A Systematic Derivation of Stochastic Taylor Methods for Stochastic Delay Differential Equations

Abstract

The physical and biological systems that behave in the presence of random effects and time delay can often be modeled via stochastic delay differential equations (SDDEs). The analytical solution of SDDEs is hard to be found; hence numerical methods provide a way to solve such problems. The researches on numerical methods for SDDEs are far from complete, although there were many results for numerical solutions of stochastic differential equations (SDEs). The main difficulty to the development of high order numerical schemes for SDDEs is the derivation of stochastic Taylor expansion with arbitrarily high orders. Taylor expansion is a fundamental and frequently used in a numerical analysis for the derivation of almost all of deterministic and stochastic numerical methods. An obvious distinction between Taylor expansion of SDDEs and SDEs is that Taylor expansion in SDDEs contains the multiple stochastic integrals with time delay that have to be approximated. This presentation demonstrates a systematic derivation of stochastic Taylor methods for solving SDDEs with a constant time lag, $r>0$. Initially, the fundamental background of SDEs and SDDEs as well as review on numerical methods currently available to solve these differential equations will be addressed. Then, stochastic Taylor expansion for SDDEs will be derived and numerical schemes of SDDEs from the truncated stochastic Taylor series will be developed. We provide the convergence proof of one-step stochastic Taylor methods when the drift and diffusion functions are Taylor expansion. It shows that the stochastic Taylor methods of SDDEs converge in mean-square sense.