Nonclassical Photons via Time- and Intensity-Dependent Coupling of Jaynes-Cumming Model

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Introduction

- Single two-level atom interacting with quantized field in a cavity is the simplest case.
- The connection between nonclassical light and collapse-revival dynamics in two-level system has been studied extensively, mainly by Banacloche [1].
- The similar work is the effect of time dependent field coupling on enhancing nonclassicality of light for atom moving through a photonic crystal with defect. [2]

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[1] Banacloche, J. G. Phys. Rev. Lett. 65 3385 (1990)
[2] Sherman, B. et. al. Phys. Rev. Lett. 69 1927 (1992)

Introduction

- However, the situation of time-dependent coupling $g(t)$ with intensity dependent has not been explored in the context of Wigner function.

- The motivation of the present work is field state control through time dependent intensity coupling.

- Such control may employ preselection and/ or postselection of the atomic state, as discussed by G. Harel et al. [3].

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[3] Harel, G. et al. Phys. Rev. A 53, 4534. (1996)

Introduction

Several related studies conducted with collaborators are:

- The study of the quantum dynamics of the two-atom two-mode two-photon JCM, generalizing the corresponding Hamiltonian by introducing an intensity-dependent coupling term.[4]

  [4] Sudha Singh, C. H. Raymond Ooi, and Amrita. Physical Review A 86 023810. (2012)

- Single-photon pulse propagation in and into a medium of two-level atoms: Microscopic Fresnel equations by Berman & Raymond Ooi.[5]

  [5] P. R. Berman and C. H. Raymond Ooi. Physical Review A 84 063851. (2011)

- Nonclassical photon correlation of nanoparticle in a Microcavity.[6]

  [6]C. H. Raymond Ooi and Qihuang Gong, Physical Review A 85, 023803. (2012)

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Present Work

- A system of two-level atom interacting with a quantized field in a high quality cavity is studied. The time- and intensity-dependent atom-field coupling are applied to the system, with different initial field states and initial atomic states.
- The dynamics of the atom and photon in the collapse-revival pattern of inversion and the Wigner function are investigated.

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Calculation

- A single two-level atom interacts with a resonant single mode field is described by the Hamiltonian:

$$\hat{V}_I(t) = \hbar g(t)(\hat{\sigma}_+\hat{R} + \hat{R}^\dagger\hat{\sigma}_-)$$

- Coupling function:

$$g(t) = g\sin^2(xgt)$$ *(Sinusoidal function)*

$$g(t) = \frac{g}{\cosh((t-t_0)/t_m)^4}$$ *(Hat function)*

- Intensity dependent:

$$\hat{R} = \hat{a}\sqrt{\hat{a}^\dagger\hat{a}}$$

$$\hat{R}^\dagger = \sqrt{\hat{a}^\dagger\hat{a}}\,\hat{a}^\dagger$$

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Calculation

- The evolution of the atom-field system can be described by the state vector

$$|\psi(t)\rangle = \sum_n [C_{a,n}(t)|a,n\rangle + C_{b,n}(t)|b,n\rangle]$$

- The interaction energy can only have transition between $|a,n\rangle$ and $|b,n+1\rangle$ so we obtain

$$\frac{d}{dt}C_{a,n}(t) = -ig(t)(n+1)e^{i\Delta t}C_{b,n+1}(t)$$

$$\frac{d}{dt}C_{b,n+1}(t) = -ig(t)(n+1)e^{-i\Delta t}C_{a,n}(t)$$

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Calculation

- For time independent coupling and finite detuning, the analytical solutions are known [7] as follows:

$$C_{a,n}(t) = e^{i\Delta t/2}[C_{a,n}(0)r_n(t) - iC_{b,n+1}(0)q_n(t)]$$
$$C_{b,n}(t) = e^{-i\Delta t/2}[C_{b,n}(0)r^*_{n-1}(t) - iC_{a,n-1}(0)q_{n-1}(t)]$$

where

$$r_n(t) = \cos\phi_n(t) - i\frac{\Delta}{\Omega_n}\sin\phi_n(t),$$
$$q_n(t) = \frac{2g(t)(n+1)}{\Omega_n}\sin\phi_n(t)$$
$$\phi_n(t) = \frac{1}{2}\int_0^t \Omega_n(t')dt'$$
$$\Omega_n^2 = \Delta^2 + (2g(t)(n+1))^2$$

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[7] Scully, M. O. and Zubairy, M. S. Quantum Optics. Cambridge University Press, UK (1997)

Calculation

- The density matrix elements ρ_{nm} were calculated

$$\rho_{nm}(t) = \langle n|\{\hat{\rho}_{aa}(t) + \hat{\rho}_{bb}(t)\}|m\rangle$$
$$= C_{a,n}(t)C^*_{a,m}(t) + C_{b,n}(t)C^*_{b,m}(t)$$

- The atomic inversion can be obtained using

$$n_{ab} = \sum_{n=0}^{\infty}(|C_{a,n}(t)|^2 - |C_{b,n}(t)|^2)$$

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Calculation

- The Wigner function, *W(t)* were plotted using the general analytical expression:

$$\frac{W(\alpha,t)}{K(\alpha)} = \sum_{m=0}^{\infty}(-1)^m L_m^0(x)\rho_{mm}(t) + \sum_{m=1}^{\infty}\sum_{n=0}^{m-1}(-1)^n\sqrt{\frac{n!}{m!}}L_n^k(x)2\mathrm{Re}\{z^k\rho_{nm}(t)\}$$

- Note that negativity of Wigner function are also found in Airy beam [8] .

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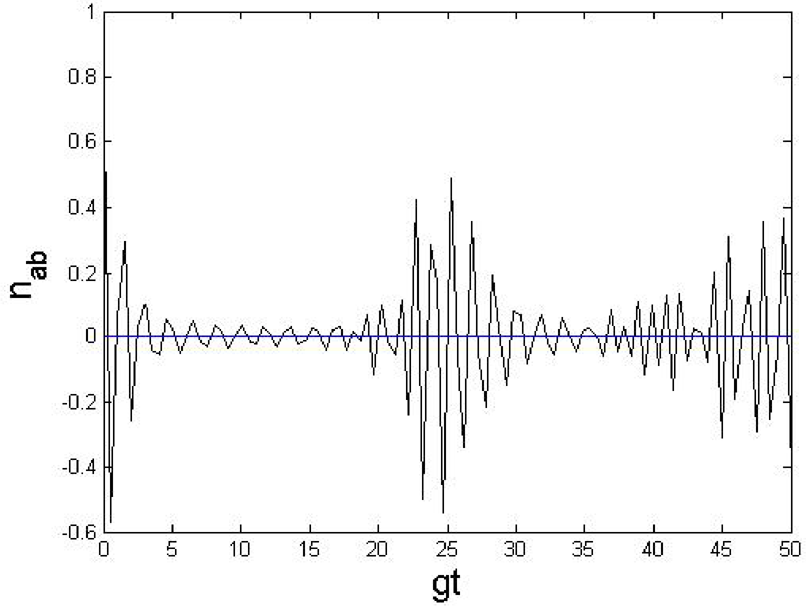
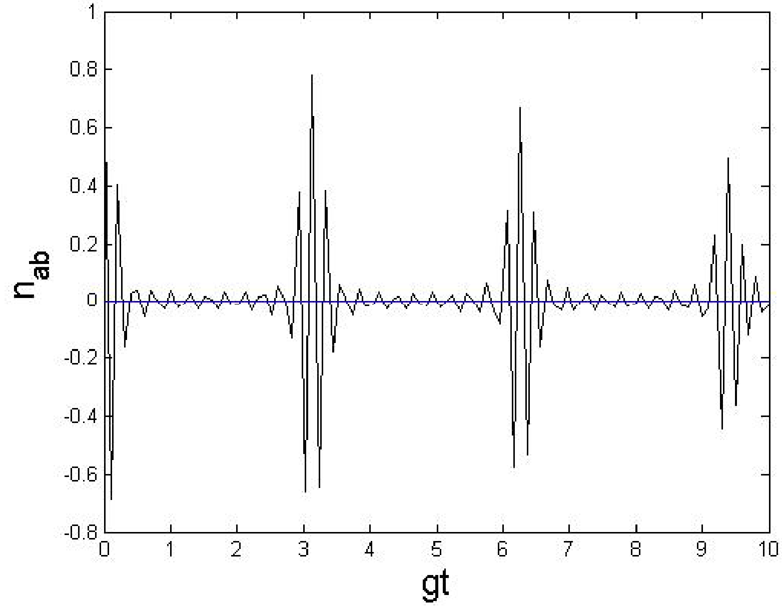
[8] R. P. Chen and C. H. Raymond. Physical Review A 84, 043846 (2011)

Results & Discussion

Case 1 :
Constant coupling, g(t)=g



Intensity independent



Intensity dependent

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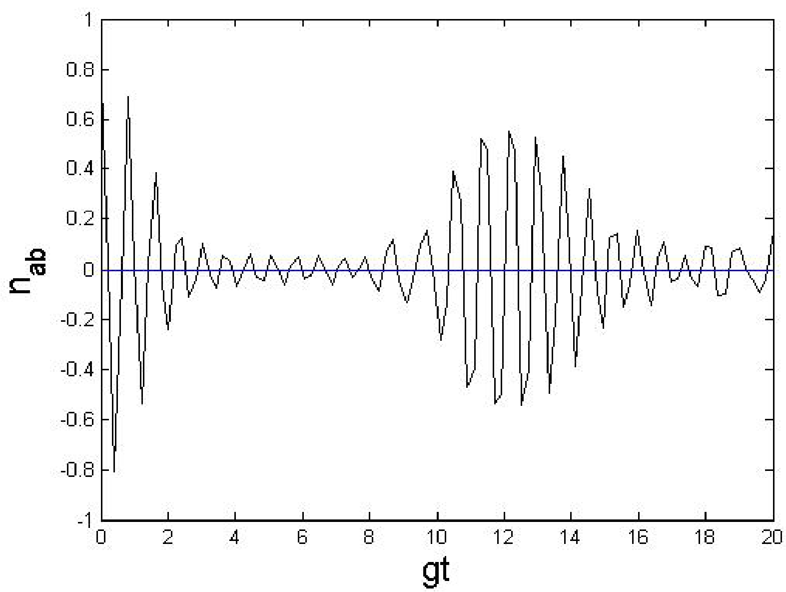
INITIAL COHERENT STATE

Results & Discussion

Case 1 :
Constant coupling, g(t)=g



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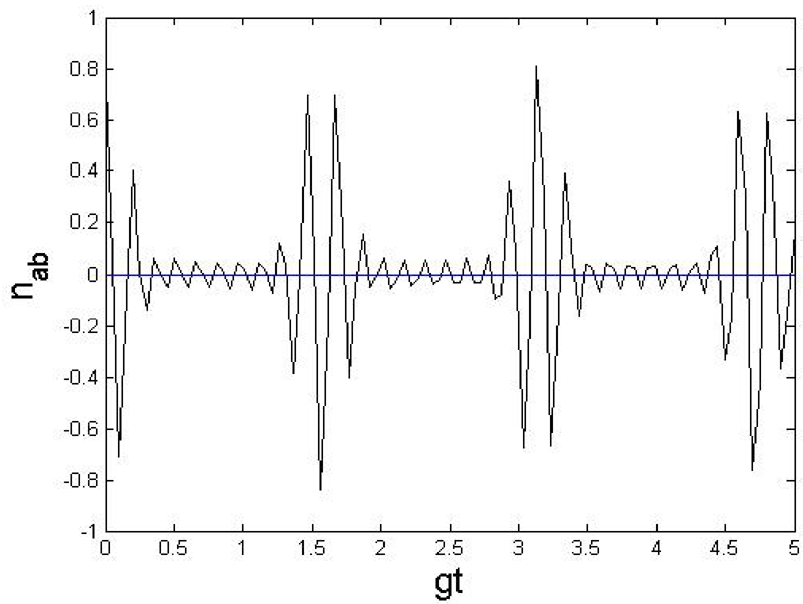

Intensity dependent

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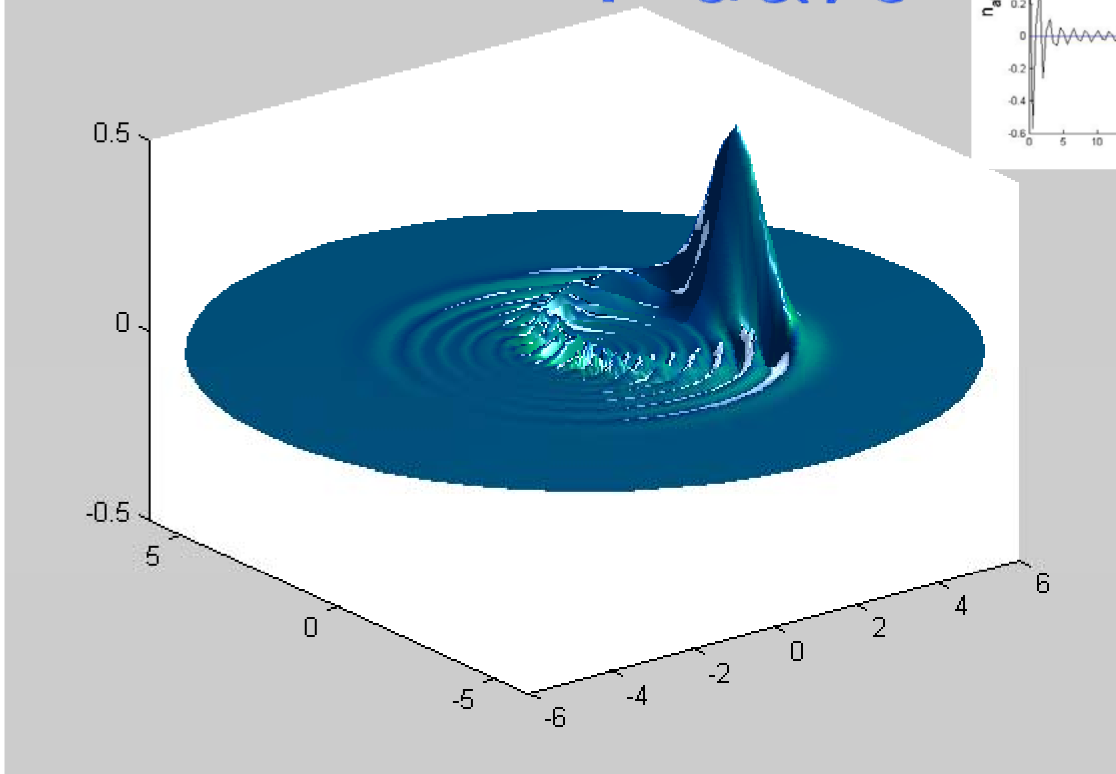
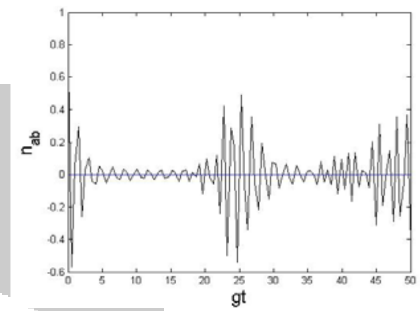
INITIAL SCHRODINGER CAT STATE

Results & Discussion

Case 1 :
Constant coupling, g(t)=g





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INITIAL COHERENT STATE (INTENSITY INDEPENDENT)

Results & Discussion

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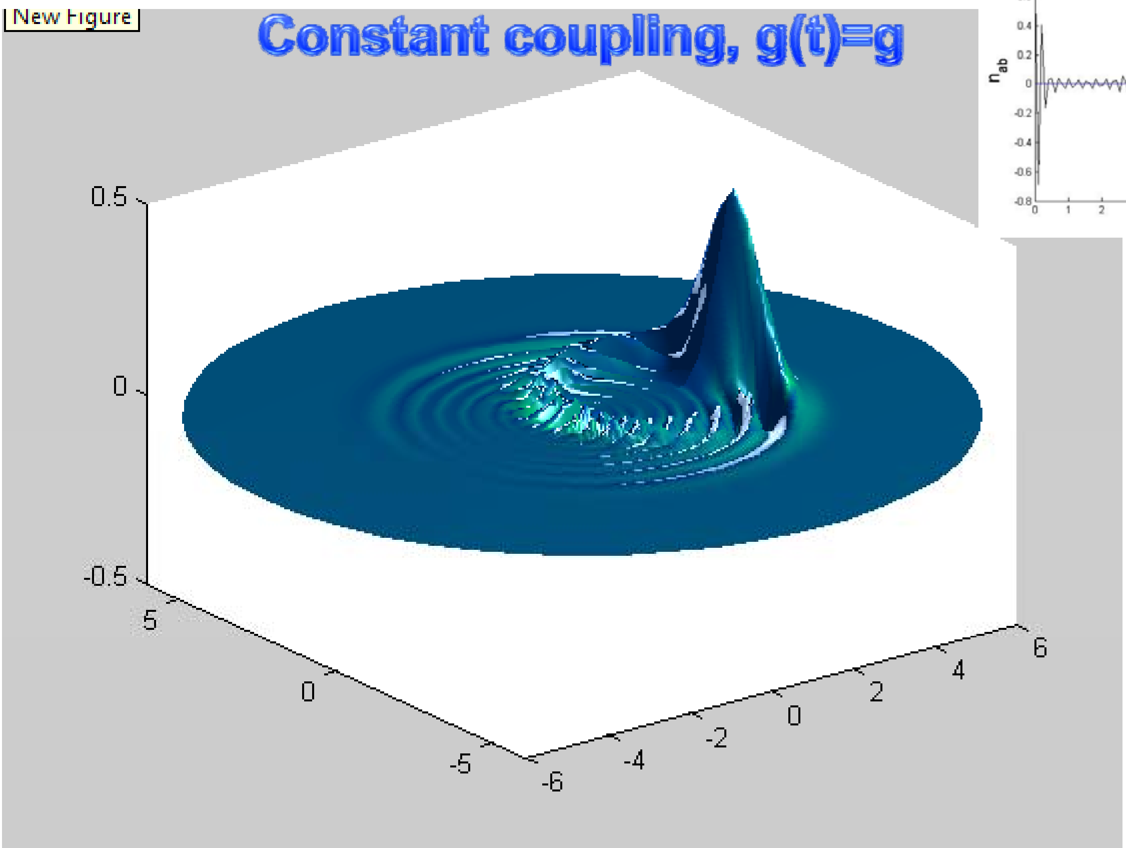
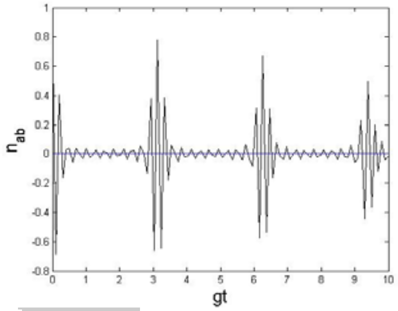
Constant coupling, g(t)=g





INITIAL COHERENT STATE (INTENSITY DEPENDENT)

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Results & Discussion

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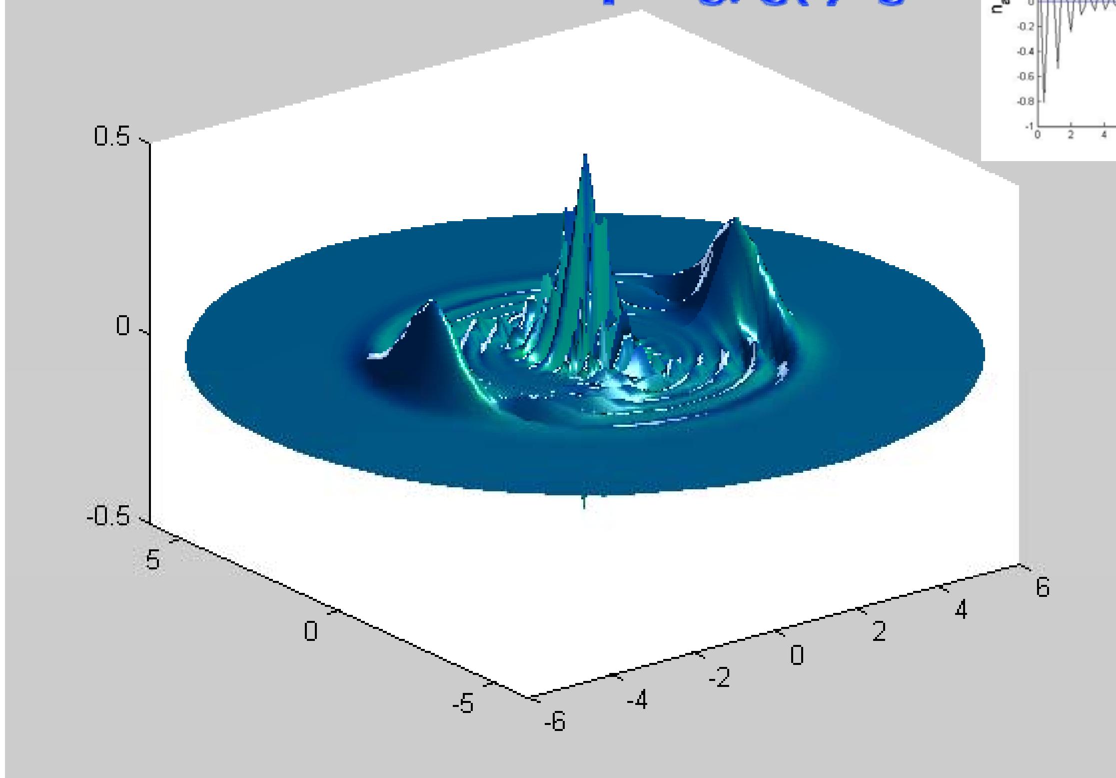
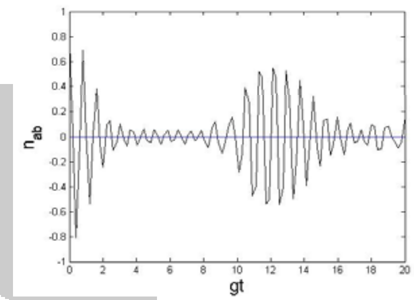
Constant coupling, g(t)=g





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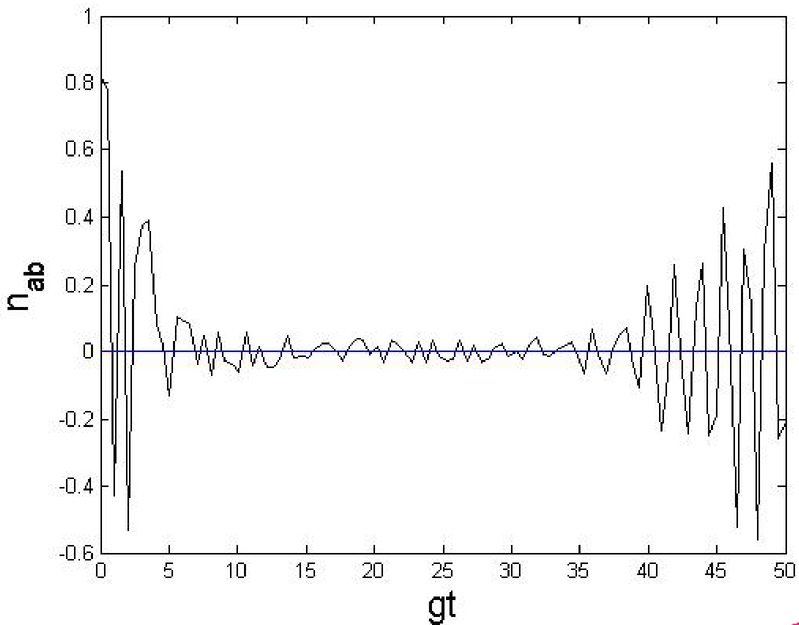
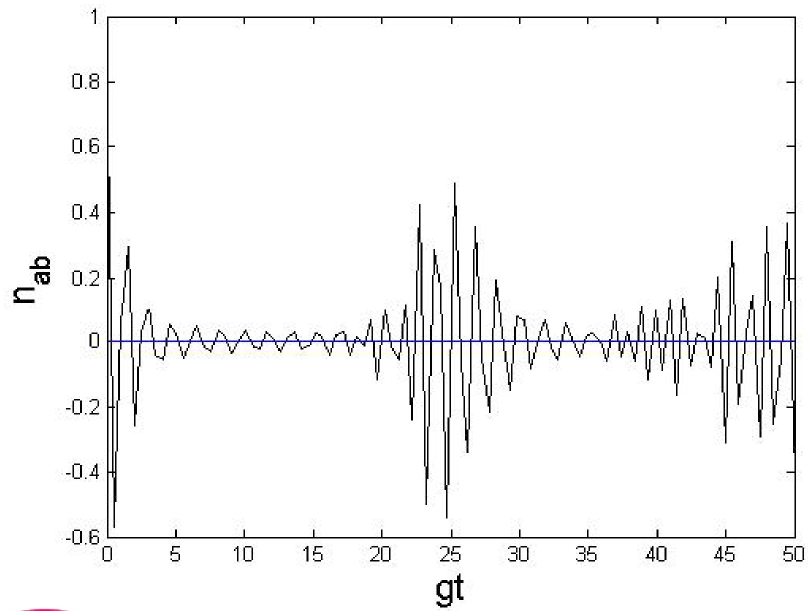
INITIAL SCHRODINGER CAT STATE (INTENSITY DEPENDENT)

Results & Discussion

Case 2 :
Sinusoidal Function, $g(t)=gsin^2(xgt)$

Intensity independent





Sinusoidal function VS **Constant coupling**

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INITIAL COHERENT STATE

Results & Discussion

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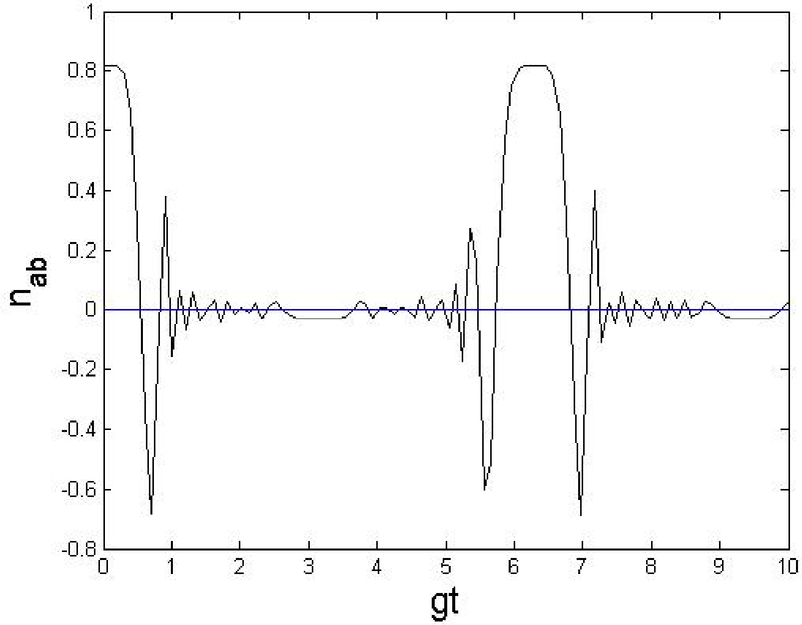

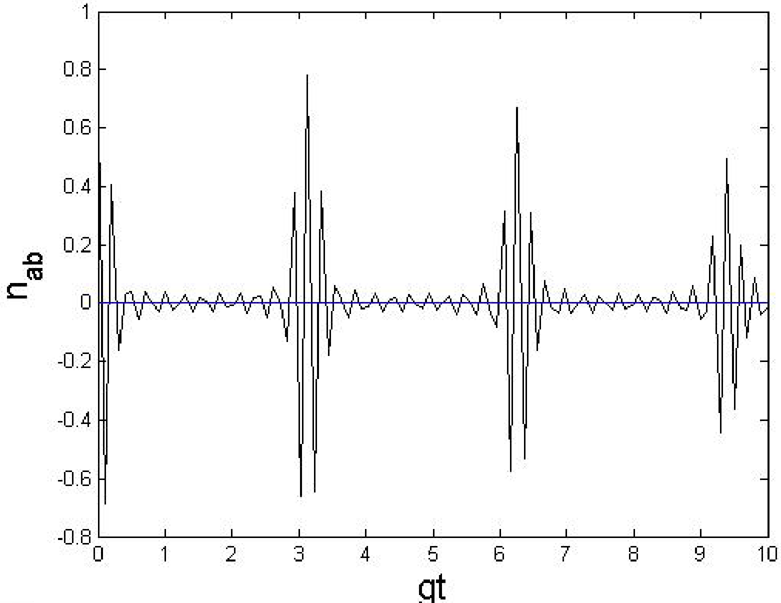

Sinusoidal function VS **Constant coupling**

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INITIAL COHERENT STATE

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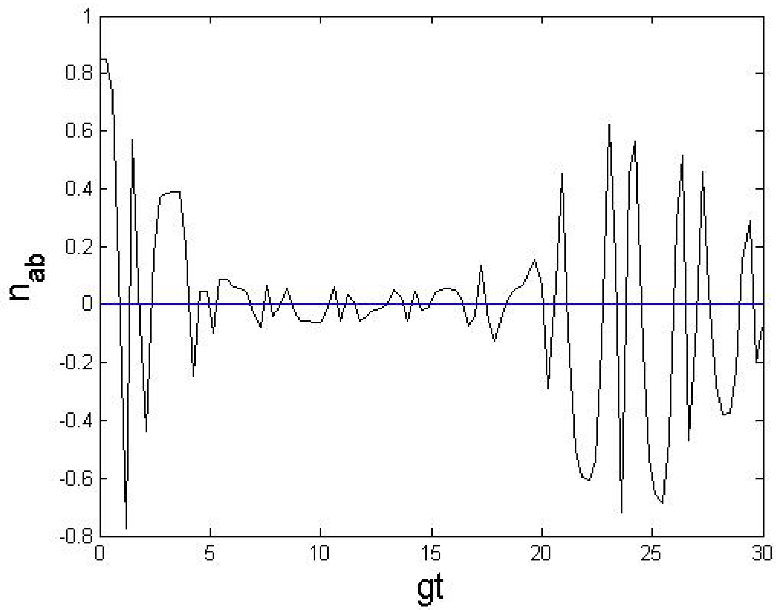

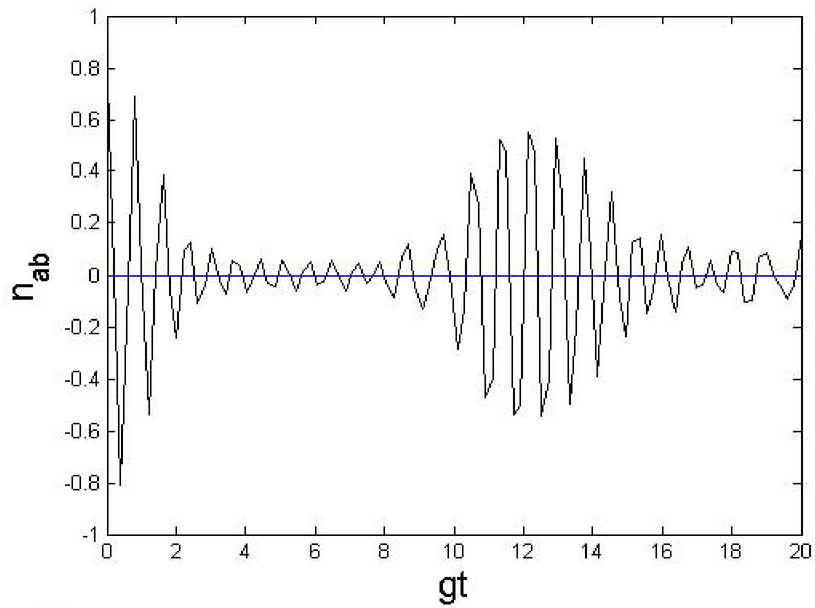

Sinusoidal function VS **Constant coupling**

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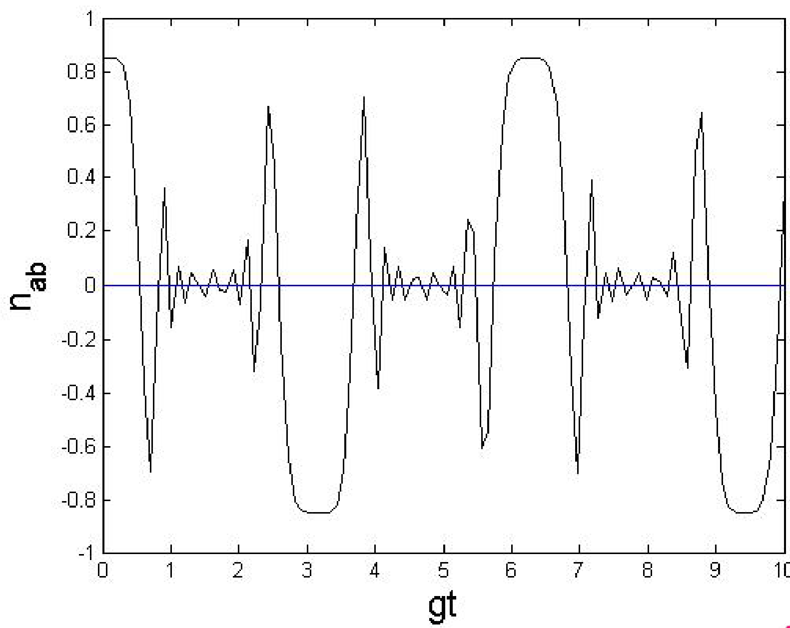

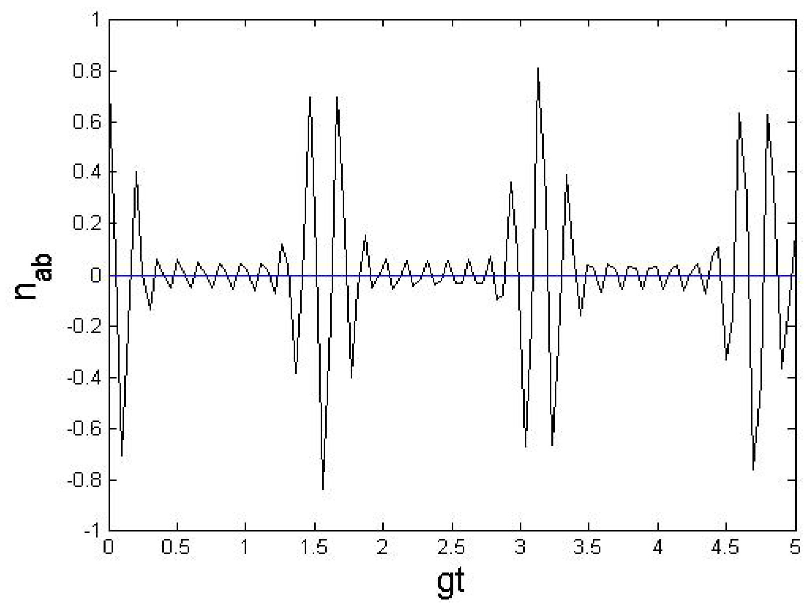

Sinusoidal function VS **Constant coupling**

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INITIAL SCHRODINGER CAT STATE

Results & Discussion

Case 2 :

Sinusoidal Function, $g(t)=g\sin^2(xgt)$

- Shown an anti-Zeno effects in analogy of Quantum Zeno effect but here it referred to the increase of collapse duration rather the decay/decoherence time as discussed in the case of anti Zeno effect.

- The modulation of coupling field stretches the collapse duration by disturbing the rephasing process towards revival via quantum interference

- Thus its underlying mechanism is different from recent work that uses the phase-space tweezer approach by Haroche's group.[9]

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[9] J. M. Raimond, C. Sayrin, S. Gleyzes, I. Dotsenko, M. Brune, S. Haroche, P. Facchi, and S. Pascazio. Phys. Rev. Lett. 105, 213601. (2010)

Results & Discussion

Case 2 :
Sinusoidal Function, $g(t)=g\sin^2(xgt)$

- When the coupling depends on intensity, the collapse and revival occurs very rapid.
- The coherent interaction scheme does not show analogy of the anti-Zeno effect, which first shown by Kofman & Kurizki [10], using certain time-dependent measurements in dissipative reservoir.
- This implies that dissipation may be an important mechanism behind the anti-Zeno effect.

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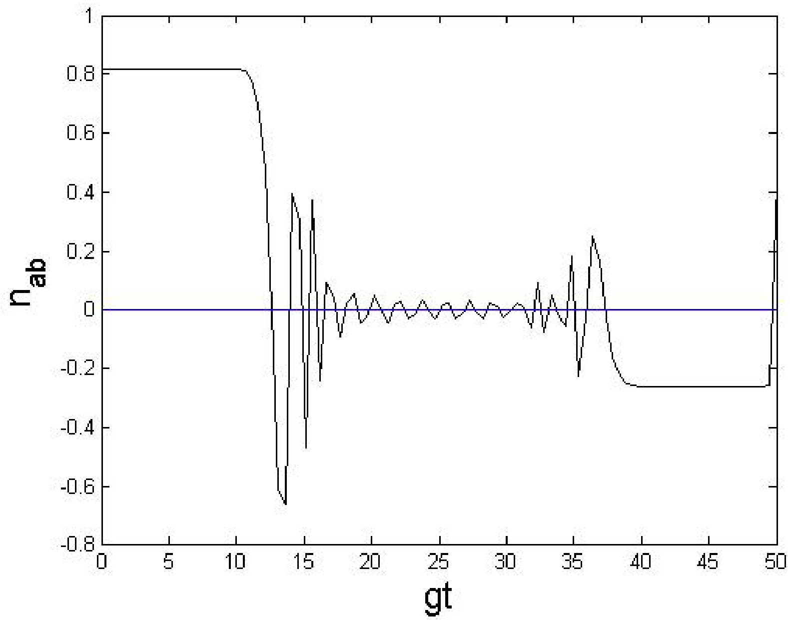
[10] A. G. Kofman and G. Kurizki. Nature 405, 546. (2000)

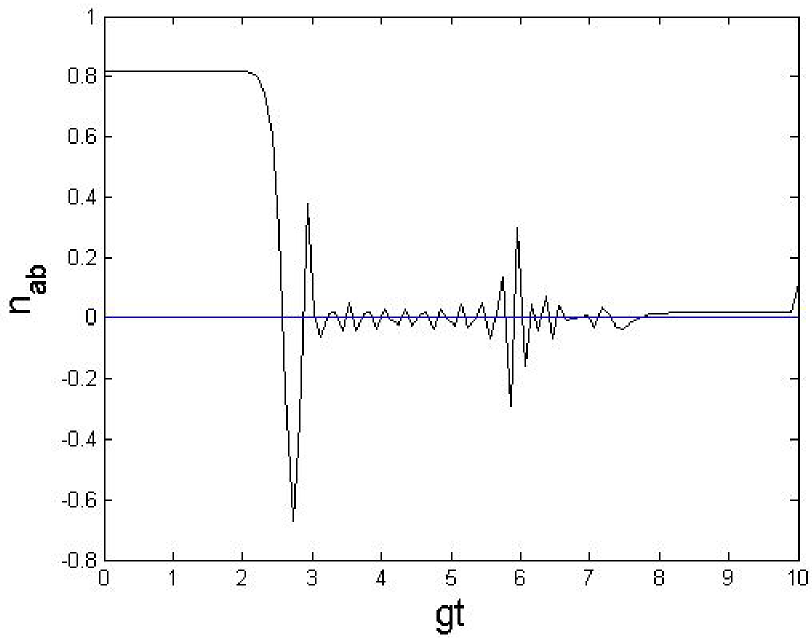
Results & Discussion

Case 3 :

Hat Function, $g(t)=gsech(t-t_o/t_m)^4$



Intensity independent



Intensity dependent

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INITIAL COHERENT STATE

Results & Discussion

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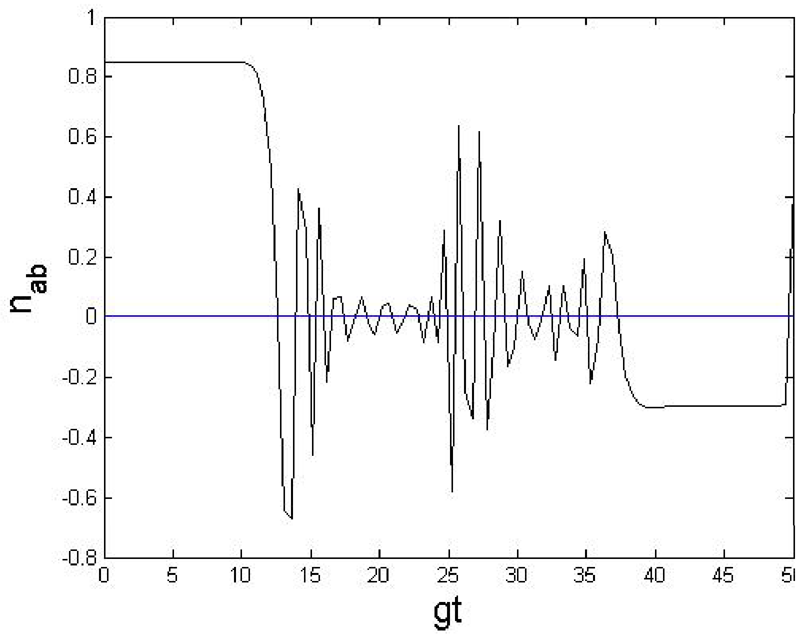
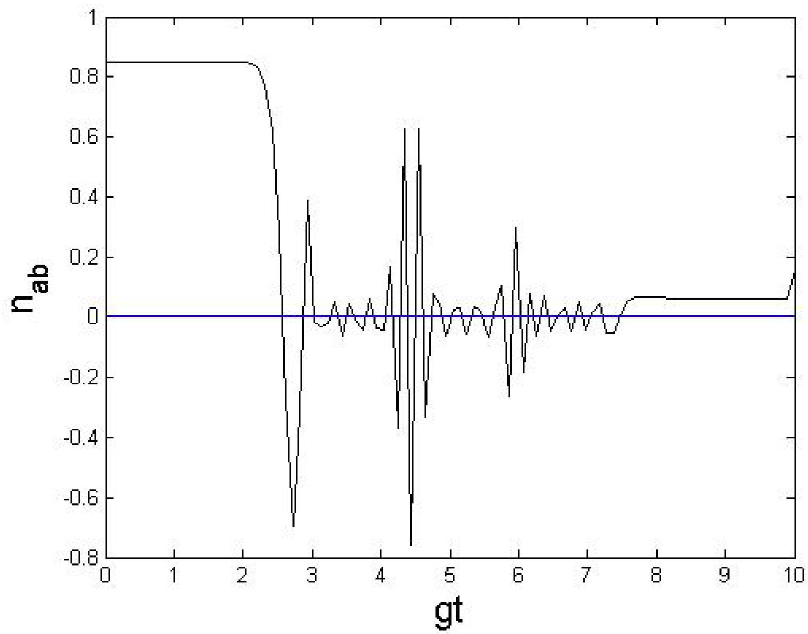


Intensity independent



Intensity dependent

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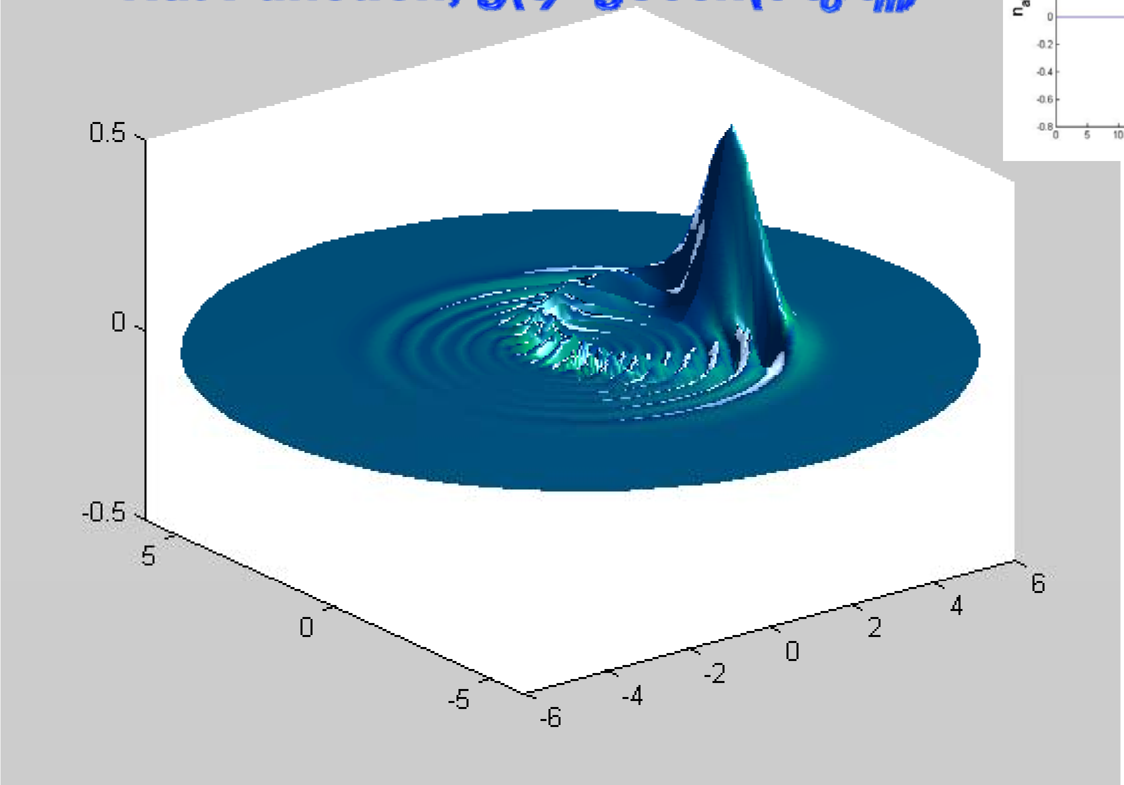
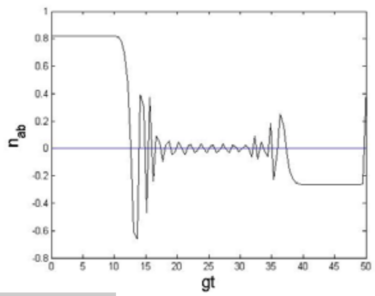
INITIAL SCHRODINGER CAT STATE

Results & Discussion

Case 3 :

Hat Function, $g(t)=g\,sech(t-t_o/t_m)^4$





INITIAL COHERENT STATE

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Results & Discussion

Case 3 :
Hat Function, $g(t)=g\,sech(t-t_o/t_m)^4$

- Frozen population can be seen and at the same point of time, the peak of Wigner function is also freezes.

- The frozen population depends on the hat duration t_m.

- This can be understood by analyzing $g(t)=g\,sech(t-t_o/t_m)^4$ which can be integrated as

$$\lambda(t)_{\text{hat}} = 2gt_m[\tan^{-1}(e^{\frac{t-t_0}{t_m}}) - \tan^{-1}(e^{-\frac{t_0}{t_m}})]$$

- For $t_o >> t_m$, the second term is negligible. At large t when the pulse is gone, $\lambda(\infty) = \pi g t_m$.

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Results & Discussion

Case 3 :
Hat Function, $g(t)=g\,sech(t-t_o/t_m)^4$

- Thus, the duration of the hat function or atomic transit can be a control parameter for creating a frozen state of light in an empty cavity.
- This result is connected to the frozen cat state studied by Sherman et. al. in [2] on photonic crystal.

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[2] Sherman, B. et. al. A. Phys. Rev. Lett. 69 1927 (1992)

Conclusion

- We have studied the dynamics of a two-level particle or system coupled to any state of light in a cavity.
- The generalization to arbitrary time-dependent coupling g(t) enables the analysis of transient effects, such as particle oscillating through the cavity and particle transit duration.
- We have analyzed the time evolutions of the atomic states (through inversion) and the cavity field (through the Wigner function) for time-dependent and intensity-dependent atom-field coupling, except that it does not involve a measurement nor dissipative mechanism.

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Conclusion

- Each time-dependent coupling function with different initial field shows quantum Zeno effect when the system include the intensity-dependent coupling.
- The anti Zeno effect only occurs in sinusoidal coupling compared with normal JCM with constant coupling.

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Further Work

- Dissipation will be considered
- Applied to more complex system such as multilevel atom, many-body system, multiphoton etc.

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