

Influence of Disorder on the Fidelity Susceptibility in the BCS-BEC Crossover

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Cooling Techniques

② BCS-BEC Crossover

Motivation

The Theory of Crossover

③ Disorder

Anderson Localization (AL)

Experiments

④ The Dirty Crossover

Continuum Model (3D)

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⑤ Conclusion

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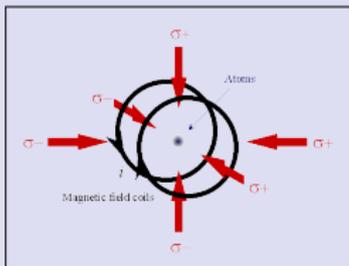
- ✓ Observation of superconductivity in 1911.
- ✓ Theoretical Prediction of Bose-Einstein Condensate in 1925.
- ✓ Theory of Superconductivity (BCS) in 1957.
- ✓ Experimental Observation of Bose-Einstein Condensate in 1995.



Image Source:

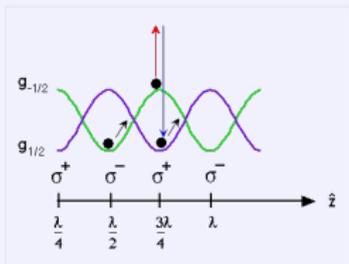
1. <http://www.quantumconsciousness.org/penrose-hameroff/anesthesiahydrophobic.html>

Doppler Cooling



- ▶ Six lasers applied opposite to each other from each direction.
- ▶ Light is red detuned to activate Doppler effect.
- ▶ Temperature attain $\lesssim 1mK$.

Sisyphus Cooling



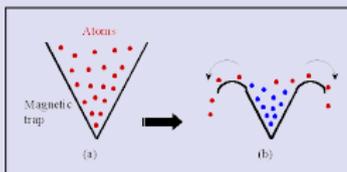
- ▶ Opposite polarization of the laser beams create a potential crest and valley.
- ▶ Takes down temperature to $\lesssim 1\mu K$.

Image Source:

2. http://www.physics.otago.ac.nz/research/jackdodd/resources/exp_aspects.html

3. <http://cold-atoms.physics.lsa.umich.edu/projects/lattice/sis1.html>

Evaporative Cooling



- ▶ Cut the higher edge of the magnetic trap with rf spectra. Atoms with higher energy will leave the pot leaving cooler atoms inside.
- ▶ Takes down temperature to nK level.

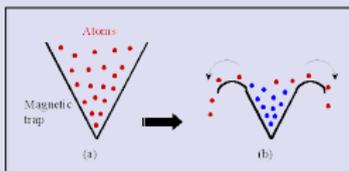
Sympathetic Cooling

- ▶ Evaporative cooling does not work well for fermions.
- ▶ Mix evaporative cooled bosons to fermions to cool the laser cooled fermions sympathetically.

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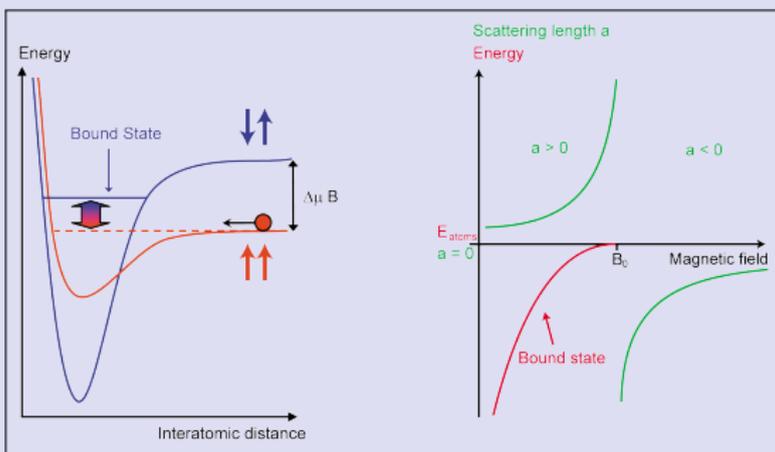
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Feshbach Resonance

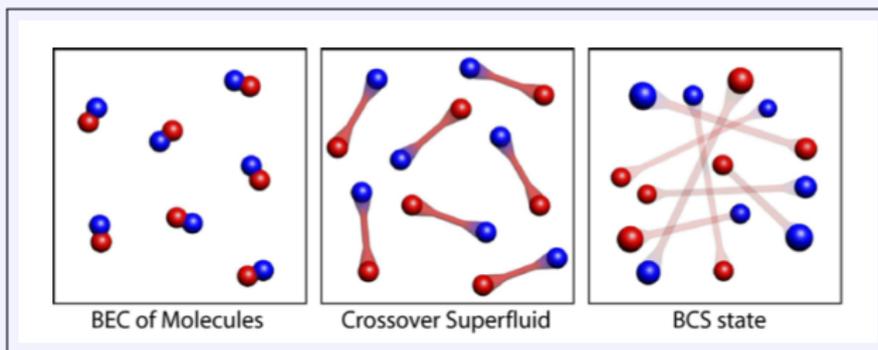


$$a = a_0 \left[1 - \frac{\Delta B}{B - B_0} \right]$$

Image Source:

5. http://cua.mit.edu/ketterle_group/experimental_setup/BEC.1/background.html

BCS-BEC Crossover



Phase diagram BCS-BEC Crossover

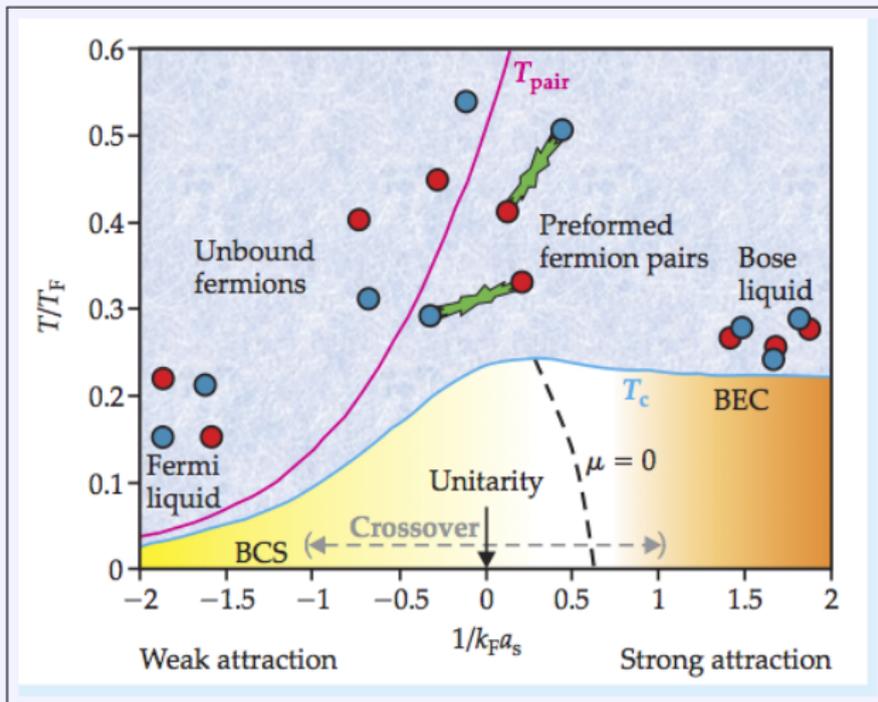


Image Source:

Beauty of BCS Wave function

- ▶ BCS ground state wave function can be extended to BEC limit under some constraint.

$$|\Psi\rangle = \prod_{\mathbf{k}} [u_{\mathbf{k}} + v_{\mathbf{k}} c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}^{\dagger}] |0\rangle$$

- ▶ Set $g_{\mathbf{k}} = v_{\mathbf{k}}/u_{\mathbf{k}}$,

$$|\Psi\rangle = \left(\prod_{\mathbf{k}'} u_{\mathbf{k}'} \right) \exp \left[\sum_{\mathbf{k}} g(\mathbf{k}) c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}^{\dagger} \right] |0\rangle$$

- ▶ Define a new operator: $b^{\dagger} = \sum_{\mathbf{k}} g(\mathbf{k}) c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}^{\dagger}$
- ▶ $[b, b^{\dagger}] = \sum_{\mathbf{k}} |g(\mathbf{k})|^2 (1 - n_{\mathbf{k}\uparrow} - n_{-\mathbf{k}\downarrow}) \neq c$ number.
- ▶ Provided $\langle n_{\mathbf{k}\sigma} \rangle \ll 1 \Rightarrow [b, b^{\dagger}] = c$ number.
- ▶ $|\Psi\rangle = \exp(b^{\dagger})|0\rangle$ represents a Bosonic coherent state.

- ▶ The appropriate Hamiltonian:

$$H_{eff} = \int d\mathbf{r} \left\{ \sum_{\alpha} \Psi^{\dagger}(\mathbf{r}\alpha) H_e(\mathbf{r}) \Psi(\mathbf{r}\alpha) + V(\mathbf{r}) \Psi^{\dagger}(\mathbf{r}\alpha) \Psi(\mathbf{r}\alpha) \right\} \\ + \int d\mathbf{r} \left\{ \Delta(\mathbf{r}) \Psi^{\dagger}(\mathbf{r}\uparrow) \Psi^{\dagger}(\mathbf{r}\downarrow) + \Delta^*(\mathbf{r}) \Psi(\mathbf{r}\downarrow) \Psi(\mathbf{r}\uparrow) \right\}$$

- ▶ Perform unitary transformation³:

$$\Psi(\mathbf{r}\uparrow) = \sum_n \left(\gamma_{n\uparrow} u_n(\mathbf{r}) - \gamma_{n\downarrow}^{\dagger} v_n^*(\mathbf{r}) \right) \\ \Psi(\mathbf{r}\downarrow) = \sum_n \left(\gamma_{n\downarrow} u_n(\mathbf{r}) + \gamma_{n\uparrow}^{\dagger} v_n^*(\mathbf{r}) \right)$$

3. P. D. de Gennes *Superconductivity of Metals and Alloys*, Addition-Wesley Publishing Company, Inc. 1989.

Bogoliubov deGennes Equation

$$\epsilon u(\mathbf{r}) = [H_e + V(\mathbf{r})] u(\mathbf{r}) + \Delta(\mathbf{r})v(\mathbf{r})$$

$$\epsilon v(\mathbf{r}) = -[H_e^* + V(\mathbf{r})] v(\mathbf{r}) + \Delta^*(\mathbf{r})u(\mathbf{r})$$

- ▶ The order parameter is $\Delta(\mathbf{r}) = g \sum_n v_n^*(\mathbf{r})u_n(\mathbf{r})(1 - 2f_n)$.

Gap & Density Equation

$$\Delta(\mathbf{r}) = g \sum_{\epsilon_n > 0} u_n(\mathbf{r})v_n^*(\mathbf{r})$$

$$n(\mathbf{r}) = 2 \sum_{\epsilon_n > 0} |v_n(\mathbf{r})|^2$$

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- ▶ Consider no external potential
- ▶ fermion-fermion interaction is mediated via short range contact potential.

Gap & Density Equation

$$-\frac{m}{4\pi a} = \sum_k \left[\frac{1}{2E_k} - \frac{1}{2\epsilon_k} \right]$$

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Definition

Beyond a critical amount of impurity, motion of the electron can come to a complete halt. The electron becomes trapped and the conductivity vanishes.

- ▶ Direct observation of AL for electron is very difficult.
 - Number of phenomena can mask single particle quantum effects genuinely induced by disorder.
 - Most of the evidences are indirect and stem from conductivity measurement.
- ▶ Cold atoms are good candidate for observation of AL:
 - Genuine quantum particles described as matter waves.
 - Single atom matter waves can be directly visualized by different imaging techniques in Bose-Einstein Condensate.

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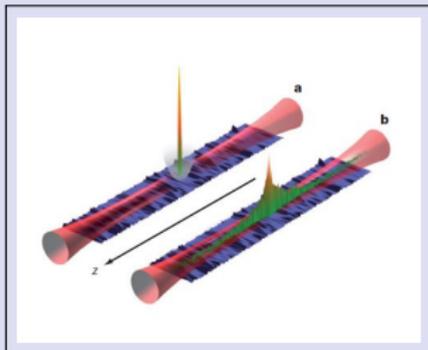
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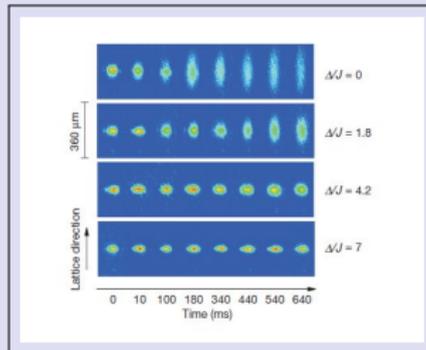
In One Dimension

Optical Speckle, ^{87}Rb



Nature **453**, 891 (2008).

Bichromatic Lattice, ^{39}K



Nature **453**, 895 (2008).

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What to Expect?

- ▶ **Disorder should not affect the BCS superfluid.**
- ▶ **Disorder should seriously affect the molecular BEC.**
- ▶ **What happens in the crossover?**

The Scheme

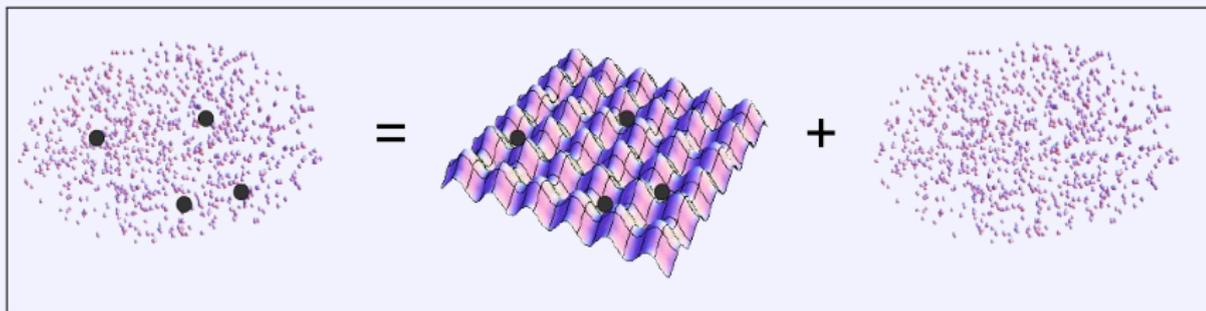


Image Source:

9. L. Han & C. A. R. Sa de Melo, *New J. Phys.* **13**, 055012 (2011).

Theory-I

The Hamiltonian

$$H = \Psi_{\sigma}^{\dagger} \left[\left(-\frac{\nabla^2}{2m} - \mu \right) + V(\mathbf{r}) \right] \Psi_{\sigma} - g \Psi_{\uparrow}^{\dagger} \Psi_{\downarrow}^{\dagger} \Psi_{\downarrow} \Psi_{\uparrow}$$

- ▶ The random potential originating from the scattering of fermions against impurity atoms:

$$V(\mathbf{r}) = \sum_j g_d \delta(\mathbf{r} - \mathbf{R}_j).$$

- ▶ Corresponding correlation function: $\langle V(-q)V(q) \rangle = \beta \delta_{i\omega_m, 0} \kappa$, where $q = (\mathbf{q}, i\omega_m)$ and κ is the disorder strength.

Theory-II

Modified gap and density equation³:

$$-\frac{m}{4\pi a} = \sum_k \left[\frac{1}{2E_k} - \frac{1}{2\epsilon_k} \right]$$

$$n = \sum_k \left[1 - \frac{\xi_k}{E_k} \right] - \frac{\partial \Omega_B}{\partial \mu}$$

Disorder induced thermodynamic potential:

$$\Omega_B = \lim_{\beta \rightarrow 0} \frac{1}{2\beta} \sum_q \ln |M| - \frac{\kappa}{2} \sum_{q, \omega_m=0} W^\dagger M^{-1} W$$

4. G. Orso, Phys. Rev. Lett. 99, 250402 (2007).

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Analytical Extensions⁵:

$$\frac{1}{k_F a} = -\frac{2}{\pi} \left[\frac{2}{3l_2(x_0)} \right]^{1/3} l_1(x_0)$$

$$\frac{\Delta}{\epsilon_F} = \left[\frac{2}{3l_2(x_0)} \right]^{2/3}.$$

Disorder induced Density Equation⁶:

$$\frac{\Delta}{\epsilon_F} = \left(\frac{2}{3l_1(x_0)} \right)^{2/3} + \frac{\eta}{\pi^2} l_3(x_0),$$

$$\frac{\Delta - \Delta(\eta = 0)}{\epsilon_F} = \frac{\eta}{\pi^2} l_3(x_0), \quad \eta = \kappa m^2 / k_F$$

5. M. Marini, F. Pistolesi, and G. C. Strinati, Eur. Phys. J. B 1, 151, (1998).

6. Personal communication with G. Orso is acknowledged.

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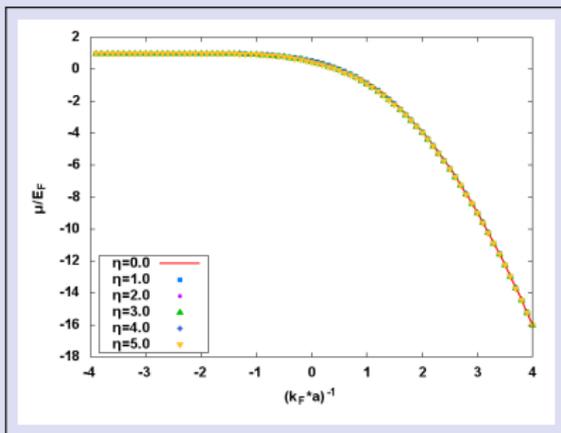
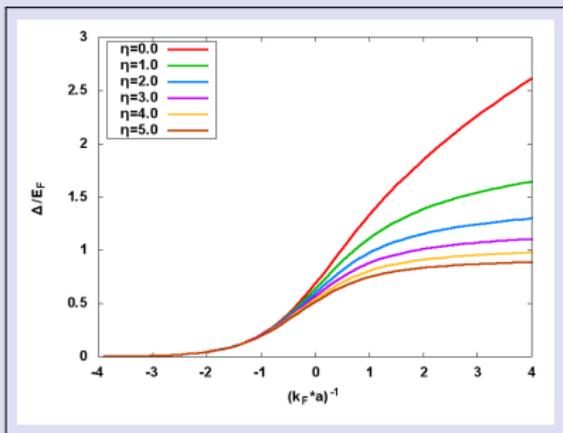
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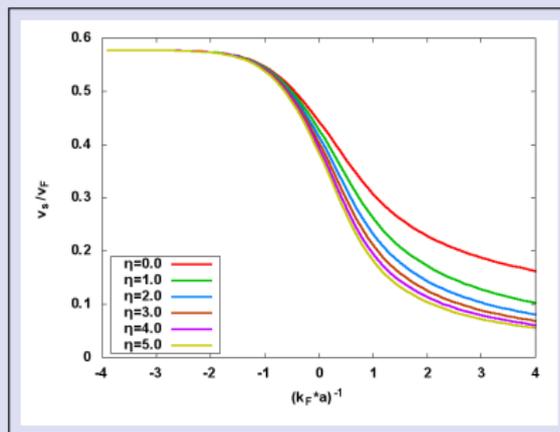
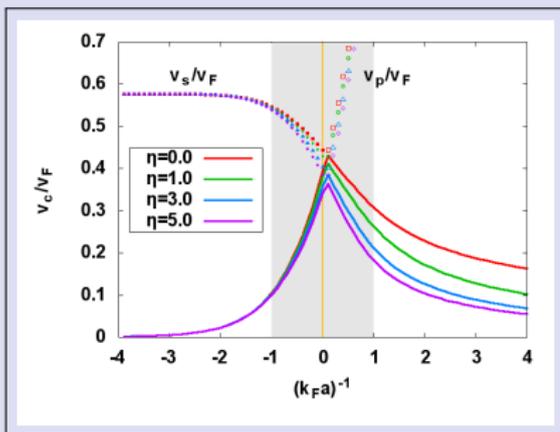
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Δ & μ 

What they say?

- Order parameter (Δ) remains unaffected by weak disorder in the BCS limit but follows the approximation to the hard core bosons in BEC limit i.e $\Delta - \Delta_0 \propto \eta/k_F a$.
- Chemical potential (μ) remains unaffected.

Continuum Model (3D)

 v_c & v_s 

What they say?

- Critical velocity (v_c) is nonmonotonic and the maxima is pinned in the vicinity of $a \rightarrow \infty$.
- Sound velocity (v_s) get depleted in the BEC side might be due to additional random scattering rendered by impurity.

Fidelity in Quantum Phase Transition

- ▶ Fidelity is the measure of closeness of two quantum states, $F = |\langle \Psi | \Phi \rangle|$ (for normalized states).
- ▶ Quantum Phase Transition is a sudden change in the ground state of a many body system when a controlling parameter λ of the Hamiltonian crosses critical value λ_c .
- ▶ There should be an abrupt change in the fidelity $F(\lambda + \delta\lambda, \lambda) = |\langle \Psi(\lambda + \delta\lambda) | \Psi(\lambda) \rangle|$ in the vicinity of λ_c .
- ▶ A hasty drop of the ground-state fidelity at the critical point will then correspond to a divergence of the fidelity susceptibility,

$$\chi(\lambda) = \frac{1}{\Omega} \frac{\partial \langle \Psi(\lambda) | \partial | \Psi(\lambda) \rangle}{\partial \lambda}.$$

Fidelity in BCS-BEC Crossover

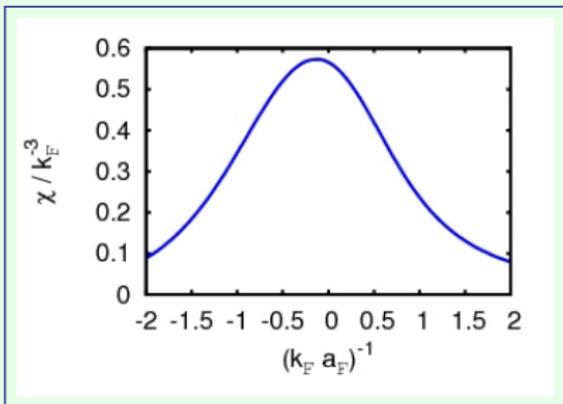
- ▶ Start with BCS ground state wave function⁷,
 $|\Psi(\lambda)\rangle = \prod_{\mathbf{k}} [u_{\mathbf{k}}(\lambda) + v_{\mathbf{k}}(\lambda)c_{\mathbf{k}\uparrow}^{\dagger}c_{-\mathbf{k}\downarrow}^{\dagger}]|0\rangle.$
- ▶ The fidelity-susceptibility is:

$$\chi(\lambda) = \int \frac{d\mathbf{k}}{(2\pi)^3} \left[\left(\frac{du_{\mathbf{k}}}{d\lambda} \right)^2 + \left(\frac{dv_{\mathbf{k}}}{d\lambda} \right)^2 \right].$$
- ▶ The dependence of $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ on λ is determined by BCS gap and density equation:

$$\Delta_{\mathbf{k}} = - \int \frac{d\mathbf{k}'}{(2\pi)^3} V_{\lambda}(\mathbf{k}, \mathbf{k}') \frac{\Delta_{\mathbf{k}'}}{2E_{\mathbf{k}'}} , \quad n = \int \frac{d\mathbf{k}}{(2\pi)^3} 2v_{\mathbf{k}}^2.$$
- ▶ The resulting fidelity susceptibility will be:

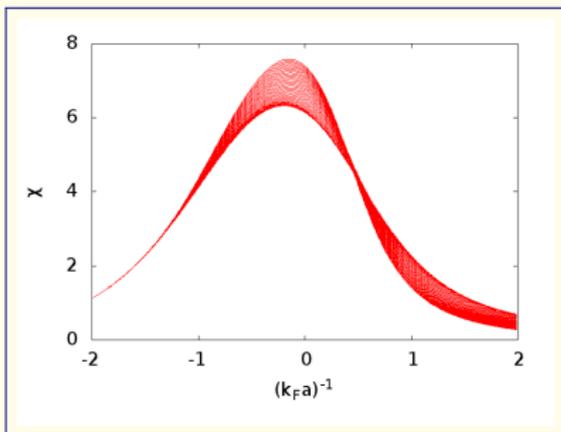
$$\chi(\lambda) = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{4E_{\mathbf{k}}^4} \left[\Delta_{\mathbf{k}} \frac{d\mu}{d\lambda} + \xi_{\mathbf{k}} \frac{d\Delta_{\mathbf{k}}}{d\lambda} \right]^2.$$

Clean Fermi Gas



A. Khan, P. Pieri, Phys. Rev. A **80**, 012303 (2009).

Dirty Fermi Gas

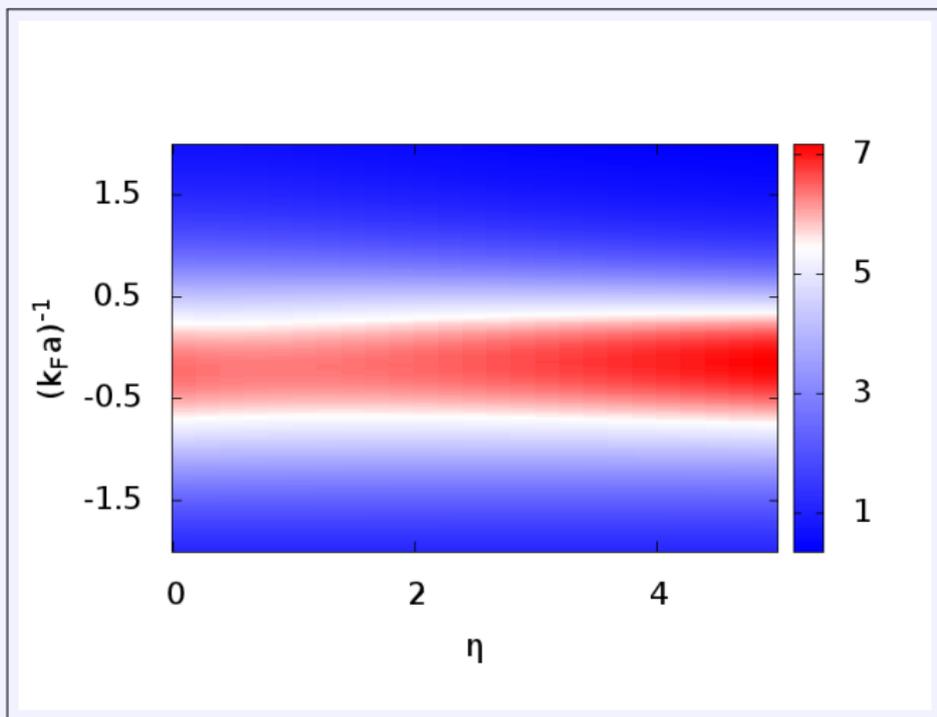


A.Khan, S. Basu and BT, *submitted in Phys. Lett. A*.

Notations

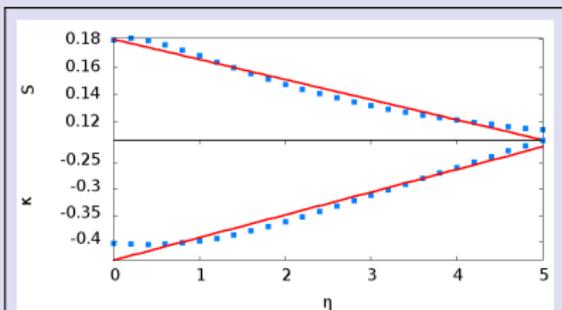
- ▶ χ is in dimensions of k_F^{-3} in both the plots.
- ▶ $\lambda = (k_F a)^{-1}$.

Fidelity Susceptibility (FS)



A.Khan, S. Basu and BT, *submitted in Phys. Lett. A.*

Statistical Analysis



A.Khan, S. Basu and BT, *submitted in Phys. Lett. A.*

Definition

- ▶ Skewness

$$S = \frac{\langle (x - \langle x \rangle)^3 \rangle}{\langle (x - \langle x \rangle)^2 \rangle^{3/2}}.$$

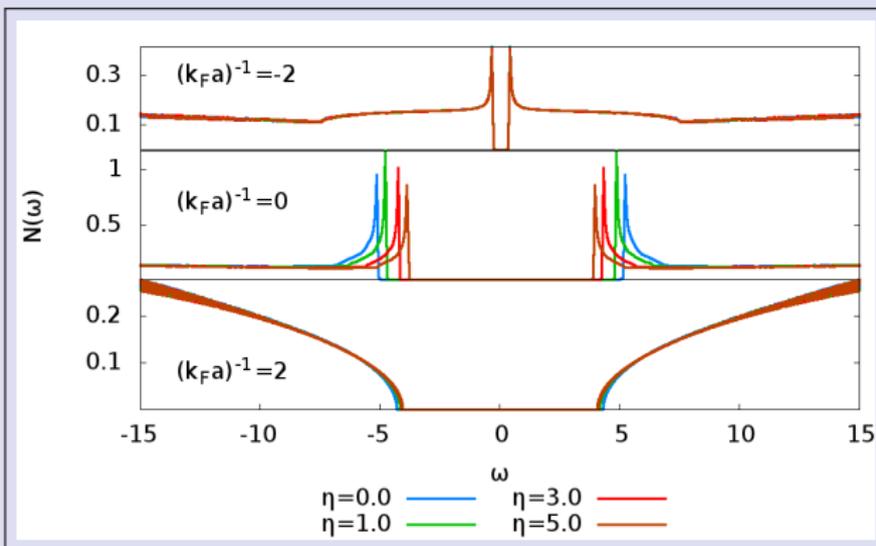
- ▶ Kurtosis

$$\kappa = \frac{\langle (x - \langle x \rangle)^4 \rangle}{\langle (x - \langle x \rangle)^2 \rangle^2} - 3.$$

- ▶ $x = (k_F a)^{-1}$.
- ▶ Both S and κ monotonically moves towards zero.
- ▶ $\eta_c = 10 \sim 13$ obtained from the linear fit of the data.

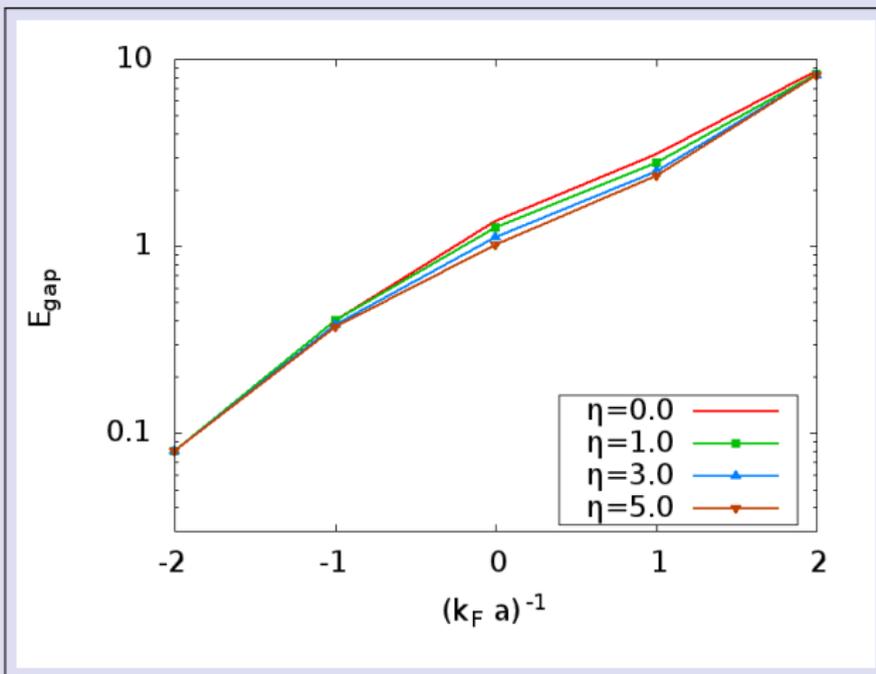
Density of States (DOS)

$$N(\omega) = \sum_k u_k^2 \delta(\omega - E_k) + v_k^2 \delta(\omega + E_k)$$



A.Khan, S. Basu and BT, *submitted in Phys. Lett. A.*

Spectral Gap



A.Khan, S. Basu and BT, *submitted in Phys. Lett. A.*

Observations

- $N(\omega)$ is low in the BCS and BEC regions but the singular pile up is high at the unitarity.
- Distinct reduction of spectral gap at the unitarity.
- S and κ data predict for a phase transition at a moderate to high disorder value.
- From the behavior of DOS as well as FS, we consider the possible phases after transition might be Anderson glass for BCS superfluid, Fermi glass for unitary superfluid and Bose glass for BEC superfluid.

- ▶ **Field of dirty crossover is introduced.**
- ▶ Effect of weak disorder in three dimensional continuum model:
 - Monotonic depletion of order parameter is observed.
 - Nonmonotonic behavior of condensate fraction is discussed.
 - Suppression of sound velocity is presented.
- ▶ Study of FS and DOS:
 - The FS loses symmetric nature in presence of disorder, associated skewness and kurtosis approach zero for large disorder strength.
 - Spectral gap is considerably reduced at unitarity where as BCS and BEC extremes remains unaffected.

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Thanks to my Collaborators

- ▶ **Collaboration of Ayan Khan with Sang Wook Kim is acknowledged.**
- ▶ **Saurabh Basu, Indian Institute of Technology Guwahati, India.**
- ▶ **Ayan Khan, Bilkent University, Turkey.**

Thank You for Your Kind Attention