New Types of Functions and Separation Axiom by $\gamma$-Open Sets in Fuzzy Bitopological Spaces

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ABSTRACT

The main aim of this paper is to introduce a new class of fuzzy irresolute functions called (i,j)-completely fuzzy $\gamma$-irresolute function in fuzzy bitopological space and to study some basic properties of the newly defined function. Then we define a weaker form of (i,j)-completely fuzzy $\gamma$-irresolute function termed as (i,j)-fuzzy $\gamma^*$-irresolute function. Finally we study a new type of fuzzy bitopological space namely (i,j)-fuzzy $\gamma$-normal space, We obtain an additional condition under which (i,j)-fuzzy $\gamma^*$-closed surjection function preserves (i,j)-fuzzy $\gamma$-normality.

Keywords: (i,j)-fuzzy $\gamma$-open set,(i,j)-fuzzy $\gamma$-irresolute function,(i,j)-completely fuzzy $\gamma$-irresolute function.
1. Introduction

The notion of bitopological spaces was introduced and investigated at the initial stage by Kelly (1963) and thereafter a large number of papers are done to generalize the topological ideas to bitopological spaces. Functions and in fact irresolute functions offer new path towards analysis. Mukherjee (1985) defined the irresolute mappings in bitopological spaces. For more details on alternative forms of irresolute functions in topology and fuzzy topological spaces see for example, Chitharanjan and Missier (2012), Ekici and Jafari (2008), Khedr and Al-Saadi (2012), Park and J. (1998), Rajesh (2011), Thakur and Malviya (1995) and Zorlutuna (2008). The notion of fuzzy set was introduced by Zadeh (1965). The potential of the introduced notion was realised by the researchers and has been applied with success for investigations in all the branches of science and technology.

Then Kandil (1989) introduced the idea of fuzzy bitopological spaces as an extension of fuzzy topological spaces and a generalization of bitopological spaces. Bhaumik and Pal (1993) studied and introduced the concepts of completely and weakly irresolute function in fuzzy topological space. Nour (1995) has studied some separation axiom in bitopological space following the pairwise separation axiom of Kelly (1963). Subsequently Thakur and Malviya (1995) introduced the ideas of pairwise fuzzy irresolute mappings. Presently Tripathy and Debnath (2013) have introduce \( \gamma \)-open set in fuzzy bitopological space and shown that every \((1, 2)\)-fuzzy pre open set may not be a \((1, 2)\)-fuzzy \( \gamma \)-open set. Tripathy and Sharma (2013) introduced pairwise strongly b-open and pairwise strongly b-closed functions in bitopological spaces and studied the connection with pairwise strongly b-normal and pairwise b-separated spaces. Also similar notions have been introduced in the area of L-fuzzy topology by Aygun and Kuldri (1999) and Mukherjee and Bhattacharya (2003). Naschie (1998, 2000) has shown that the notion of fuzzy topology may be relevant to quantum particle physics in connections with string theory and \( \epsilon^{\infty} \) theory.

The aim of this paper is to introduce a new type of irresolute function namely \((i,j)\)-completely fuzzy \( \gamma \)-irresolute function in fuzzy bitopological space. Some basic properties and characterization of this newly defined function are mentioned. In this paper we tend to outline the ideas of \((i,j)\)-fuzzy \( \gamma \)-irresolute function, \((i,j)\)-fuzzy \( \gamma^* \)-closed mapping, \((i,j)\)-fuzzy \( \gamma \)-normal spaces and try to investigate the properties of this newly defined ideas. Also our special concentration is to show that every \( \tau_1 \)-fuzzy open set may not be a \((1,2)\)-fuzzy \( \gamma \)-open set and every \( \tau_2 \)-fuzzy open set may not be a \((2,1)\)-fuzzy \( \gamma \)-open set.
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Throughout this paper $X$ and $Y$ denote the fuzzy bitopological spaces $(X, \tau_1, \tau_2)$ and $(Y, \sigma_1, \sigma_2)$ respectively. A fuzzy subset $\mu$ of a fuzzy bitopological space $(X, \tau_1, \tau_2)$ is said to be:

i. $(i,j)$-fuzzy preopen (cite Kumar 1994) if $\mu \leq i\text{-int}(j\text{-cl}(\mu))$, where $i \neq j$, $i,j = 1,2$.

ii. $(i,j)$-fuzzy $\gamma$-open (Tripathy 2013) if $\mu \cap \beta$ is $(i,j)$-fuzzy preopen for every $(i,j)$-fuzzy preopen set $\beta$ in $X$, where $i \neq j$, $i,j = 1,2$.

iii. $(i,j)$-fuzzy regular open (Khedr and Al-Saadi 2005) if $\mu = i\text{-int}(j\text{-cl}(\mu))$, where $i \neq j$, $i,j = 1,2$.

iv. A function $f: X \to Y$ from a fuzzy topological space $(X, \tau)$ to another fuzzy topological space $(Y, \sigma)$ is said to be fuzzy completely irresolute function (Bhaumik and Pal 1993) iff $f^{-1}(\alpha)$ is fuzzy regular open subset of $X$ for every fuzzy semi-open subset $\alpha$ in $Y$.

For a function $f: X \to Y$, a function $g: X \to XX Y$, defined by $g(x) = (x, f(x))$ for each $x \in X$ is called the graph function of $f$.

The subset $\{(x, f(x)) : x \in X\}$ is called the graph of $f$ and is denoted by $G(f)$. The family of all $(i,j)$-fuzzy regular open (resp. $(i,j)$-fuzzy $\gamma$-open) sets of a fuzzy bitopological space $(X, \tau_1, \tau_2)$ is denoted by $(i,j)-\text{FR}O(X)$ (resp. $(i,j)-\text{FG}O(X)$).

2. Properties of $(i,j)$-Completely Fuzzy $\gamma$-Irresolute Functions

In this section we define $(i,j)$-completely fuzzy $\gamma$-irresolute function and study some of their basic properties and characterizations.

**Definition 2.1.** A function $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called $(i,j)$-completely fuzzy $\gamma$-irresolute function if the inverse image of each $(i,j)$-fuzzy $\gamma$-open set in $Y$ is $(i,j)$-fuzzy regular open set in $X$.

**Remark 2.1.** In a fuzzy bitopological space an $(i,j)$-fuzzy $\gamma$-continuous function may not be a $(i,j)$-completely fuzzy $\gamma$-irresolute function as seen in the following example:

**Example 2.1.** Let $X = \{x,y\}$ and $Y = \{a,b\}$. Consider the fuzzy set on $X$ are $\overline{0}, \overline{1}, A, B$, where $A = \{(x,0.4), (y,0.8)\}$ and $B = \{(x,0.2), (y,0.3)\}$. Also consider the fuzzy sets in $Y$ are $\overline{0}, \overline{1}, C$, where $C = \{(a,0.5), (b,0.5)\}$.
Now let $\tau_1=\{\emptyset, T, A\}, \tau_2=\{\emptyset, T, B\}, \sigma_1=\{\emptyset, T\}$ and $\sigma_2=\{\emptyset, T, C\}$. We get (i,j)-F$\gamma$O($Y$)=$\{\emptyset, T, \{(a,\alpha),(b,\beta)\}\}$ where $\alpha>0.95$, and $\beta>0.95$ and also we get (i,j)-FRO($X$)=$\{\emptyset, T\}$.

We define a function $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ such that $f(x)=a$ and $f(y)=b$. Thus the inverse image of each $\sigma_1$-fuzzy open set in $Y$ is (i,j)-fuzzy $\gamma$-open set in $X$. But under this function the inverse image of (i,j)-fuzzy $\gamma$-open set in $Y$ is (i,j)-fuzzy $\gamma$-open set in $X$. Hence $f$ is (i,j)-completely fuzzy $\gamma$-irresolute function.

**Theorem 2.1.** For a function $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ the following properties are equivalent:

1. $f$ is (i,j)-completely fuzzy $\gamma$-irresolute function.
2. $f^{-1}(V)\subseteq(i,j)$-FRint($f^{-1}(i,j)$-F$\gamma$cl($V$)) $\forall V \subseteq(i,j)$-F$\gamma$O($Y$) and
3. (i,j)-FRcl($f^{-1}(V))\subseteq f^{-1}((i,j)$-F$\gamma$cl($V$)).

**Proof.** (i)$\Rightarrow$(ii) Let $V \subseteq(i,j)$-F$\gamma$O($Y$) and $x \in f^{-1}(V)$. Thus $f(x) \in V$. Now since $f$ is (i,j)-completely fuzzy $\gamma$-irresolute function, so there exist a $U \subseteq$ FRO($X,x$) such that $f(U) \subseteq V \Rightarrow f(U) \subseteq(i,j)$-F$\gamma$cl($V$). Therefore $x \in U \subseteq f^{-1}((i,j)$-F$\gamma$cl($V$)). This shows that $f^{-1}(V) \subseteq(i,j)$-FRint($f^{-1}(i,j)$-F$\gamma$cl($V$)).

(ii)$\Rightarrow$(iii) Let $V \subseteq(i,j)$-F$\gamma$O($Y$). Suppose $x \notin f^{-1}((i,j)$-F$\gamma$cl($V$)). Thus there exist a (i,j)-fuzzy $\gamma$-open set $W$ in $Y$ containing $f(x)$ such that $W \cap V = \emptyset$.

Hence (i,j)-F$\gamma$cl($W$)$\cap V = \emptyset$. $\Rightarrow f^{-1}((i,j)$-F$\gamma$cl($W$)$)\cap f^{-1}(V) = \emptyset$. Now since $x \in f^{-1}(W)$ then by (ii) we get $x \in(i,j)$-FRint($f^{-1}(i,j)$-F$\gamma$cl($W$)). Thus there exist a $U \subseteq$ FRO($X,x$) such that $U \subseteq f^{-1}(i,j)$-F$\gamma$cl($W$)). Thus we get $U \subseteq f^{-1}(V) = \emptyset \Rightarrow x \notin(i,j)$-FRcl($f^{-1}(V)$).

Hence the proof. (iii)$\Rightarrow$(i) Suppose $x \in X$ and $V \subseteq(i,j)$-F$\gamma$O($Y,f(x)$). Thus $f(x) \notin(i,j)$-F$\gamma$cl($Y-V$). Therefore $x \notin f^{-1}((i,j)$-F$\gamma$cl($Y-V$)) and by (iii) we have $x \notin(i,j)$-FRcl($f^{-1}(Y-V)$). Therefore there exist $U \subseteq$ FRO($X,x$) such that $U \subseteq f^{-1}(Y-V) = \emptyset$. $\Rightarrow U \subseteq f^{-1}(V)$. This shows that $f$ is (i,j)-completely fuzzy $\gamma$-irresolute function. □
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**Definition 2.2.** A function $f:(X,\tau_1,\tau_2)\rightarrow (Y,\sigma_1,\sigma_2)$ is called $(i,j)$-fuzzy $\gamma$-irresolute function if the inverse image of each $(i,j)$-fuzzy $\gamma$-open set in $Y$ is $(i,j)$-fuzzy $\gamma$-open set in $X$.

**Proposition 2.1.** In a fuzzy bitopological space every $(i,j)$-completely fuzzy $\gamma$-irresolute function is a $(i,j)$-fuzzy $\gamma$-irresolute function.

**Remark 2.2.** Converse of the above proposition may not be true as seen in the following example:

**Example 2.2.** Let $X=\{x,y\}$ and $Y=\{a,b\}$. Consider the fuzzy set on $X$ are $0,1,A$ and $B$ where $A=\{(x,0.4),(y,0.3)\}$ and $B=\{(x,0.2),(y,0.3)\}$. Also consider the fuzzy sets in $Y$ are $0,1,C,D$ where $C=\{(a,0.2),(b,0.3)\}$ and $D=\{(a,0.1),(b,0.3)\}$. Now let $\tau_1=\{0,1,A\},\tau_2=\{0,1,B\},\sigma_1=\{0,1,C\}$ and $\sigma_2=\{0,1,D\}$.

We get $(i,j)\text{-}F_\gamma O(X) = \{(x,\alpha),(y,\beta)\}$, for $0 \leq \alpha \leq 0.4, 0 \leq \beta \leq 0.3$

We get $(i,j)\text{-}F_\gamma O(Y) = \{(a,\alpha),(b,\beta)\}$, for $0 \leq \alpha \leq 0.2, 0 \leq \beta \leq 0.3$

Now let $f:(X,\tau_1,\tau_2)\rightarrow (Y,\sigma_1,\sigma_2)$ such that $f(x)=a$ and $f(y)=b$. Here inverse image of every $(i,j)$-fuzzy $\gamma$-open set in Y is $(i,j)$-fuzzy $\gamma$-open set in X although $f^{-1}(D)=\{(x,0.1),(y,0.3)\}$ which is not a $(i,j)$-fuzzy regular open set in X.

Now we find an additional condition and using it we show that the converse of the above Proposition 2.1 holds true in the following theorem:

**Theorem 2.2.** If $f:(X,\tau_1,\tau_2)\rightarrow (Y,\sigma_1,\sigma_2)$ is a $(i,j)$-fuzzy $\gamma$-irresolute function and every subset is a $(i,j)$-fuzzy pre-closed set in X then f is $(i,j)$-completely fuzzy $\gamma$-irresolute function.

**Proof.** Let $A$ be a $(i,j)$-fuzzy $\gamma$-open set in Y. Now since f is $(i,j)$-fuzzy $\gamma$-irresolute function, therefore $f^{-1}(A)$ is $(i,j)$-fuzzy $\gamma$-open set in X. This implies $f^{-1}(A)$ is $(i,j)$-fuzzy pre-open set in X. Again from the hypothesis $f^{-1}(A)$ is $(i,j)$-fuzzy pre-closed set in X. Thus $f^{-1}(A)$ is $(i,j)$-fuzzy regular open set in X. Hence f is a $(i,j)$-completely fuzzy $\gamma$-irresolute function.

**Theorem 2.3.** For the function $f:(X,\tau_1,\tau_2)\rightarrow (Y,\sigma_1,\sigma_2)$ the following statements are equivalent:

1. $f$ is $(i,j)$-completely fuzzy $\gamma$-irresolute and
ii. \( f^{-1}(U) \) is \((i,j)\)-fuzzy regular closed set in \( X \) for every \((i,j)\)-fuzzy \( \gamma \)-closed set \( U \) of \( Y \).

**Proof.** The proof is straight forward. \( \square \)

**Theorem 2.4.** If a function \( f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2) \) is \((i,j)\)-completely fuzzy \( \gamma \)-irresolute and \( A \) is \( \tau_i \)-fuzzy open set in \( X \), then the restriction function \( f|_A:A\to Y \) is \((i,j)\)-completely fuzzy \( \gamma \)-irresolute.

**Proof.** Let \( U \) be any \((i,j)\)-fuzzy \( \gamma \)-open set in \( Y \). Since \( f \) is a \((i,j)\)-completely fuzzy \( \gamma \)-irresolute, so \( f^{-1}(U) \) is \((i,j)\)-fuzzy regular open set in \( X \). Now since \( (f^{-1}(U)\cap A) \) is \((i,j)\)-fuzzy regular opens set in the subspace \((A, (\tau_i, \tau_j)_A)\) so \( f^{-1}(U)\cap A=(f|_A)^{-1}(U) \). Hence \( f|_A \) is \((i,j)\)-completely fuzzy \( \gamma \)-irresolute. \( \square \)

**Theorem 2.5.** A function \( f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2) \) is \((i,j)\)-completely fuzzy \( \gamma \)-irresolute if the graph function \( g:X\to XXY \) is \((i,j)\)-completely fuzzy \( \gamma \)-irresolute.

**Proof.** Let \( x\in X \) and \( V \) be a \((i,j)\)-fuzzy \( \gamma \)-open set containing \( f(x) \). Thus \( XXV \) is a \((i,j)\)-fuzzy \( \gamma \)-open set of \( XXY \) containing \( g(x) \). Now \( g^{-1}(XXV)=f^{-1}(V) \) is a \((i,j)\)-fuzzy regular open set containing \( x \). This shows that \( f \) is \((i,j)\)-completely fuzzy \( \gamma \)-irresolute.

The next theorem is established to study the composition of \((i,j)\)-completely fuzzy \( \gamma \)-irresolute, \((i,j)\)-fuzzy \( \gamma \)-irresolute and \((i,j)\)-fuzzy \( \gamma \)-continuous functions.

**Theorem 2.6.** For a function \( f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2) \) and \( g:(Y,\sigma_1,\sigma_2)\to(Z,\eta_1,\eta_2) \) the following statements hold true:

i. If \( f \) is \((i,j)\)-completely fuzzy \( \gamma \)-irresolute and \( g \) is \((i,j)\)-completely fuzzy \( \gamma \)-irresolute, then \( gof \) is \((i,j)\)-completely fuzzy \( \gamma \)-irresolute.

ii. If \( f \) is \((i,j)\)-completely fuzzy \( \gamma \)-irresolute and \( g \) is \((i,j)\)-fuzzy \( \gamma \)-irresolute, then \( gof \) is \((i,j)\)-completely fuzzy \( \gamma \)-irresolute.

iii. If \( f \) is \((i,j)\)-completely fuzzy \( \gamma \)-irresolute and \( g \) is \((i,j)\)-fuzzy \( \gamma \)-continuous, then \( gof \) is \((i,j)\)-completely fuzzy \( \gamma \)-irresolute.

**Proof.** Proof is obvious. \( \square \)
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**Definition 2.3.** A sub set $B$ in a fuzzy bitopological space $(X,\tau_1,\tau_2)$ is called $(i,j)$-fuzzy generalized closed set if for any $\tau_i$-fuzzy open set $U$, $\tau_j$-$Fcl(B)\subset U$ whenever $B\subset U$.

**Theorem 2.7.** Let $f:(X,\tau_1,\tau_2)\to (Y,\sigma_1,\sigma_2)$ be a map from $X$ to $Y$ and if $f^{-1}(A)$ is both $\tau_i$-fuzzy open set and $(i,j)$-fuzzy generalized closed set in $X$ then $f$ is a $(i,j)$-completely fuzzy $\gamma$-irresolute function.

**Proof.** Let $B$ be a $(i,j)$-fuzzy $\gamma$- open set in $Y$. Therefore from the hypothesis, we have $f^{-1}(B)$ is $\tau_i$-fuzzy open set in $X$. Thus $f^{-1}(B)\subseteq i\text{-int}(j\text{-cl}(f^{-1}(B)))$. Again from the hypothesis we get $i\text{-int}(j\text{-cl}(f^{-1}(B)))\subseteq f^{-1}(B)$.

Therefore from the above two relations we got $f^{-1}(B)=i\text{-int}(j\text{-cl}(f^{-1}(B)))$. Thus $f^{-1}(B)$ is a $(i,j)$-fuzzy regular open set in $X$.

Hence $f$ is $(i,j)$-completely fuzzy $\gamma$ -irresolute function.

3. (i,j)-Fuzzy $\gamma^*$-Closed Mapping and (i,j)-Fuzzy-$\gamma$-Normal Spaces

In this section we introduce and study the concepts of $(i,j)$-fuzzy $\gamma^*$-closed mapping and $(i,j)$-fuzzy $\gamma$-normal spaces. Also some of their properties are investigated.

**Definition 3.1.** A function $f:(X,\tau_1,\tau_2)\to (Y,\sigma_1,\sigma_2)$ is called $(i,j)$-fuzzy $\gamma^*$-closed mapping if the image of each $(i,j)$-fuzzy $\gamma$-closed set of $X$ is $(i,j)$-fuzzy $\gamma$-closed set in $Y$.

**Theorem 3.1.** If a mapping $f:(X,\tau_1,\tau_2)\to (Y,\sigma_1,\sigma_2)$ is $(i,j)$-fuzzy $\gamma^*$-closed mapping then for each subset $B$ of $Y$ and each $(i,j)$ -fuzzy $\gamma$-open set $U$ of $X$ containing $f^{-1}(B)$there exists a $(i,j)$-fuzzy $\gamma$ -open set $V$ containing $B$, such that $f^{-1}(V)\subset U$.

**Proof.** Let $f$ is $(i,j)$-fuzzy $\gamma^*$- closed mapping. Let $B \subset Y$ and $U$ be any $(i,j)$-fuzzy $\gamma$-open set in $X$ such that $f^{-1}(B)\subset U$. But since $f$ is a $(i,j)$-fuzzy $\gamma^*$-closed mapping, so $(f(U^c))^c=V$ is $(i,j)$-fuzzy $\gamma$-open set in $Y$ containing $B$ such that $f^{-1}(V)\subset U$.

**Theorem 3.2.** Let $(X,\tau_1,\tau_2)$ and $(Y,\sigma_1,\sigma_2)$ be two fuzzy bitopological spaces. If $f:(X,\tau_1,\tau_2)\to (Y,\sigma_1,\sigma_2)$ is bijective mapping then followings are equivalent:
i. $f$ is $(i,j)$-fuzzy $\gamma^*$-closed mapping and

ii. $f^{-1}$ is $(i,j)$-fuzzy $\gamma$-irresolute mapping.

Proof. (i) $\rightarrow$ (ii) Let $U$ be any $(i,j)$-fuzzy $\gamma$-closed set in $X$. Now since $f$ is a $(i,j)$-fuzzy $\gamma^*$-closed mapping, then $(f^{-1})^{-1}(U) = f(U)$ is $(i,j)$-fuzzy $\gamma$-closed set in $Y$. Hence $f^{-1}$ is $(i,j)$-fuzzy $\gamma$-irresolute mapping.

(ii) $\rightarrow$ (i) Let $V$ be any $(i,j)$-fuzzy $\gamma$-closed set in $X$. Now since $f^{-1}$ is $(i,j)$-fuzzy $\gamma$-irresolute mapping, so $(f^{-1})^{-1}(V^c) = f(V^c)$ is $(i,j)$-fuzzy $\gamma$-open set in $Y$. Thus $f(V)$ is $(i,j)$-fuzzy $\gamma$-closed set in $Y$. Hence $f$ is $(i, j)$-fuzzy $\gamma^*$-closed mapping.

Theorem 3.3. Let $f:(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g:(Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be two mappings. If $gof$ is $(i,j)$-fuzzy $\gamma^*$-closed mapping and $f$ is $(i,j)$-fuzzy $\gamma$-irresolute surjection, then $g$ is also $(i,j)$-fuzzy $\gamma^*$-closed map.

Proof. Let $A$ be any $(i,j)$-fuzzy $\gamma$-closed set in $Y$. Thus from the hypothesis, $f^{-1}(A)$ is $(i,j)$-fuzzy $\gamma$-closed set in $X$ and $gof(f^{-1}(A)) = g(A)$ is $(i,j)$-fuzzy $\gamma$-closed set in $Z$. Hence $g$ is $(i,j)$-fuzzy $\gamma^*$-closed map.

Theorem 3.4. Let $f:(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a mapping between two fuzzy bitopological spaces. Then followings are equivalent:

i. $f$ is $(i,j)$-fuzzy $\gamma^*$-closed mapping and

ii. $(i,j)-F_{\gamma}cl(f(A)) \subset f((i,j)-F_{\gamma}cl(A))$ for every subset $A$ of $X$.

Proof. (i) $\rightarrow$ (ii) $A \subset (i,j)-F_{\gamma}cl(A)$

$\Rightarrow f(A) \subset f((i,j)-F_{\gamma}cl(A))$

$\Rightarrow (i,j)-F_{\gamma}cl(A) \subset (i,j)-F_{\gamma}cl(f((i,j)-F_{\gamma}cl(A)) = f((i,j)-F_{\gamma}cl(A))$ [by (i)].

(ii) $\rightarrow$ (i) Let $B$ be a $(i,j)$-fuzzy $\gamma$-closed set in $X$. Now by (ii),

$(i,j)-F_{\gamma}cl(f(B)) \subset f((i,j)-F_{\gamma}cl(B)) = f(B)$.

$\Rightarrow (i,j)-F_{\gamma}cl(f(B)) \subset f(B)$.

This shows that $f$ is $(i,j)$-fuzzy $\gamma^*$-closed map.

Definition 3.2. A space $(X, \tau_1, \tau_2)$ is said to be $(i,j)$-fuzzy normal if for each pair of disjoint $\tau_i$-fuzzy closed set $A$ and $\tau_j$-fuzzy closed set $B$ there exists two disjoint sets $U$ and $V$ such that $U$ is $\tau_j$-fuzzy open and $V$ is $\tau_i$-fuzzy open, $A \subset U$ and $B \subset V$, where $i \neq j$ and $i, j = 1, 2$. 

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Example 3.1. Let $X=\{a,b\}$ and consider the fuzzy sets on $X$ are $\mathcal{U},\mathcal{T},A=\{(a,0.8),(b,1]\}, B=\{(a,0),(b,0.7]\} , C=\{(a,1),(b,0.7]\} and $D=\{(a,0.3),(b,0]\}$. Also let $\tau_1=\{\mathcal{U},\mathcal{T},A,B\}$ and $\tau_2=\{\mathcal{U},\mathcal{T},C,D\}$ be two fuzzy topologies on $X$. Here $(X-A)$ and $(X-C)$ be two disjoint $\tau_1$-fuzzy closed set and $\tau_2$-fuzzy closed set in $X$ which are contained in respectively two disjoint $\tau_2$-fuzzy open set and $\tau_1$-fuzzy open set $B$ and $D$.

Definition 3.3. A space $(X,\tau_1,\tau_2)$ is said to be $(i,j)$-fuzzy $\gamma$-normal if for any disjoint subsets $F_1, F_2 \in (i,j)-F_{\gamma}C(X)$ there exist disjoints subsets $U, V \in (i,j)-F_{\gamma}O(X)$ such that $F_1 \subset U$ and $F_2 \subset V$, where $i \neq j$ and $i, j = 1, 2$.

Remark 3.1. The next example shows that every $\tau_1$-fuzzy open set may not be a $(1,2)$-fuzzy $\gamma$-open set. Also every $\tau_2$-fuzzy open set may not be $(2,1)$-fuzzy $\gamma$-open set.

Example 3.2. Let $X=\{a,b\}$ and consider the fuzzy sets on $X$ are $\mathcal{U},\mathcal{T},A=\{(a,0.2),(b,0.3]\}, B=\{(a,0.9),(b,0.7]\}$. Also let $\tau_1=\{\mathcal{U},\mathcal{T},A\}$ and $\tau_2=\{\mathcal{U},\mathcal{T},B\}$ be two fuzzy topologies on $X$. Here $\tau_1$-fuzzy open set $A$ is not a $(1,2)$-fuzzy $\gamma$-open set. Again $\tau_2$-fuzzy open set $B$ is not a $(2,1)$-fuzzy $\gamma$-open set.

From this above example we can say that every $(i,j)$-fuzzy normal space may not be $(i,j)$-fuzzy $\gamma$-normal space, where $i \neq j$ and $i, j = 1, 2$.

Question 3.1. Is there an example of disjoint $(i,j)$-fuzzy $\gamma$-open sets showing that $(i,j)$-fuzzy $\gamma$-normality does not imply $(i,j)$-fuzzy normality.

The next theorem is given for the condition under which $(i,j)$-fuzzy $\gamma$-normality is preserved.

Theorem 3.5. If $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is $(i,j)$-fuzzy $\gamma$-irresolute and $(i,j)$-fuzzy $\gamma^*$-closed surjection and $X$ is $(i,j)$-fuzzy $\gamma$-normal then $Y$ is $(i,j)$-fuzzy $\gamma$-normal.

Proof. Let $F_1,F_2 \in (i,j)-F_{\gamma}C(Y)$ such that $F_1 \cap F_2 = \emptyset$. Since $f$ is a $(i,j)$-fuzzy $\gamma$-irresolute, therefore $f^{-1}(F_1) \cap f^{-1}(F_2) = \emptyset$ and $f^{-1}(F_1), f^{-1}(F_2) \in (i,j)-F_{\gamma}C(X)$. Again from the hypothesis $X$ is a $(i,j)$-fuzzy $\gamma$-normal so there exist disjoints subsets $U, V \in (i,j)-F_{\gamma}O(X)$ such that $f^{-1}(F_1) \subset U$ and $f^{-1}(F_2) \subset V$. Again since $f$ is a $(i,j)$-fuzzy $\gamma^*$-closed surjection, therefore $f(U^c)$ and $f(V^c) \in (i,j)-F_{\gamma}C(Y)$ and $f^{-1}(F_1) \subset U$ implies $F_1 \subset G, F_2 \subset H$ such that $G \cap H = \emptyset$.

4. Concluding Remarks

A $(i,j)$-fuzzy $\gamma$-irresolute function may not be a $(i,j)$-completely fuzzy $\gamma$-irresolute function but in this paper we created a condition under which the fact
is true. We have obtained some sufficient conditions which are useful for further study on $(i, j)$-completely fuzzy $\gamma$- irresolute function and related concepts. Also in this paper we have study the notion of $(i, j)$-fuzzy $\gamma^*$- closed mapping and shown that its inverses is a $(i, j)$-fuzzy $\gamma$- irresolute mapping. Finally it is found that $(i, j)$-fuzzy $\gamma$-normality is preserved under $(i, j)$-fuzzy $\gamma^*$- closed mapping.

References


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