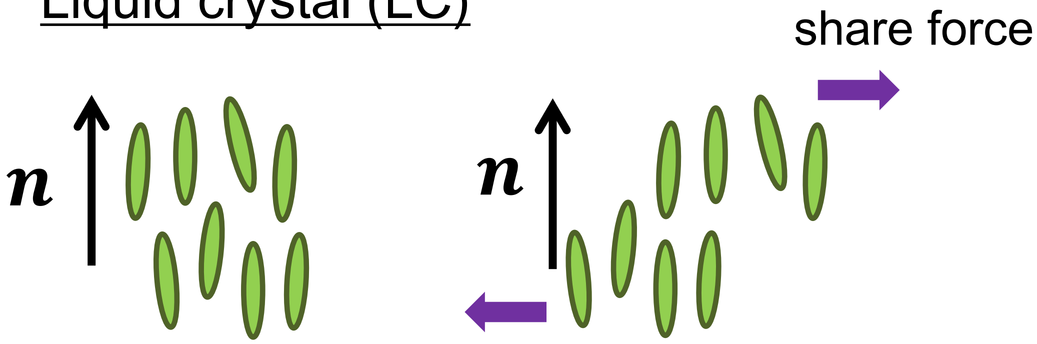
2. Elastic energy under deformation

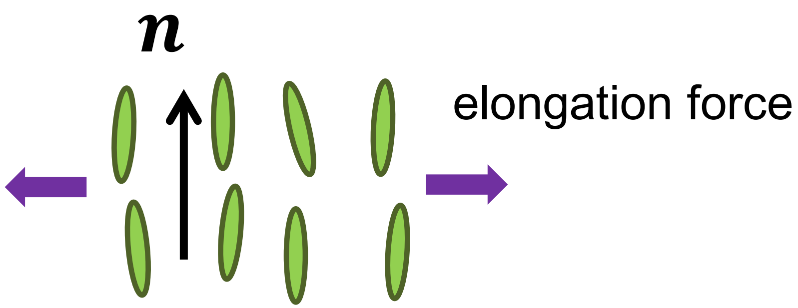
Elastic solid





Liquid crystal (LC)





Continuum elastic approximation for LC

Spatial variation in the director orientation $\boldsymbol{n}(\boldsymbol{r})$ pushes up the elastic energy:

$$F_{\mathrm{Fr}} = \tfrac{1}{2}K_1(\operatorname{div}\boldsymbol{n})^2 + \tfrac{1}{2}K_2(\boldsymbol{n}\cdot\operatorname{curl}\boldsymbol{n})^2 + \tfrac{1}{2}K_3(\boldsymbol{n}\times\operatorname{curl}\boldsymbol{n})^2$$

spray twist bend

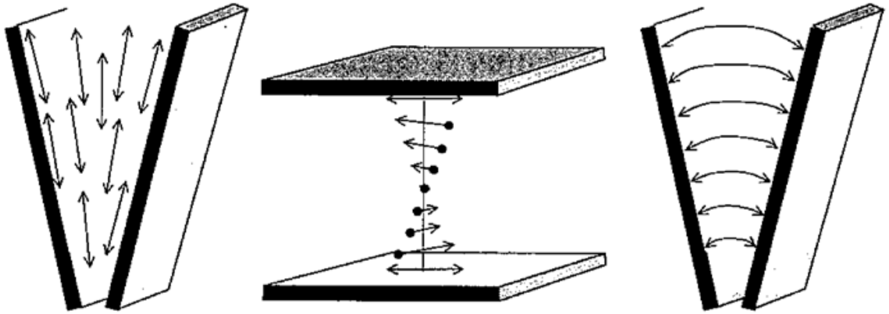
K_i : elastic constants (i=1,2,3)

Lubensky & Prost, J. Phys. II (France) (1992)

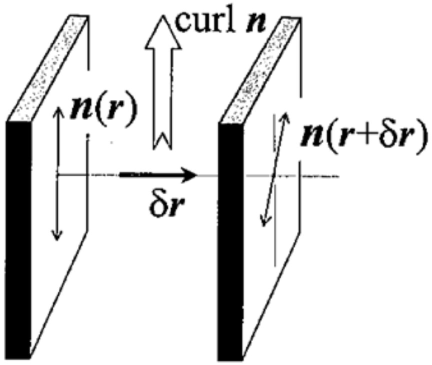
2. Elastic energy under deformation

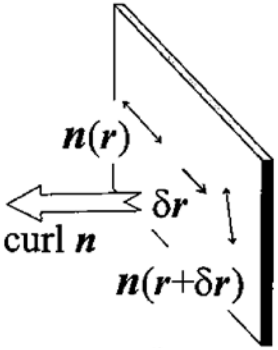
$$F_{\mathrm{Fr}} = \frac{1}{2}K_1(\mathrm{div}\,\boldsymbol{n})^2 + \frac{1}{2}K_2(\boldsymbol{n}\cdot\mathrm{curl}\,\boldsymbol{n})^2 + \frac{1}{2}K_3(\boldsymbol{n}\times\mathrm{curl}\,\boldsymbol{n})^2$$

spray twist bend



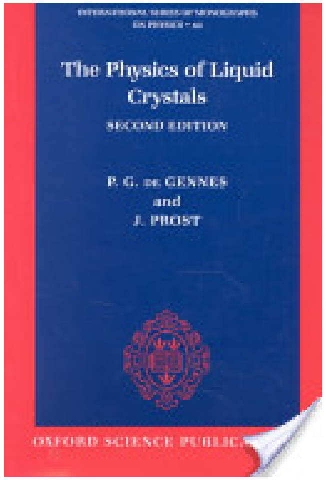
spray twist bend





twist: $\boldsymbol{n}\parallel\mathrm{curl}\,\boldsymbol{n}$

bend: $\boldsymbol{n}\perp\mathrm{curl}\,\boldsymbol{n}$



The Physics of Liquid Crystals
SECOND EDITION
P. G. de GENNES
and
J. PROST
OXFORD SCIENCE PUBLIC

de Gennes & Prost, 1995

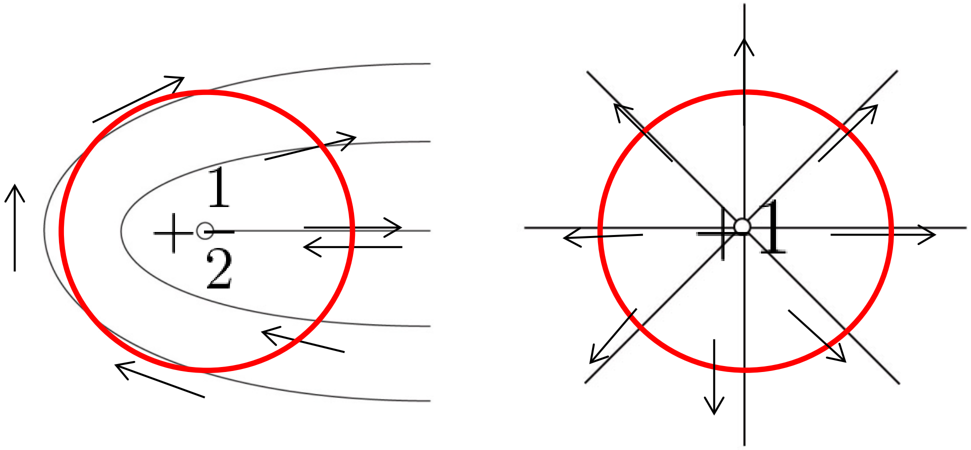
In a 3D space
- spray: O
- twist: O
- bend: O

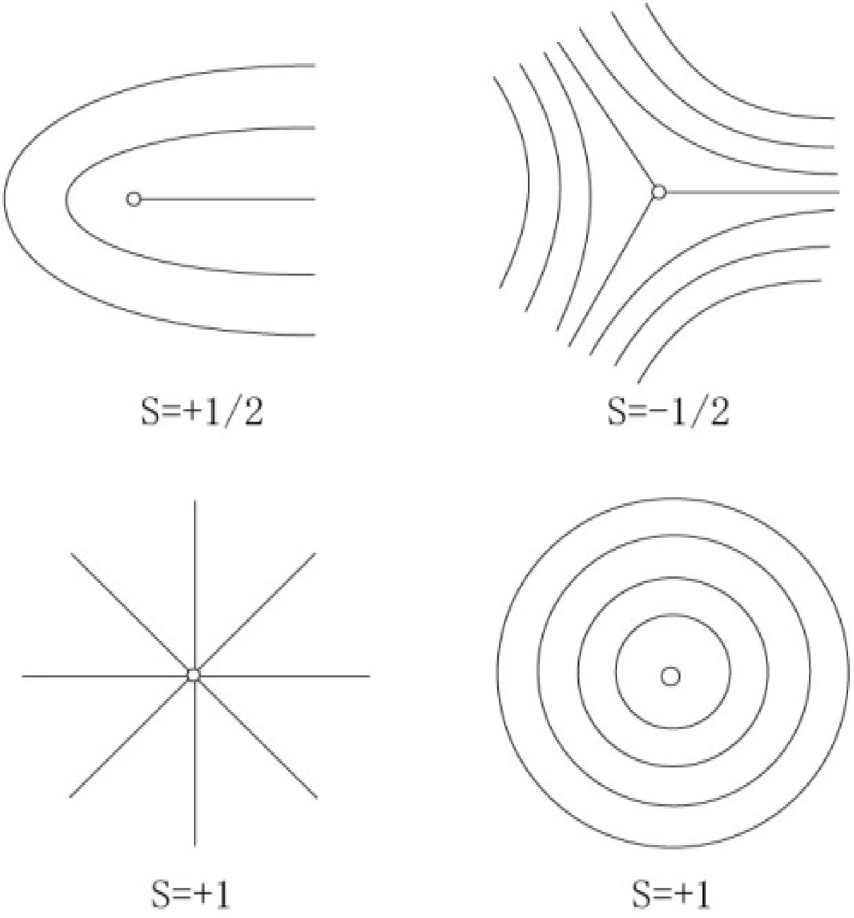
On a 2D surface (both flat and curved),
- spray: O
- twist: X
- bend: O

3. Point-defect and-Line defect

Defect = Singularity of the director field

- Point-like defect (with strength s)
- Line defect




S=+1/2

S=−1/2

S=+1

S=+1

Director's angle:

$$\psi = (s-1)\theta + \psi_0$$

Disclination strength:

$$s = \pm 1 \text{ or } \pm\frac{1}{2}$$

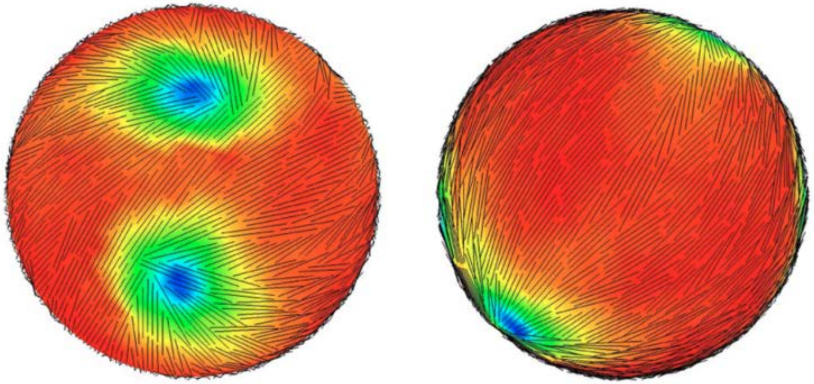
3. LC on curved surface

$$F = \frac{1}{2}\int d^2r[K_1(\vec{\nabla}\cdot\vec{n})^2 + K_3(\vec{\nabla}\times\vec{n})^2]$$

$$F = \frac{1}{2}K\int d^2x\sqrt{g(x)}[\partial_i n^j + \Gamma^j_{ki}n^k]^2$$

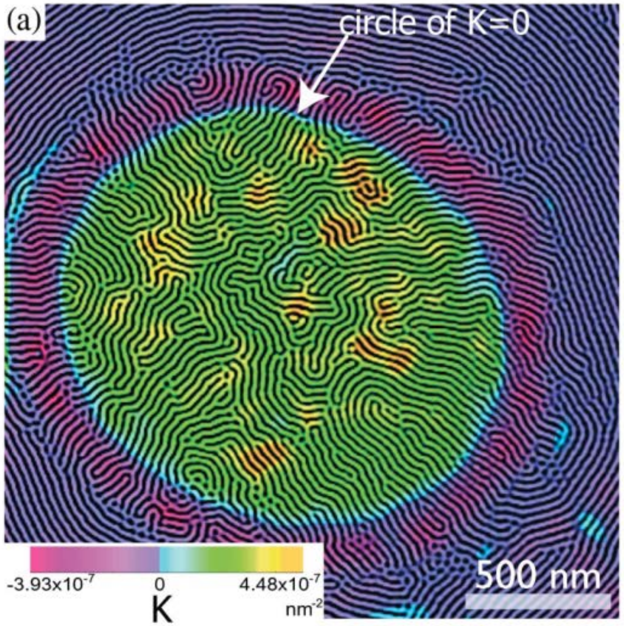
On a sphere

Shin et al., Phys. Rev. Lett. 101 (2008) 037802.



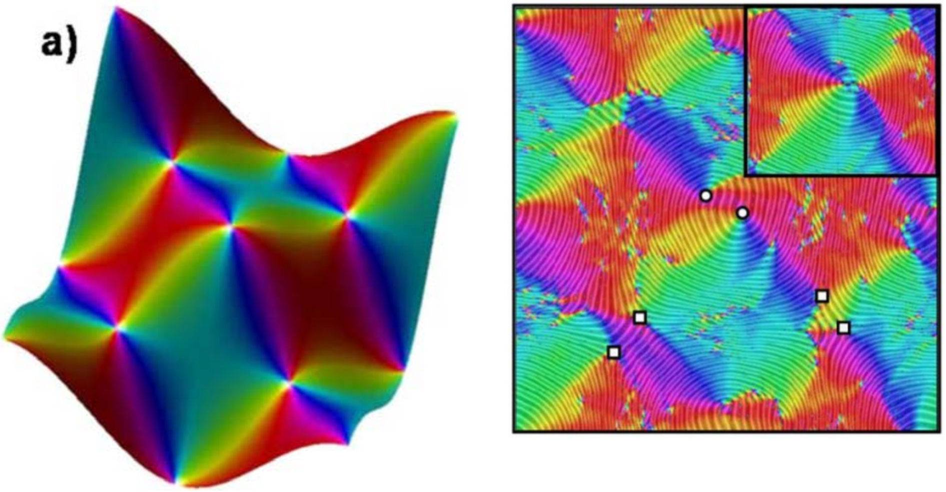
On a Gaussian bump

Santangelo et al., Phys. Rev. Lett. 99 (2007) 017801.



On a corrugated surface

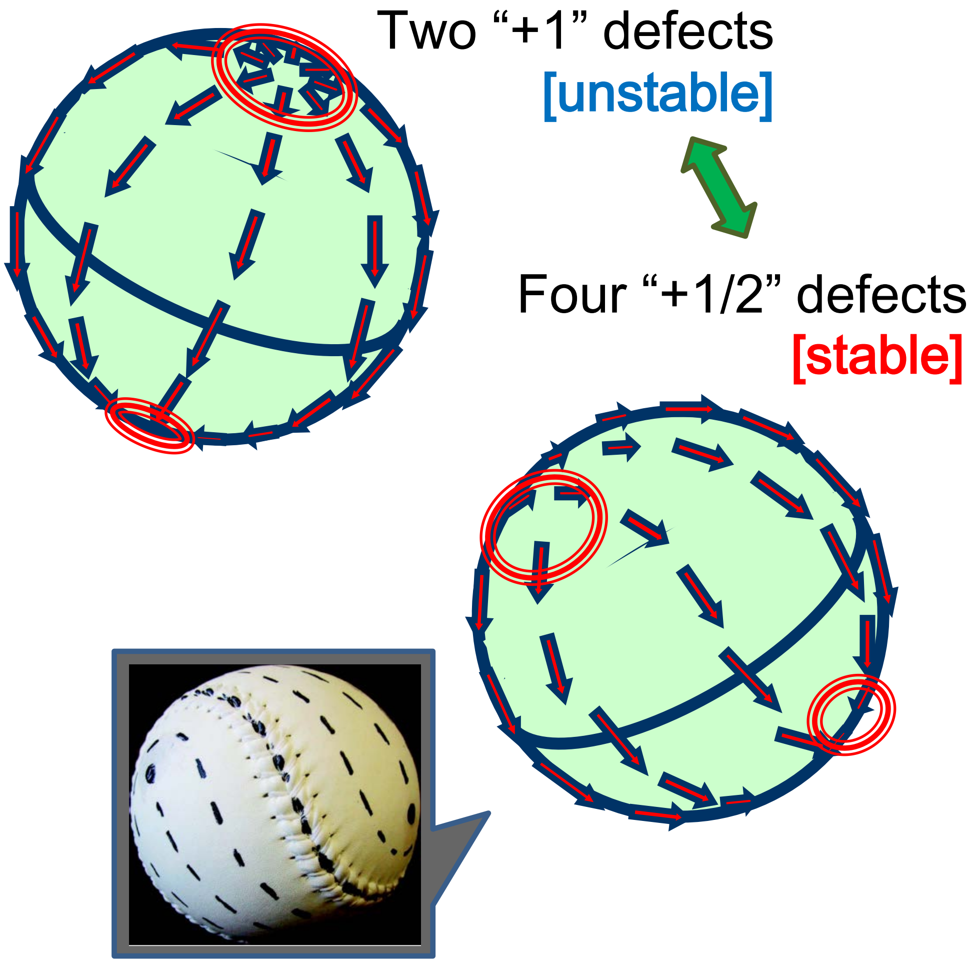
Gómez et al., Phys. Rev. E 79 (2009) 031701.



3. LC on curved surface

LC ordering in a spherical surface:

… in which, defects arises inevitably as a result of topological constraint.

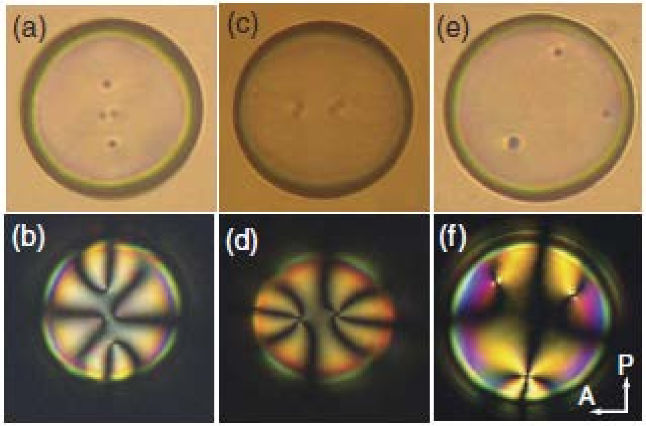


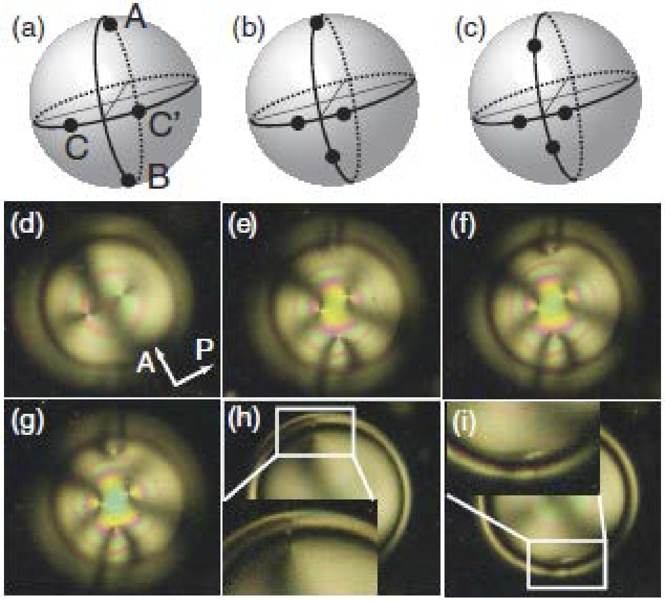
Two “+1” defects
[unstable]

Four “+1/2” defects
[stable]

Experimental observations:

Phys. Rev. Lett. **99** (2007) 157801

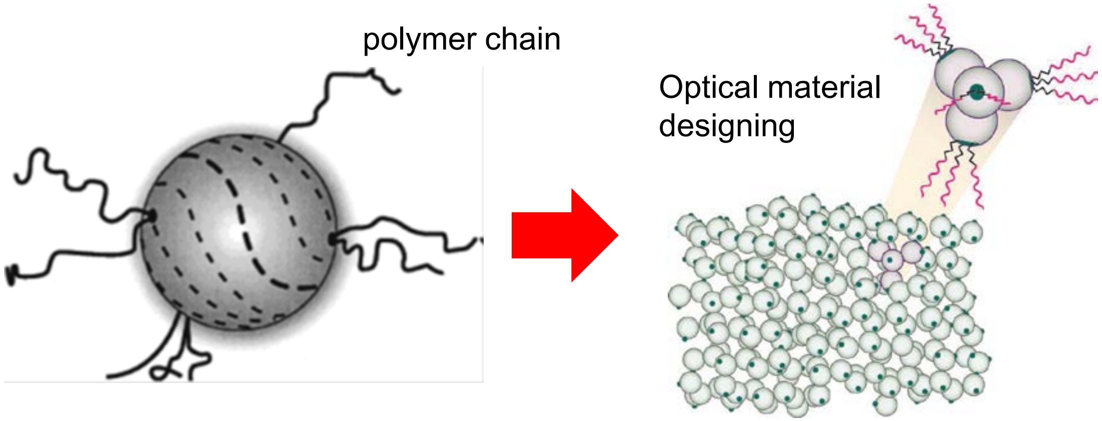




4. Engineering application

As an optical material:

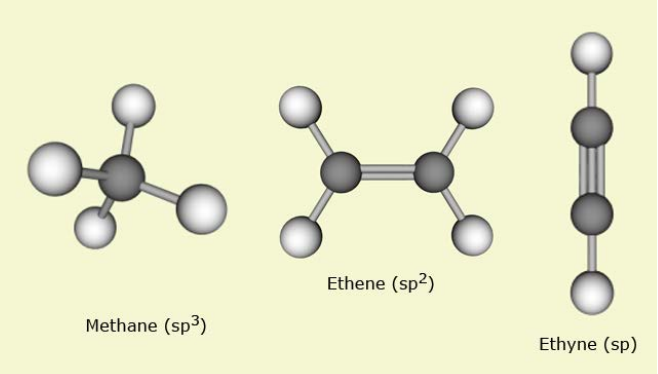
Nelson, Nano Lett. (2002)



polymer chain

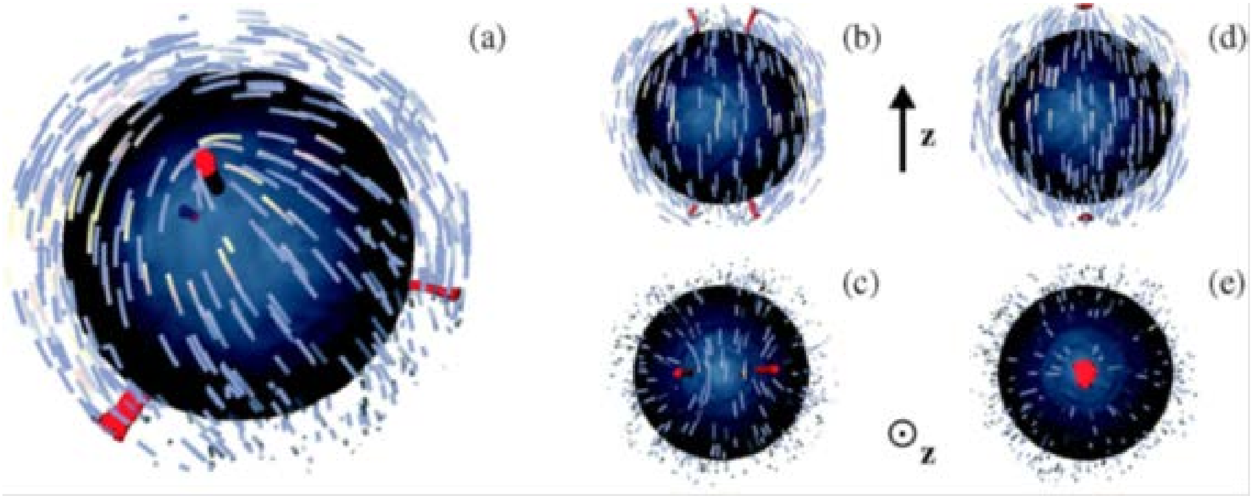
Optical material designing

Mimicking the atom-scale chemical bonding by micron-scale LC-coated colloids



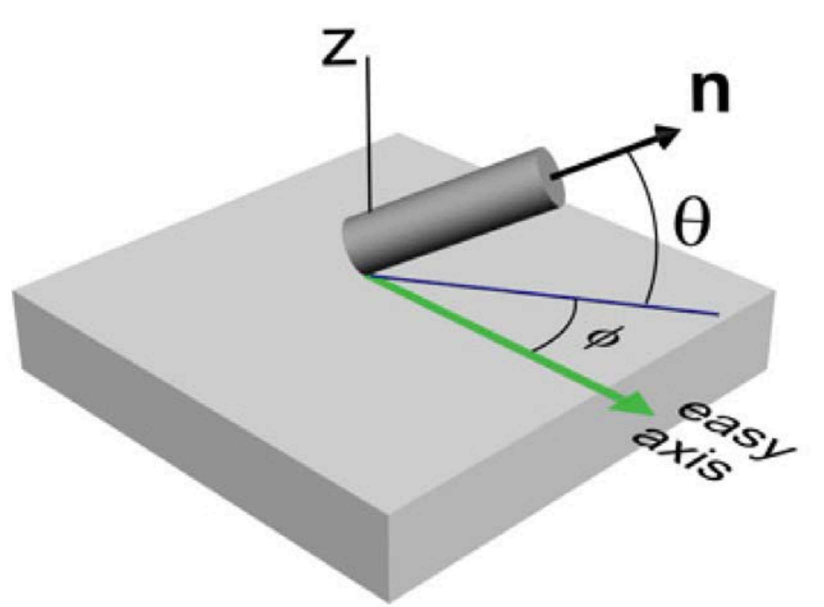
Positional control by an electric field:

Skacej & Zannoni, Phys. Rev. Lett. (2008)



5. Tilted angle

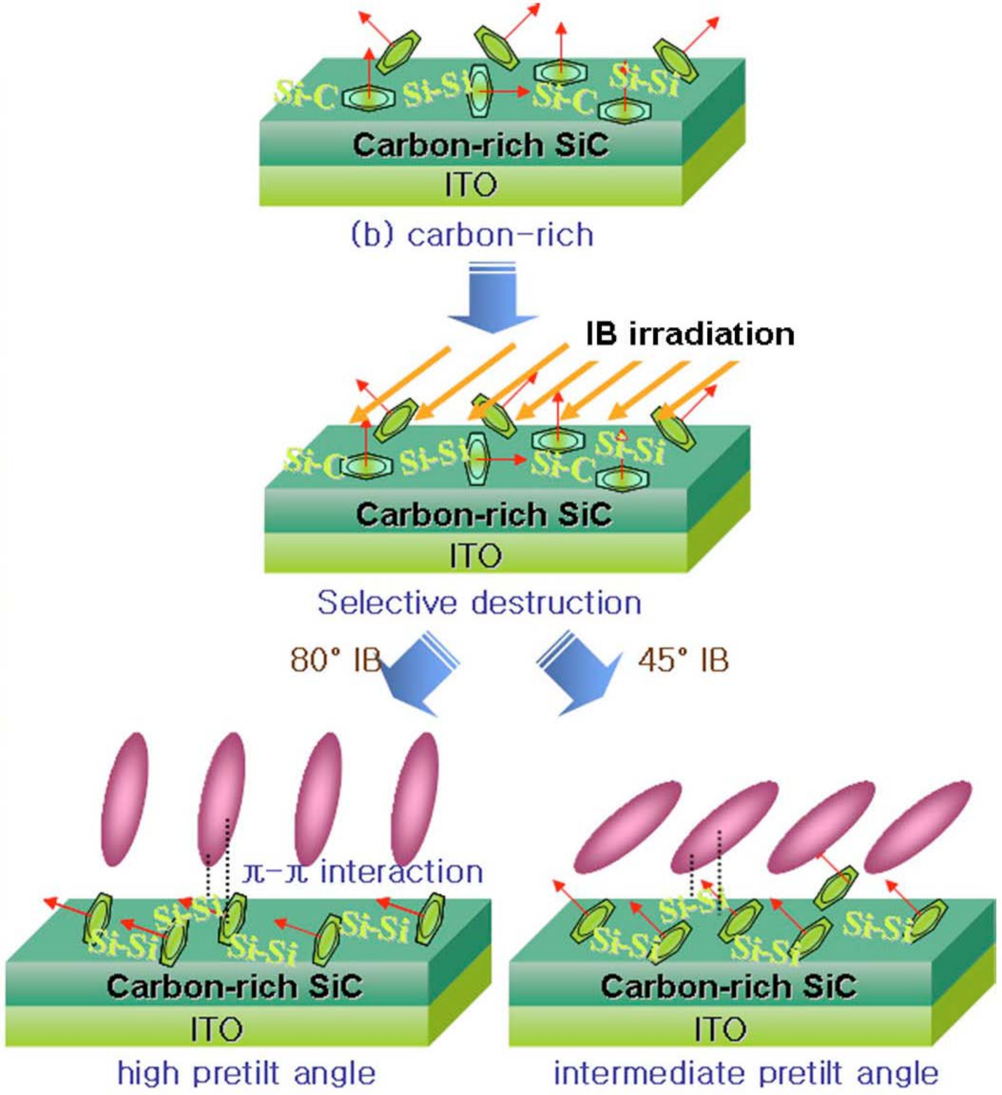
… Near the substrate, the director orients in a specific direction determined by mechanical/chemical conditions .



The preferred direction of n is labeled by the two angles
- polar angle θ
- azimuthal angle φ

A method of controlling θ and φ:

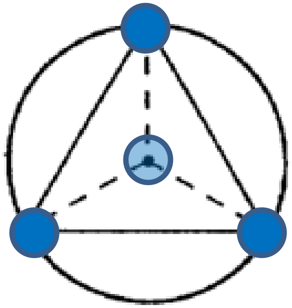
J.B. Kim, et al., Appl. Phys. Lett. **90**, 043515 (2007)



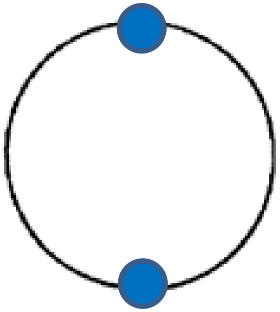
6. Anchoring effect on Defect configuration

Under the **planar** anchoring condition, we have…

Stable (Fd small)



Un-stable (Fd large)

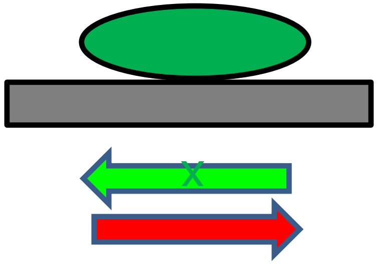


$$
\begin{aligned}
F_d = \int_V dV[&K_1(\nabla\cdot \boldsymbol{n})^2 \\
&+ K_2(\boldsymbol{n}\cdot\nabla\times\boldsymbol{n})^2 \\
&+ K_3(\boldsymbol{n}\times\nabla\times\boldsymbol{n})^2]
\end{aligned}
$$

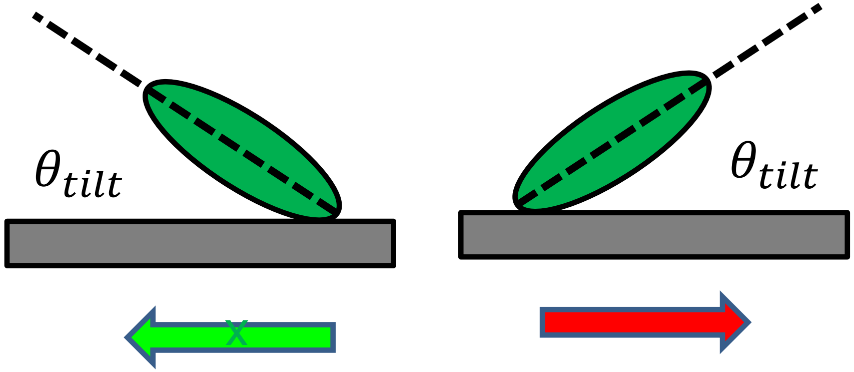
T. C. Lubensky and J. Prost,
J. Phys. II (France) **2**, 371(1992)

Question: What configuration under the **tilted** anchoring?

$\theta_{tilt} = 0$



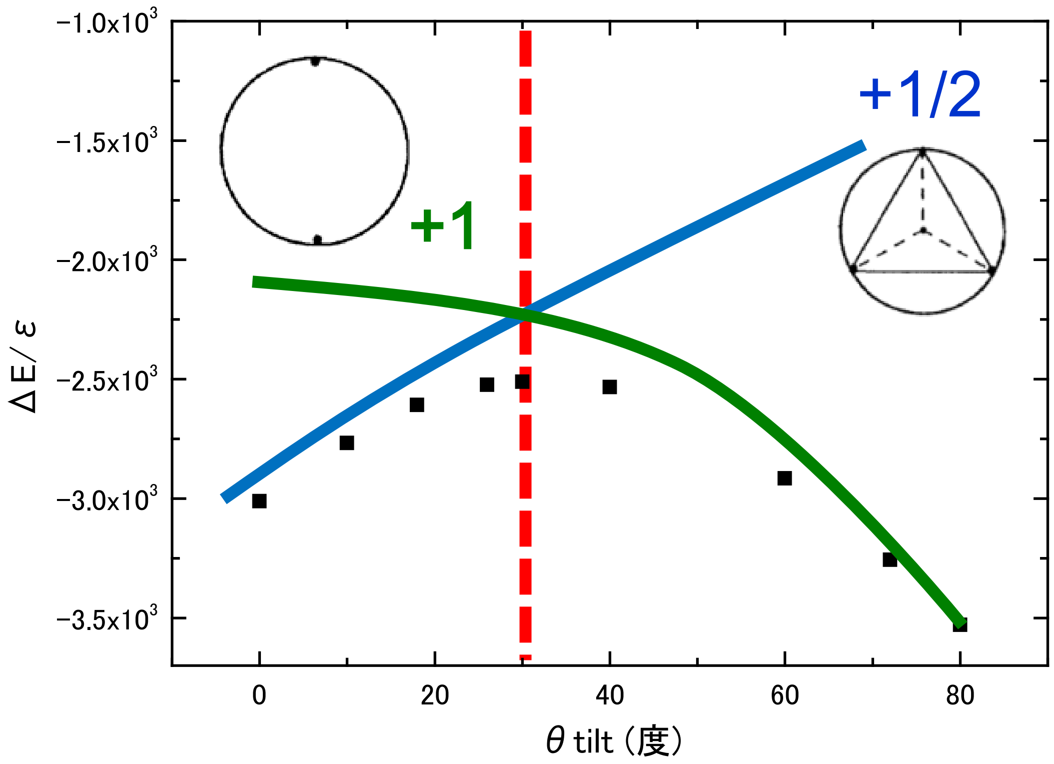
$\theta_{tilt} > 0$ (Symmetry-Breaking)



6. Anchoring effect on Defect configuration

θ_{tilt}-dependence of the elastic energy

K. Saito and H. Shima (unpublished)



$\theta_{tilt} < 30^\circ$ **Tetrahedral** (+1/2 defects)

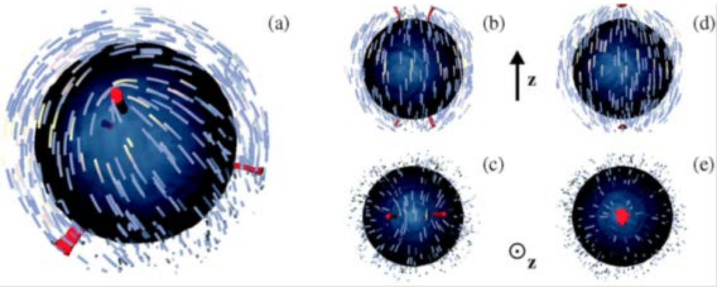
$\theta_{tilt} > 30^\circ$ **Bipolar** (+1 defects)

Structural phase transition

Tetrahedral ⟺ Bipolar

… implies the defect control by tuning θ_{tilt} (i.e., surface anchoring condition).

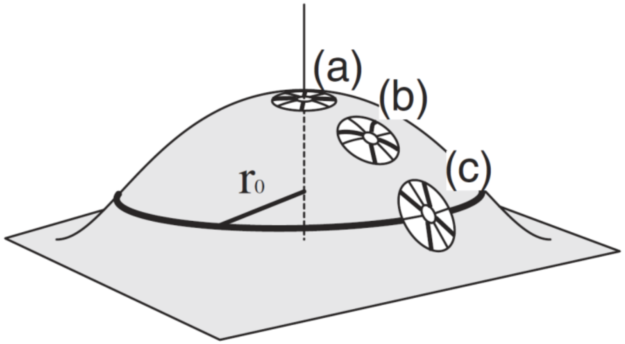
.. complementary to the control by the electric field application.

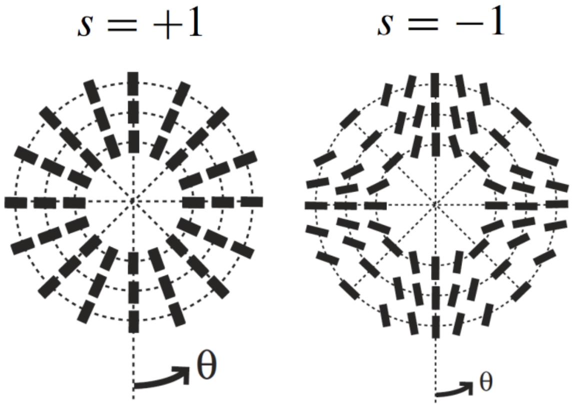


G. Skacej and C. Zannoni,
Phys.Rev.Lett. **100**, 197802 (2008)

7. Point-defect haloing

Hasegawa & HS, J. Phys. Soc. Jpn (2010)
Hasegawa & HS, Mod. Phys. Lett. B (2010)



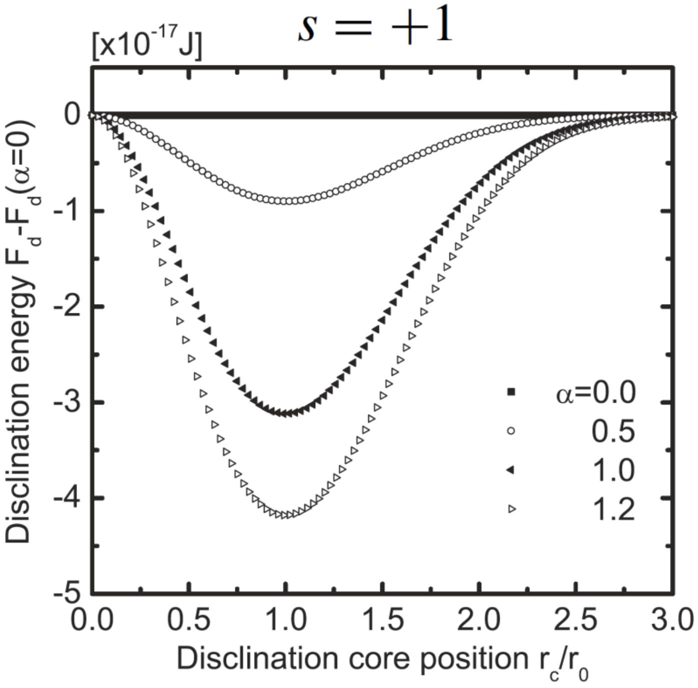


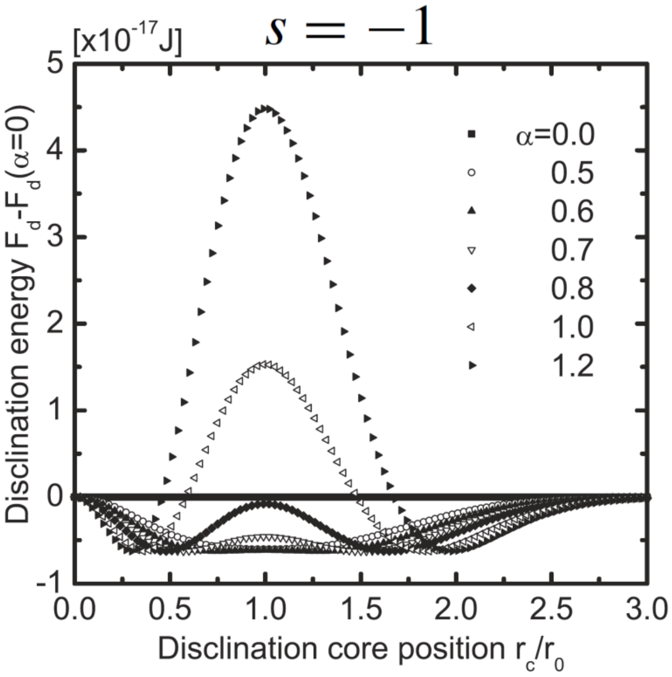
Director's angle:

$$\psi = (s-1)\theta + \psi_0$$

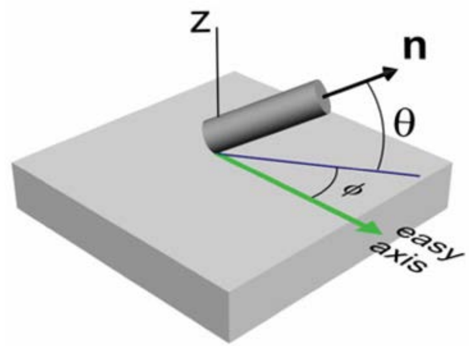
Disclination strength:

$$s = \pm 1$$



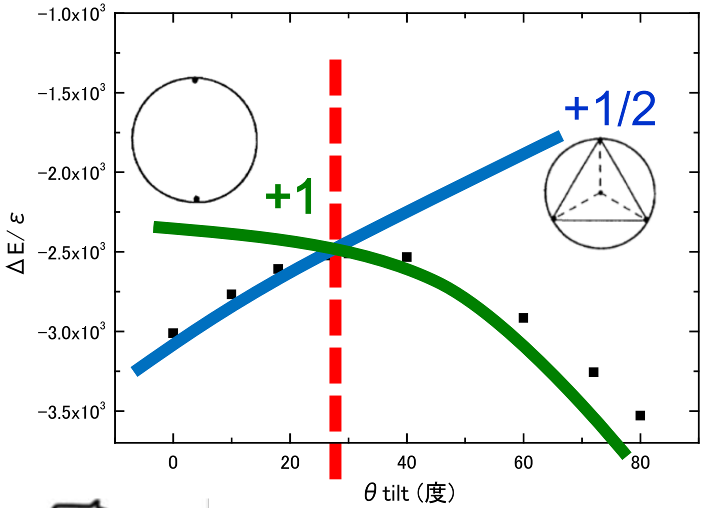


8. Summary of LC topic



LC ordering on a sphere

- -Surface anchoring conditions strongly affect defect configuration on a spherical surface
- -Bipolar-Tetrahedral phase transition occurs at the tilted angle of 30
- -It implies defect control by chemical treatment, complementary to the control by electric field.



LC ordering on a Gaussian surface

J. Phys. Soc. Jpn. **79**, 074607 (2010).

- - Defects are trapped at the region of zero-Gaussian curvature