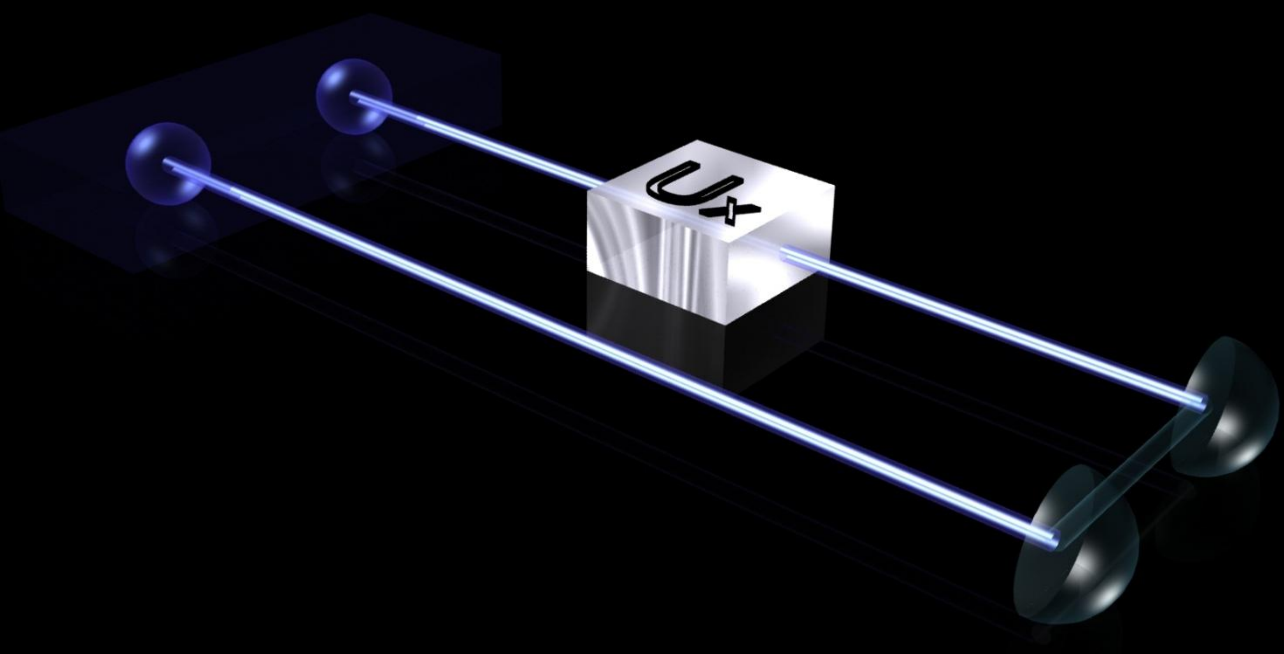
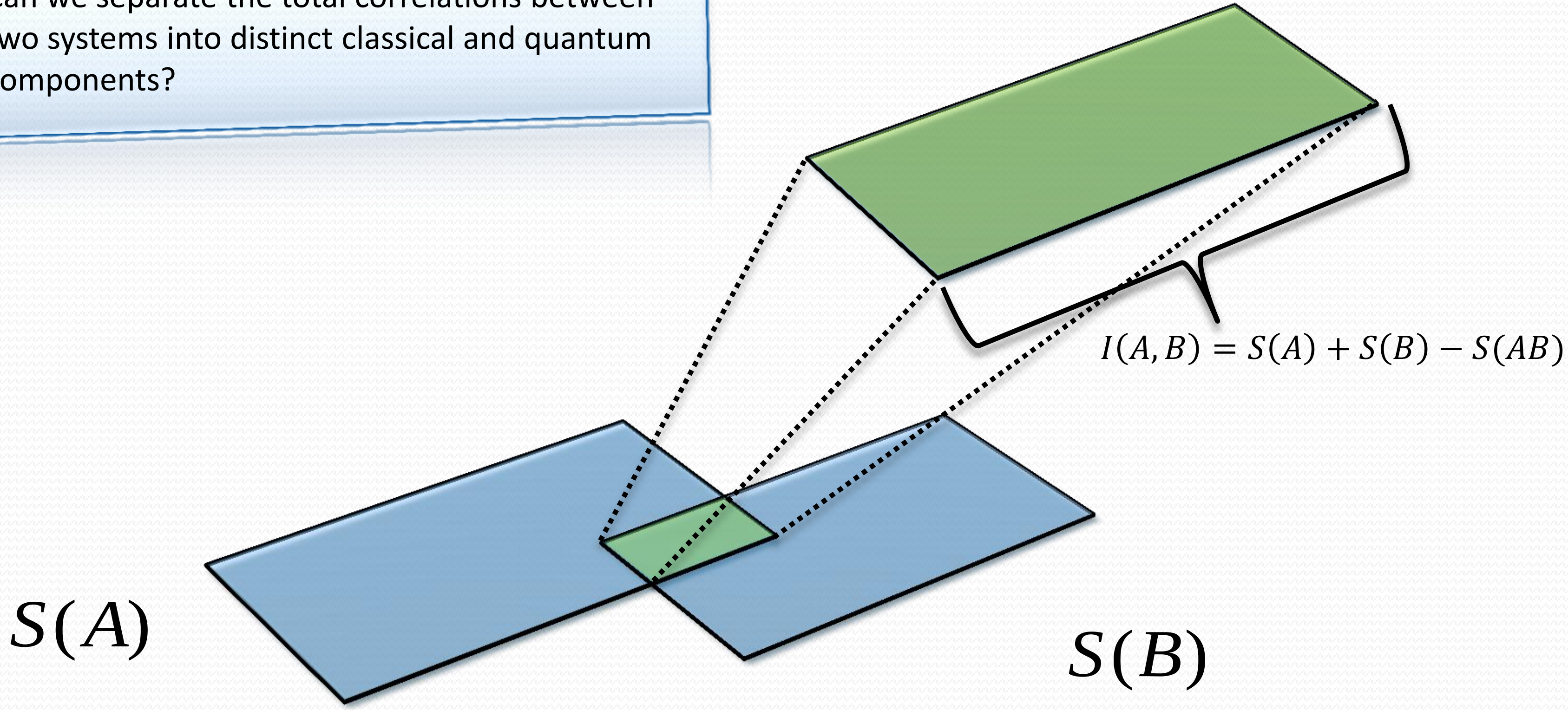
Operational Significance of Discord and its Consumption



M.Gu
H. Chrzanowsk
S. Assad
T. Symul
K. Modi
T. Ralph
P. K. Lam
V. Vedral

Separating Classical and Quantum Correlations

Can we separate the total correlations between two systems into distinct classical and quantum components?



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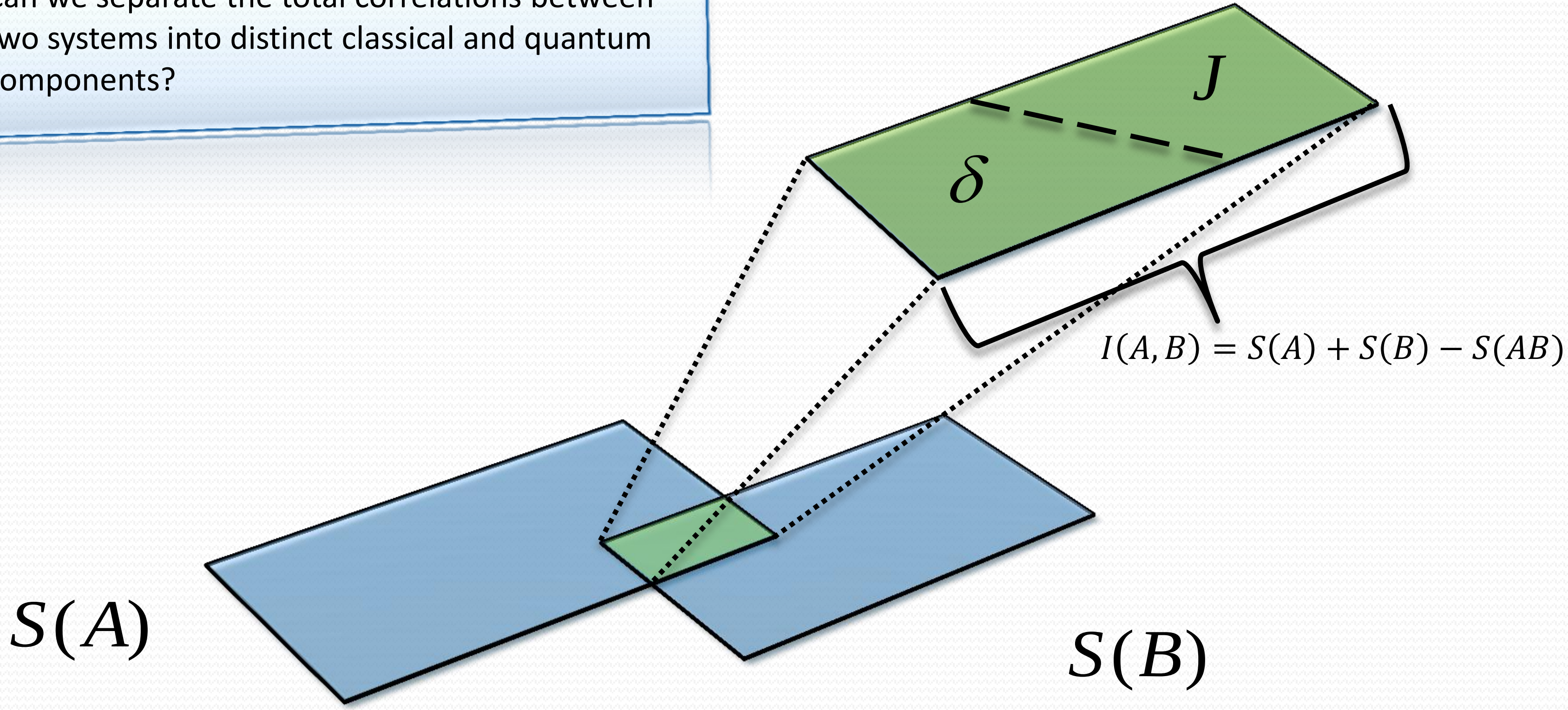
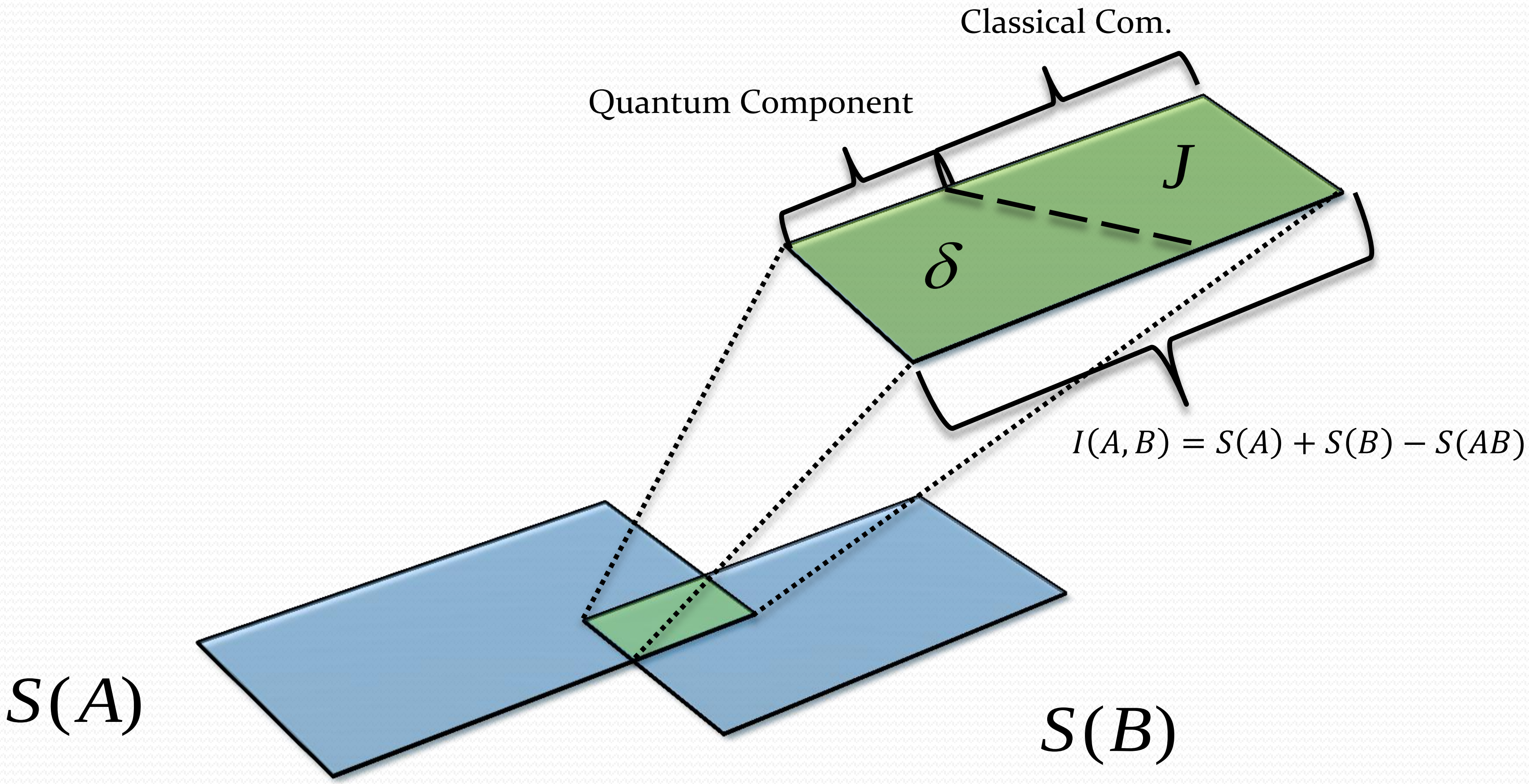


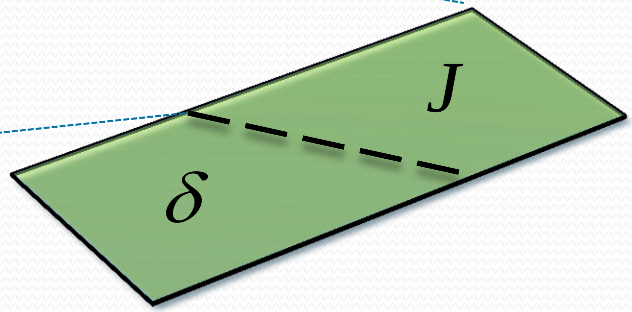
Separating Classical and Quantum Correlations



Separating Classical and Quantum Correlations

Classical Correlations $J(A|B)$

The amount of information Alice can learn about system A by performing a measurement on system B.



$$J(A|B) = S(A) - \min_{\Pi} S(A|B_{\Pi})$$

Uncertainty about A before measurement

Optimize over some set of feasible measurements

Uncertainty about A after measurement in basis Π

Separating Classical and Quantum Correlations

Classical Correlations $J(A|B)$

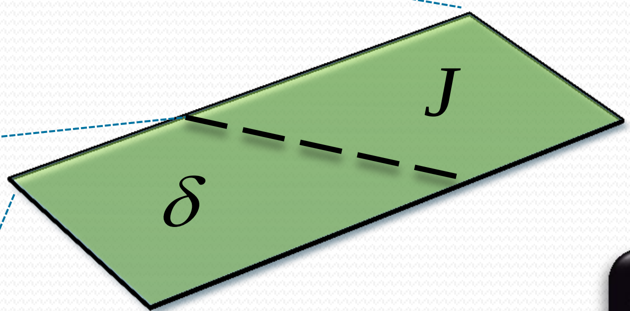
The amount of information Alice can learn about system A by performing a measurement on system B.

$$\mathrm{J(A|B)} = S(A) - \min_{\Pi} S(A|B_{\Pi})$$

Quantum Correlations

If we subtract the classical correlations from the total correlations, what remains should be considered quantum

$$\delta\mathrm{(A|B)} = \mathrm{I}(A,B) - \mathrm{J(A|B)}$$



Journal of Physics A (2001)

V. Vedral

W. H. Zurek

Physical Rev Lett. 88, 017901 (2002)

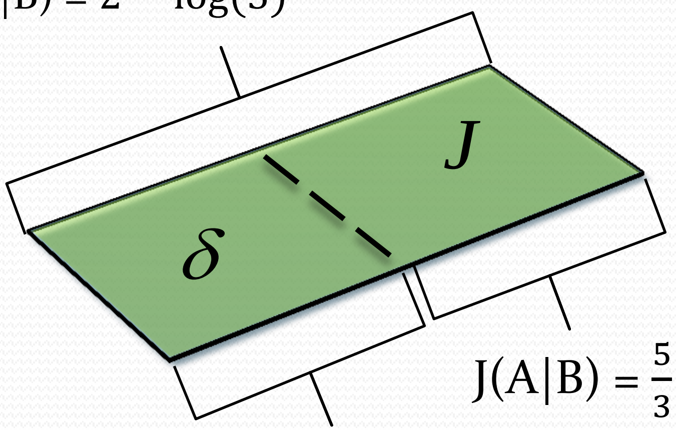
$\delta\mathrm{(A|B)}$ is given then name *'quantum discord'*

Discord without Entanglement

Example: A mixture of 3 One-Time Pads:

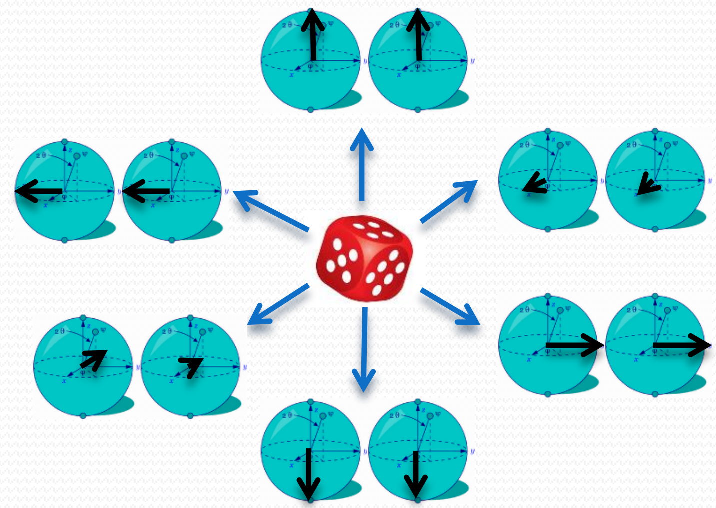
$$\rho = \sum_{\sigma=X,Y,Z} |0_\sigma 0_\sigma\rangle\langle 0_\sigma 0_\sigma| + |1_\sigma 1_\sigma\rangle\langle 1_\sigma 1_\sigma|$$

$\mathrm{I(A|B)} = 2 - \log(3)$



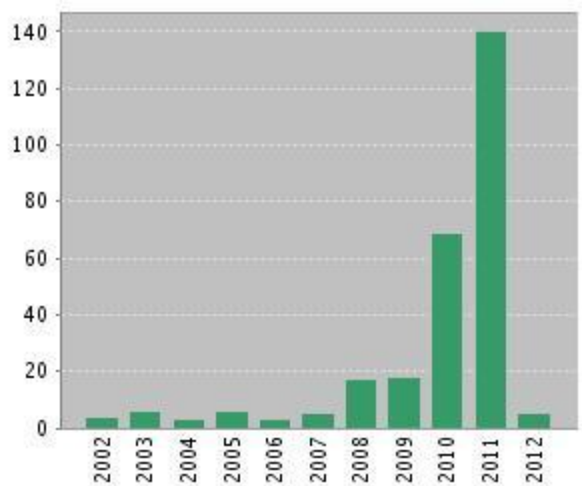
$\mathrm{J(A|B)} = \frac{5}{3} - \log(3)$

$\delta(A|B) = I - J = 1/3$



If non-entangled states can have quantum correlations, then is entangled necessary for quantum behaviour?

The first half decade....



Physical Rev Lett. 88, 017901 (2002)

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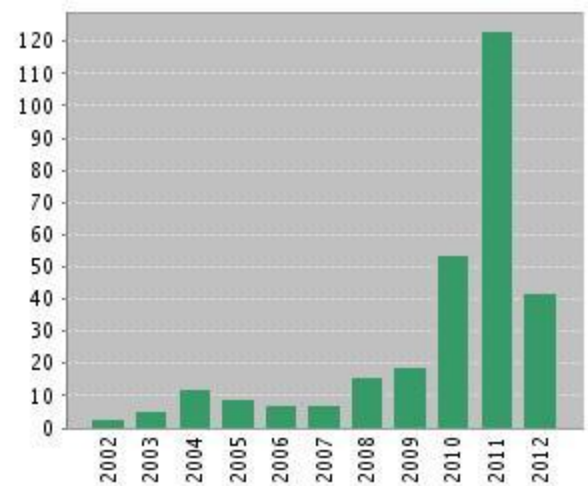
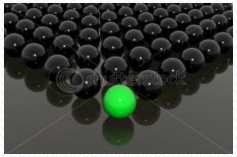


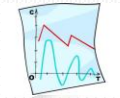
Journal of Physics A (2001)

V. Vedral

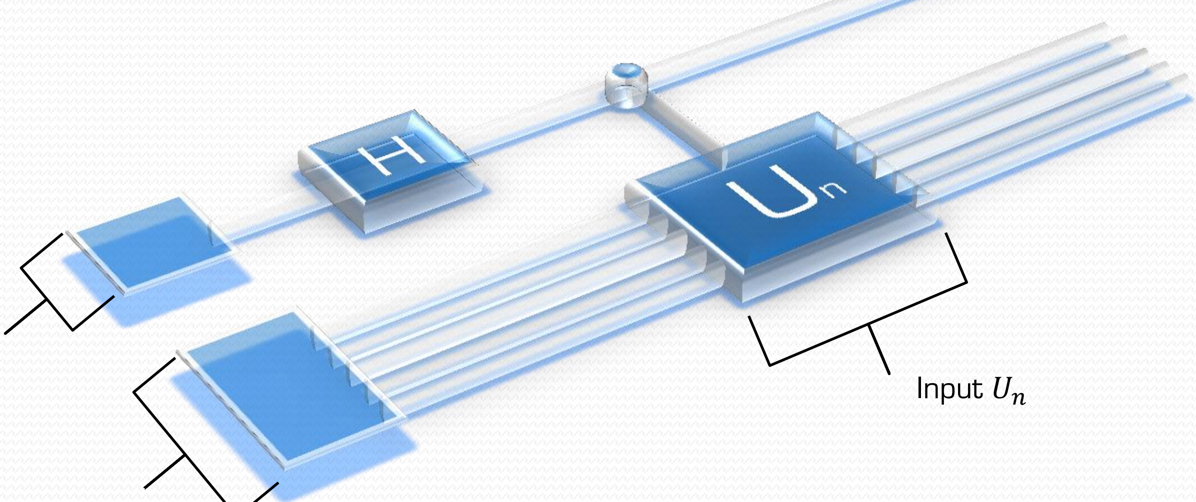
The Power of One



Computes the normalized trace of U_n exponential faster than best known classical algorithm using only A single pure qubit .



Measurement retrieves information about U_n



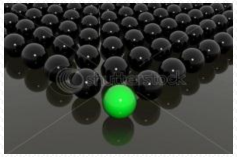
Single pure qubit $|0\rangle\langle 0|$

Input U_n

n completely mixed qubits $I_n/_d$

1998: Algorithm Discovered. (Knill and Laflamme)

The Power of One (DQC1)



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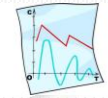
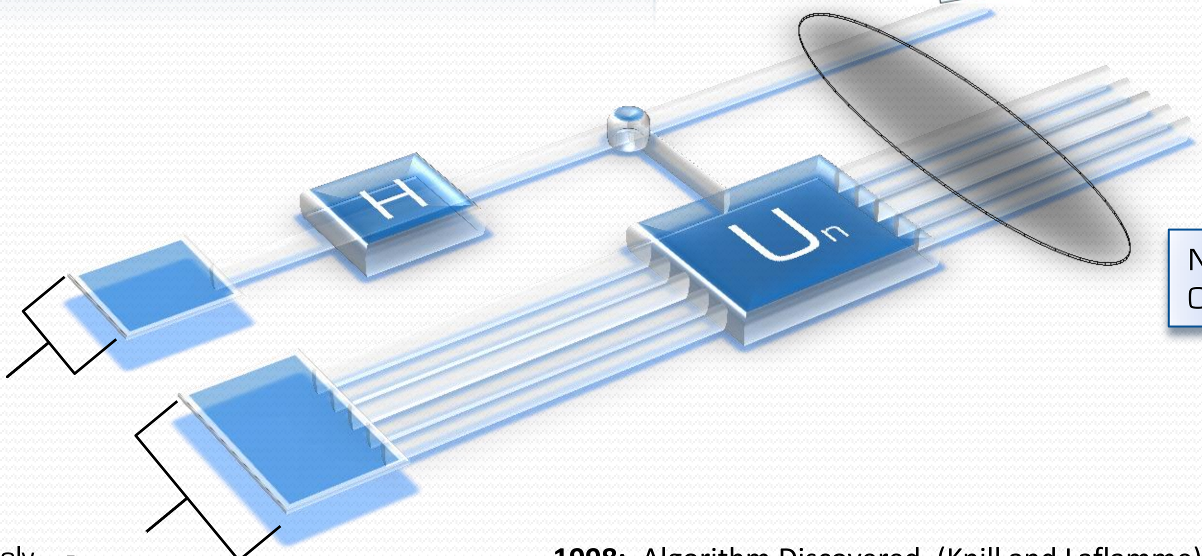


Measurement retrieves information about U_n



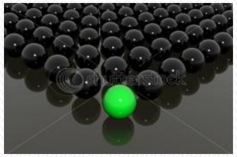
Negligible entanglement Observed!

Single pure qubit $|0\rangle\langle 0|$

n completely mixed qubits $I_n/_d$

1998: Algorithm Discovered. (Knill and Laflamme)
2005: DQC1 contains negligible entanglement (Datta, Flammia, Caves)

The Power of One (DQC1)



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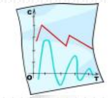

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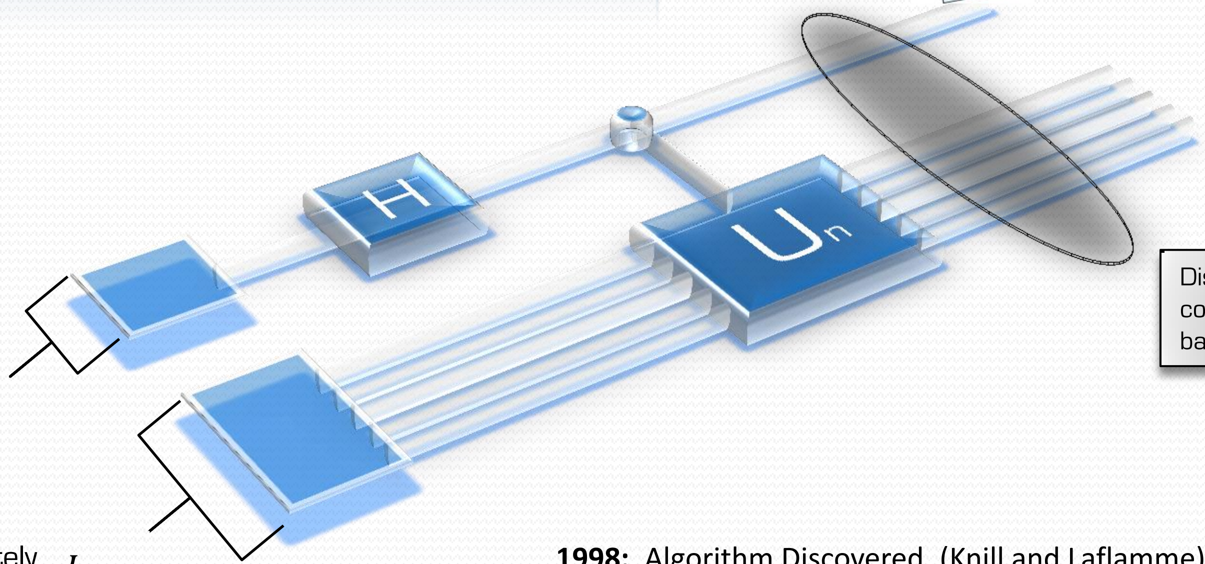


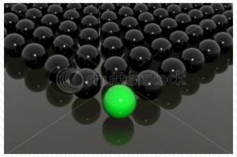
Discord between the control qubit and the bath exists

Single pure qubit $|0\rangle\langle 0|$

n completely mixed qubits $I_n/_d$

1998: Algorithm Discovered. (Knill and Laflamme)
2005: DQC1 contains negligible entanglement (Datta, Flammia, Caves)
2008: Discord présent in DQC1 (Datta, Shaji, Caves)

The Power of One (DQC1)



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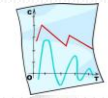
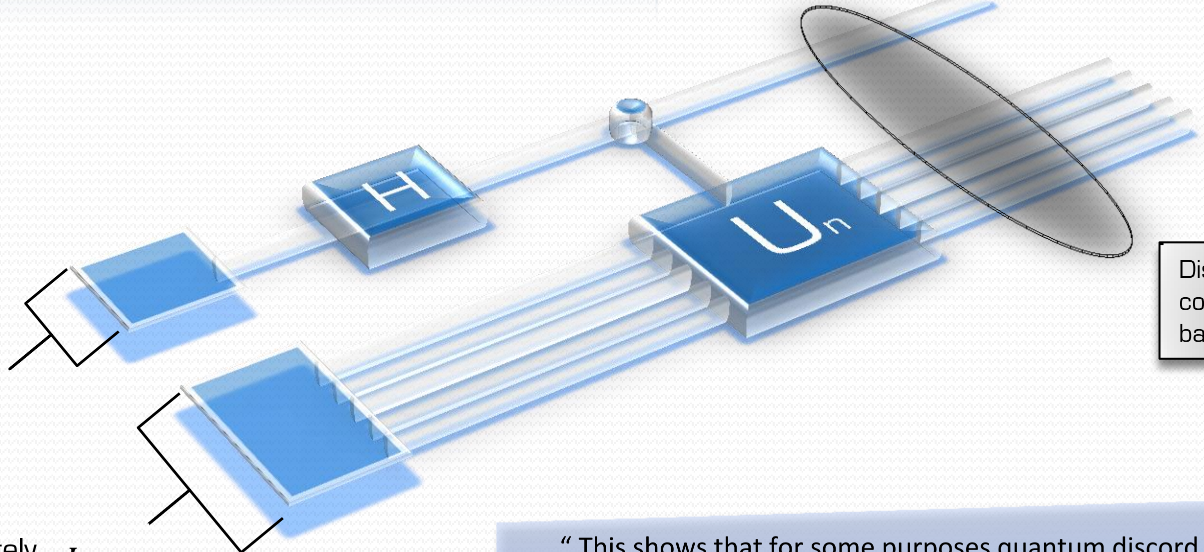


Measurement retrieves information about U_n



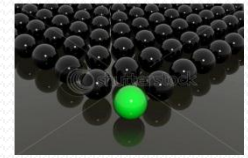
Discord between the control qubit and the bath exists

Single pure qubit $|0\rangle\langle 0|$

n completely mixed qubits I_n/d

“ This shows that for some purposes quantum discord might be a better figure of merit for characterizing the quantum resources available to a quantum information processor. ” - *(A. Datta, A. Shaji, and C. Caves, Phys. Rev. Lett. 100, 050502, 2008)*

The Power of One (DQC1)



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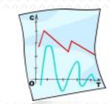
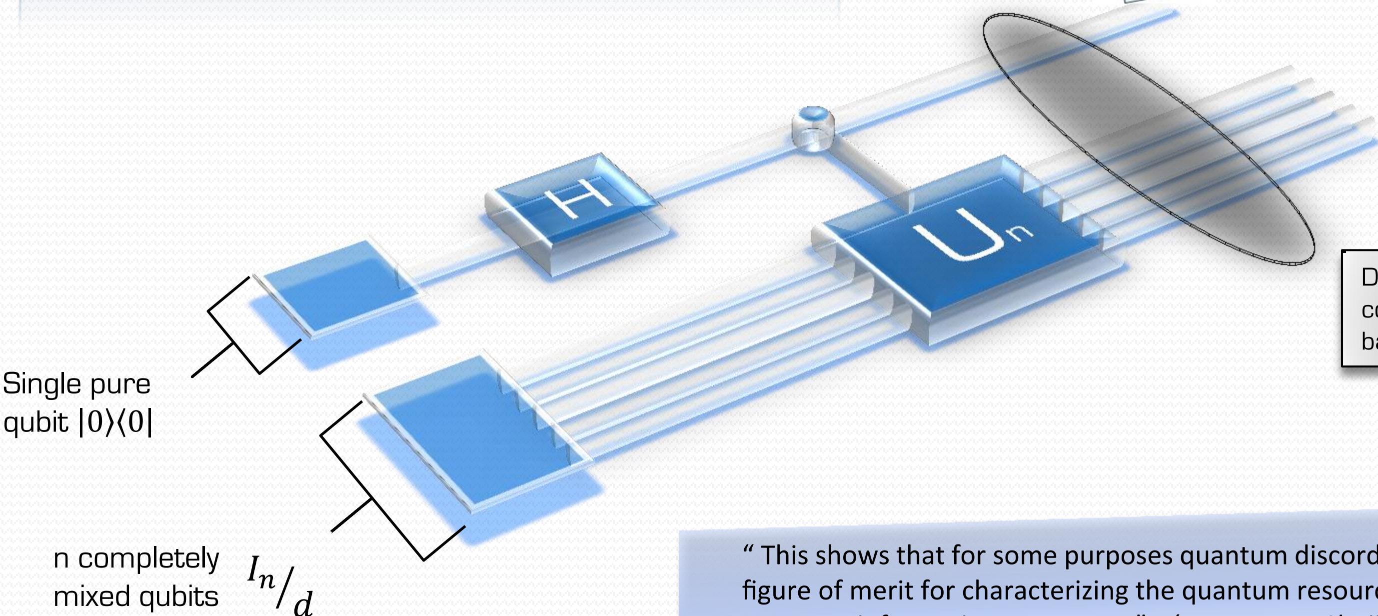


Measurement retrieves information about U_n

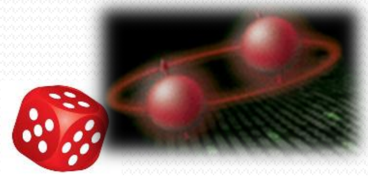


Discord between the control qubit and the bath exists

" This shows that for some purposes quantum discord might be a better figure of merit for characterizing the quantum resources available to a quantum information processor. " - *(A. Datta, A. Shaji, and C. Caves, Phys. Rev. Lett. 100, 050502, 2008)*

Is Discord a Quantum Resource?

Did DQC1 really exploit discord? Or was the presence of discord a mere coincidence?

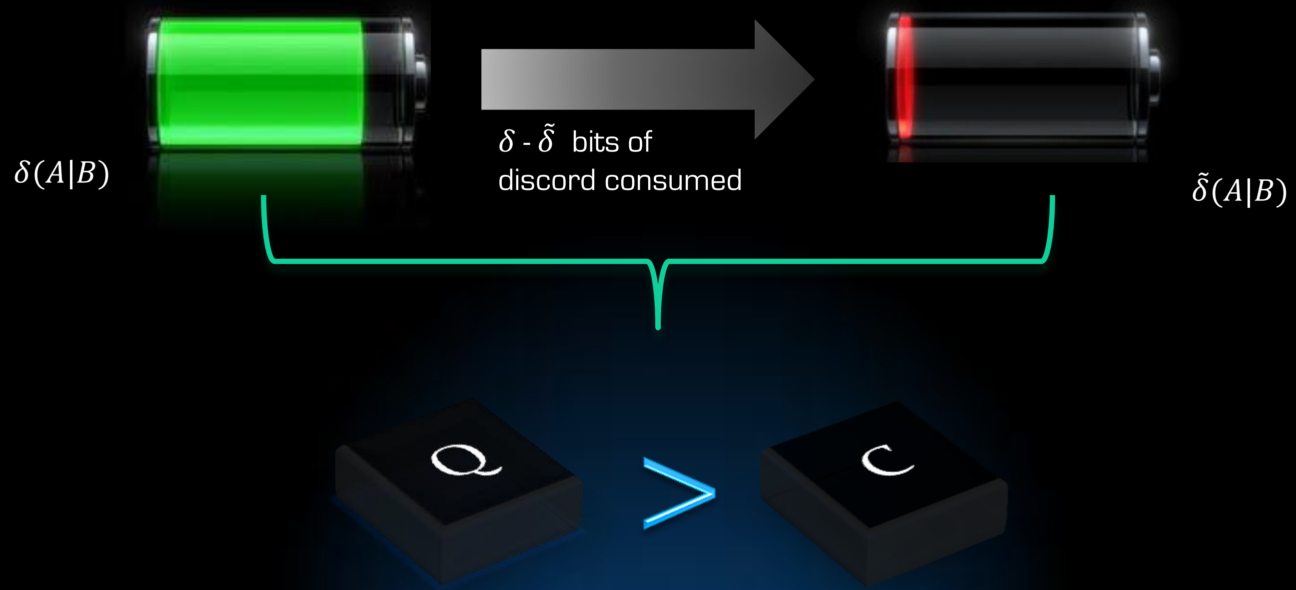


A state that is picked out at random has non-zero discord

“Typically a state picked out at random has positive discord and, given a state with zero discord, a generic arbitrarily small perturbation drives it to a positive-discord state. These results hold for any Hilbert-space dimension.”

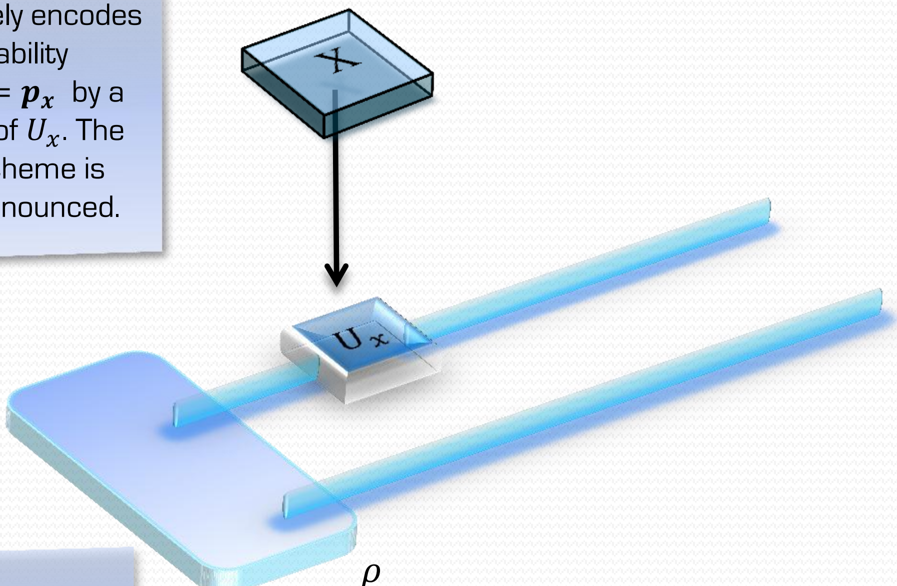
A. Ferraro, L. Aolita, D. Cavalcanti, F. M. Cucchietti, and A. Acín
Phys. Rev. A 81, 052318 (2010)

Can Discord be considered a resource for quantum processing?



The Protocol

Alice privately encodes **X** with probability $P(\boldsymbol{X} = \boldsymbol{x}) = \boldsymbol{p_x}$ by a application of U_x. The encoding scheme is publically announced.



Alice begins with initial state ρ with discord $\delta(A|B)$,

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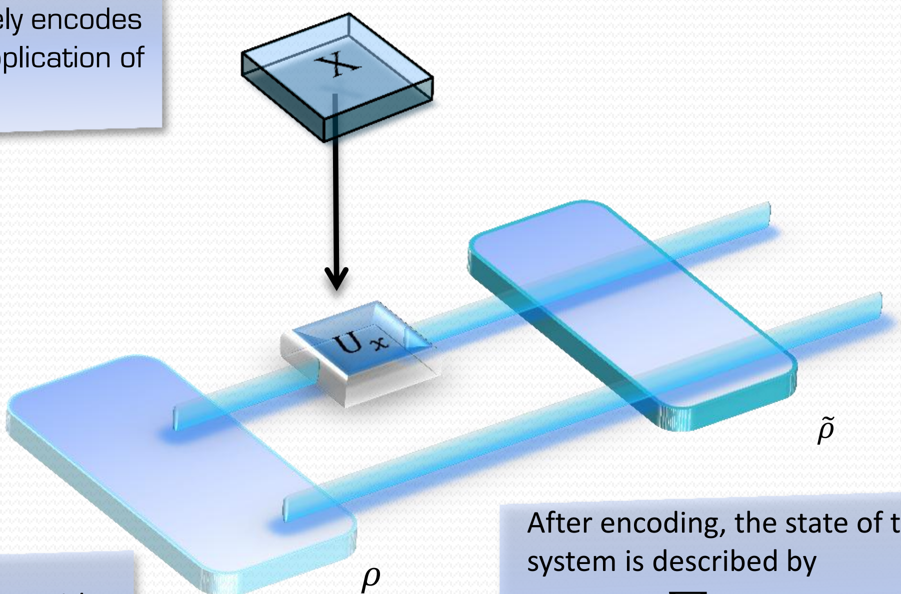


Alice begins with initial state ρ with discord $\delta(A|B)$,

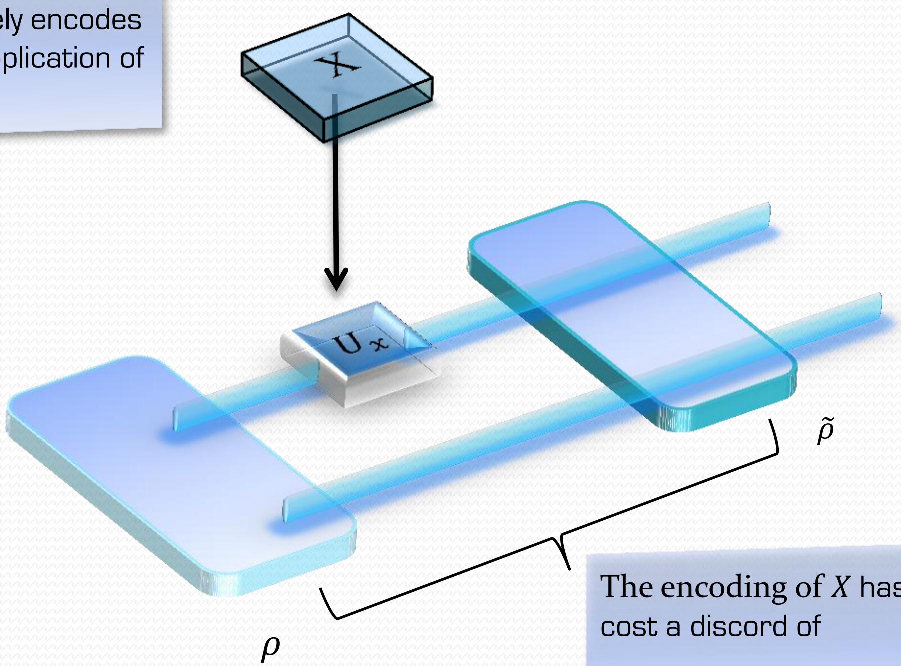
After encoding, the state of the system is described by

$$\tilde{\rho} = \sum p_k U_k \rho U^{\dagger}{}_k$$

With Discord $\tilde{\delta}(A|B)$

The Protocol

Alice privately encodes **X** with by application of U_x.

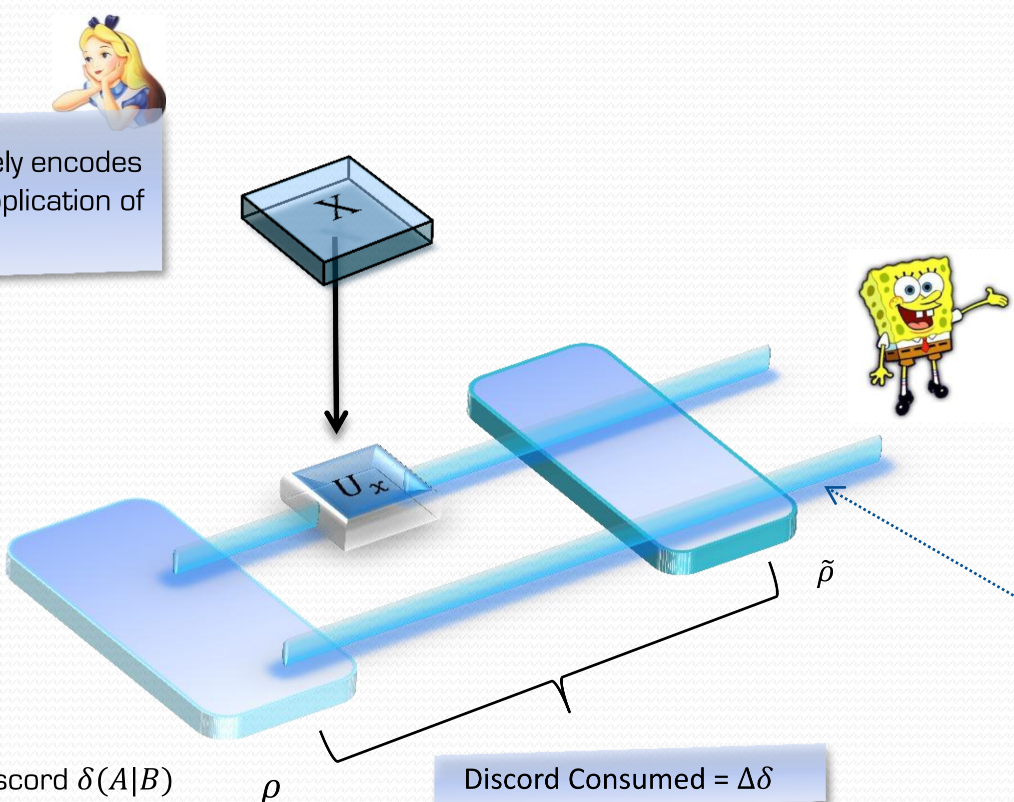


The encoding of X has cost a discord of

$$\Delta\delta = \delta(A|B) - \tilde{\delta}(A|B)$$

The Protocol

Alice privately encodes **X** with by application of U_x.



Alice gives the resulting state to Bob challenges Bob to generate an estimate of X from $\tilde{\rho}$

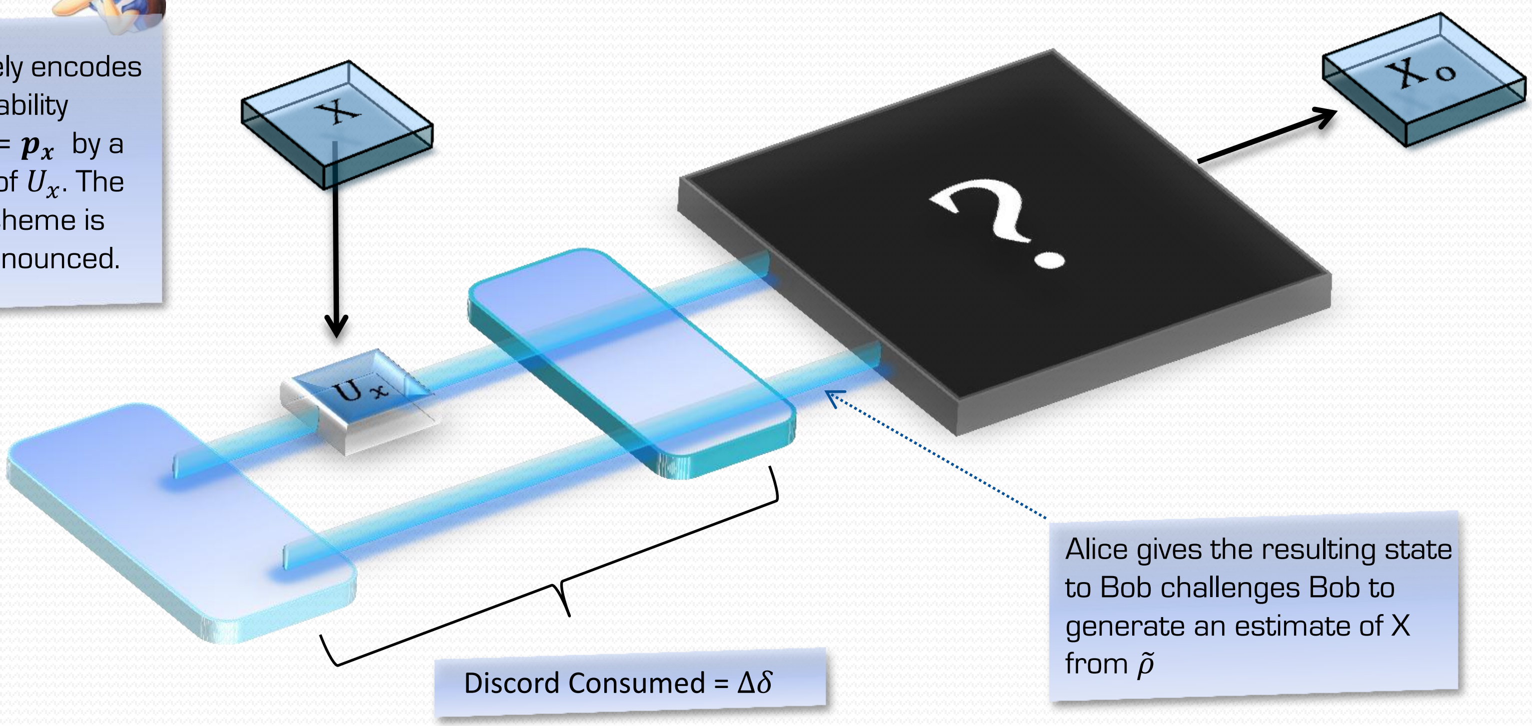
Initial Discord $\delta(A|B)$

Discord Consumed = $\Delta\delta$

The Protocol

Bob applies some computational process to retrieve an estimate X_0, and sends this back to Alice

Alice privately encodes **X** with probability $P(\boldsymbol{X} = \boldsymbol{x}) = \boldsymbol{p_x}$ by a application of U_x. The encoding scheme is publically announced.



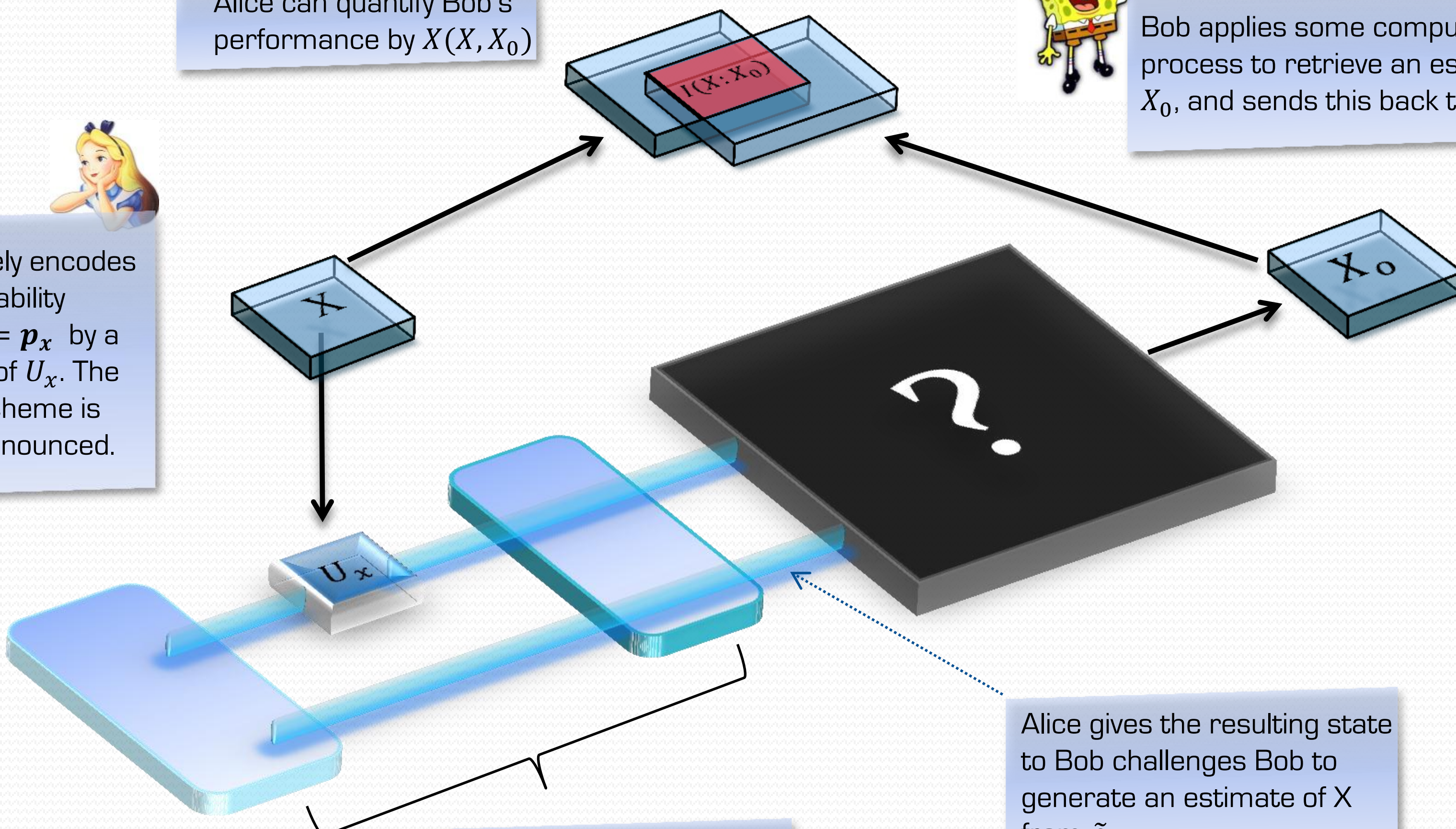
Alice gives the resulting state to Bob challenges Bob to generate an estimate of X from $\tilde{\rho}$

The Protocol

Alice can quantify Bob's performance by $X(X, X_0)$

Bob applies some computational process to retrieve an estimate X_0, and sends this back to Alice

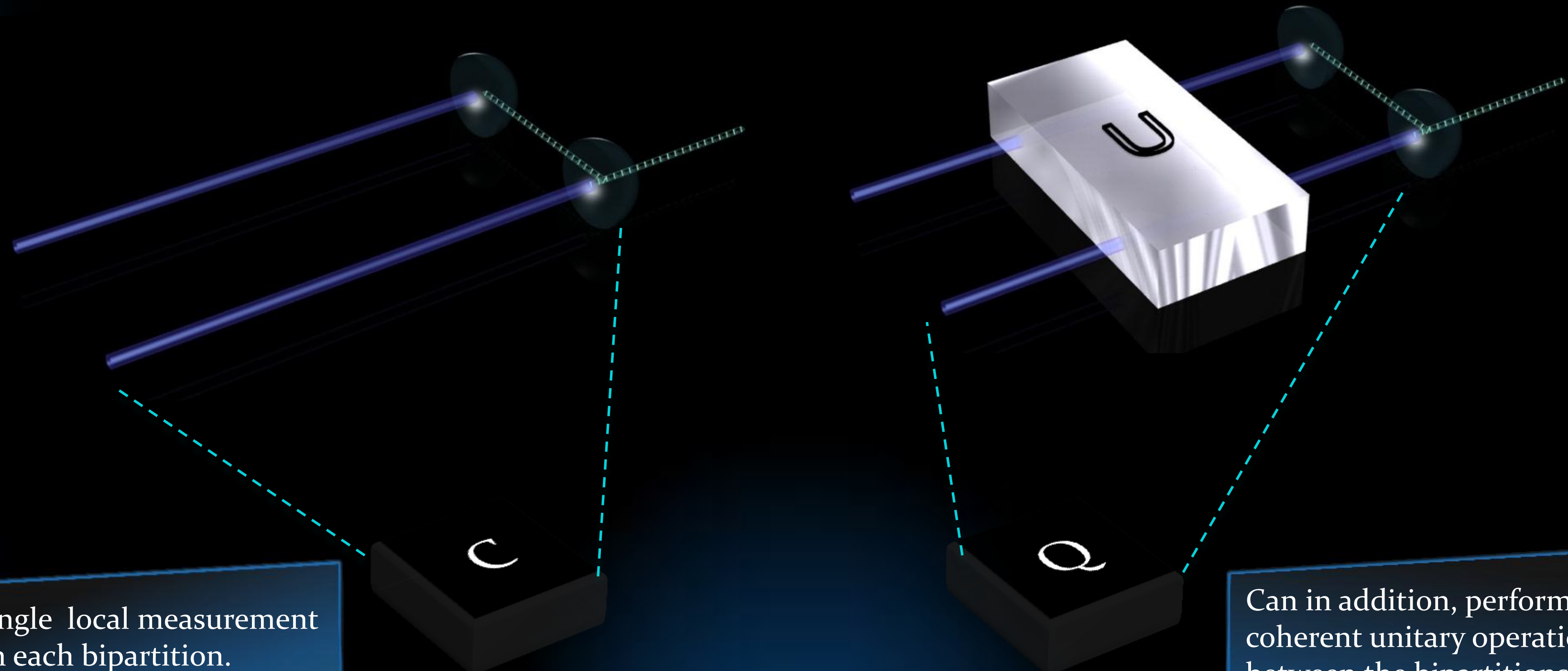
Alice privately encodes **X** with probability $P(\boldsymbol{X} = \boldsymbol{x}) = \boldsymbol{p_x}$ by a application of U_x. The encoding scheme is publically announced.



Alice gives the resulting state to Bob challenges Bob to generate an estimate of X from $\tilde{\rho}$

Discord Consumed = $\Delta\delta$

Incoherent vs. Quantum Processers



Single local measurement on each bipartition.

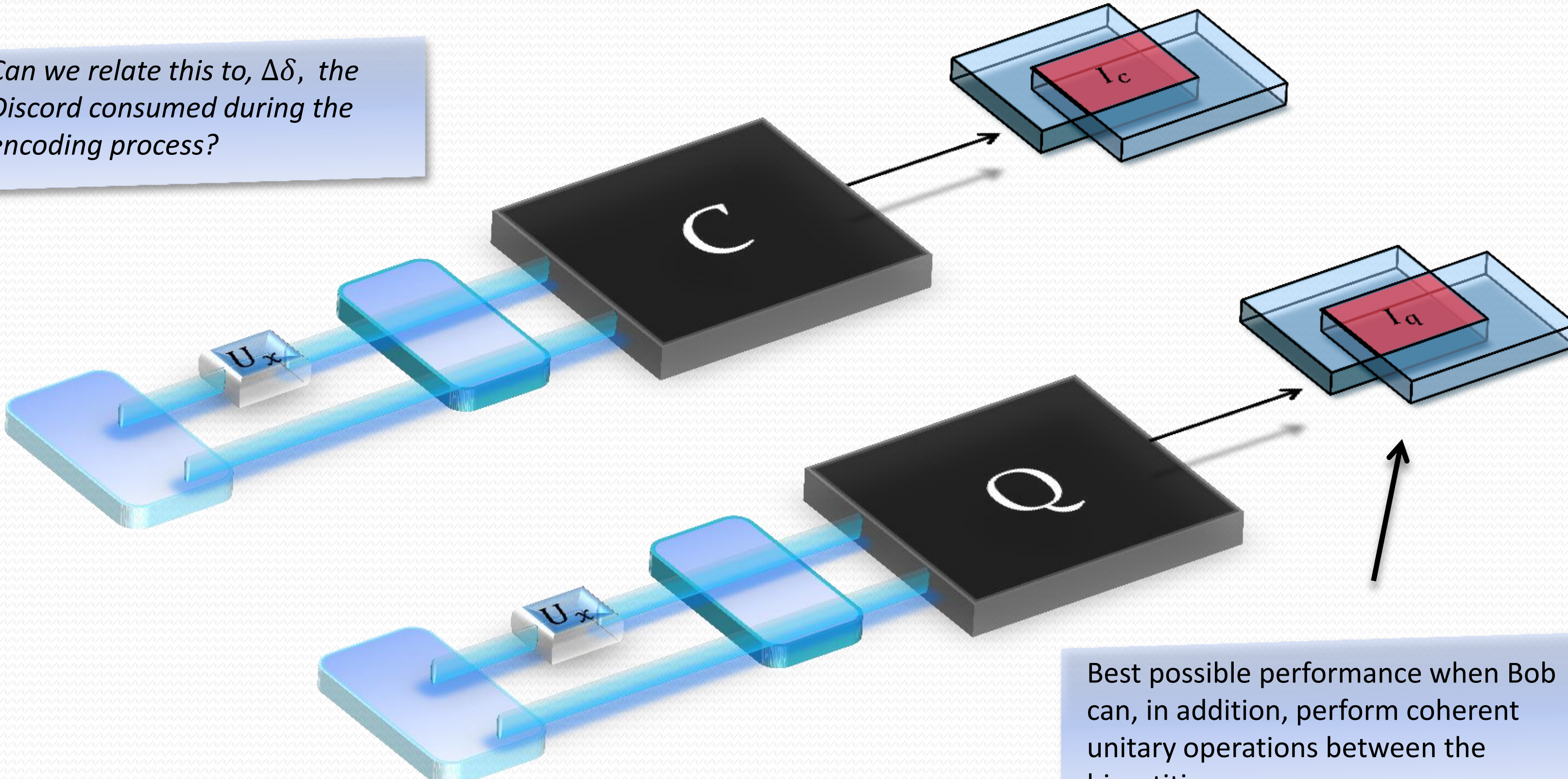
Can in addition, perform coherent unitary operations between the bipartitions

The Protocol

$\Delta I = I_c - I_q$ characterizes the advantage of having coherent interactions.

Best possible performance when Bob is limited to a single local measurement on each bipartition.

Can we relate this to, $\Delta\delta$, the Discord consumed during the encoding process?

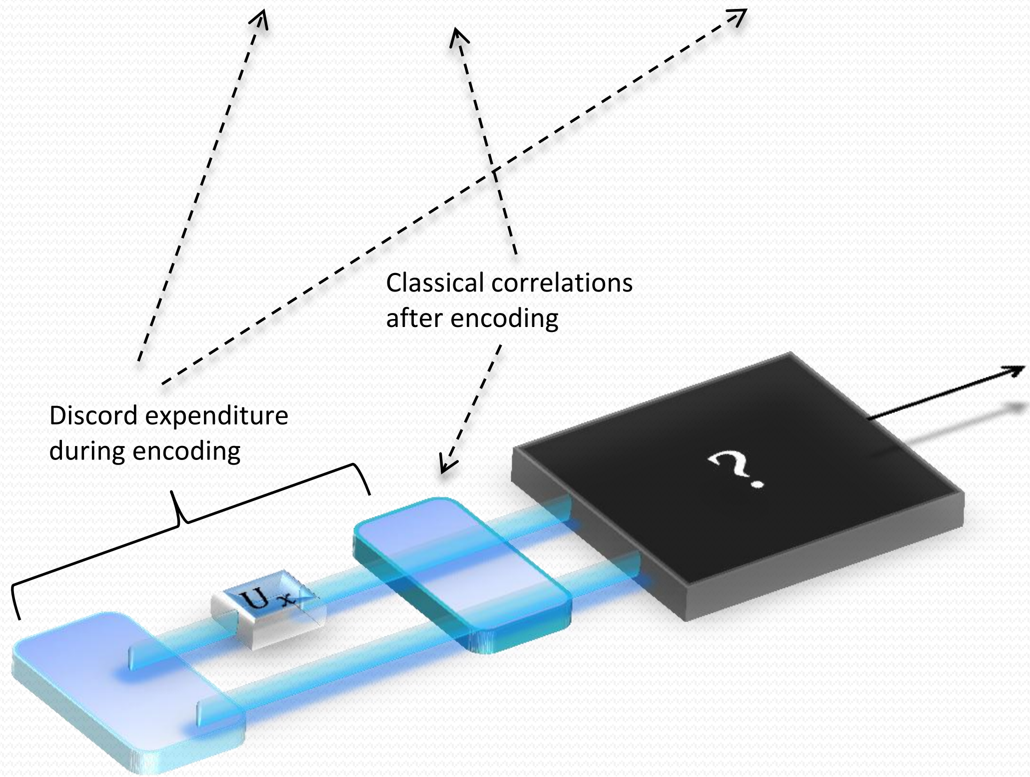


Best possible performance when Bob can, in addition, perform coherent unitary operations between the bipartitions

Discord Induced Quantum Advantage

$$\Delta\delta(A|B) - \tilde{J}(A|B) \leq \Delta I \leq \Delta\delta(\mathrm{A}|\mathrm{B})$$

Can we relate this to, $\Delta\delta$, the Discord consumed during the encoding process?



Discord Induced Quantum Advantage

$$\Delta\delta(A|B) - \tilde{J}(A|B) \leq \Delta I \leq \Delta\delta(A|B)$$

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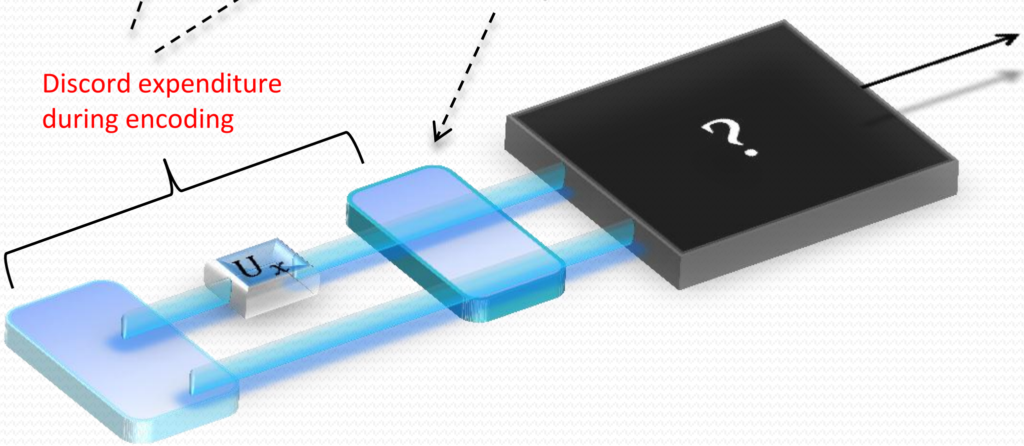
The advantage of coherent interactions is bound by the expenditure of discord during encoding



Discord Induced Quantum Advantage

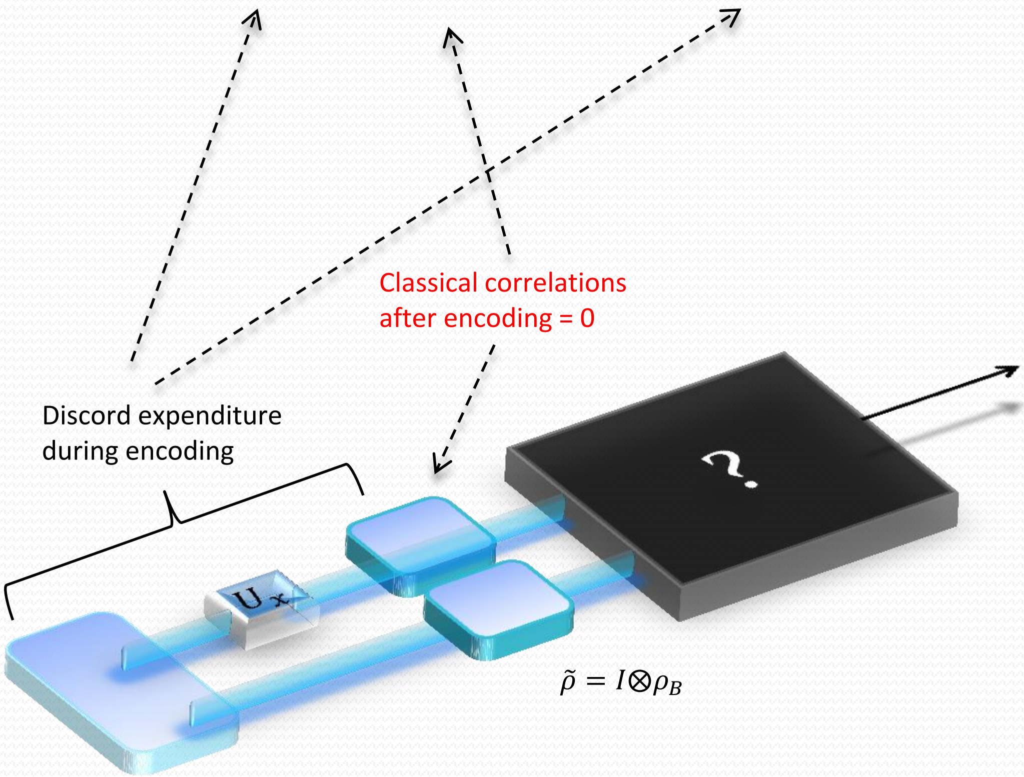
$$\Delta\delta(A|B) - \tilde{J}(A|B) \leq \Delta \leq \Delta\delta(\mathrm{A|B})$$

Can we relate this to, $\Delta\delta$, the Discord consumed during the encoding process?

The advantage of coherent interactions is bound by the expenditure of discord during encoding

Alice can always choose an encoding that expends all available discord, such that:

$$\Delta I = \Delta\delta = \delta(\mathrm{A|B})$$

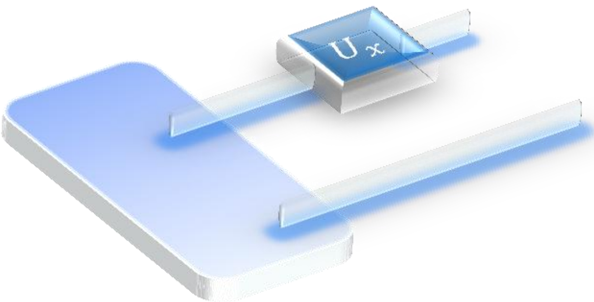


Alice can always choose a maximal encoding
Such that $\tilde{J}(A|B) = 0$

Experimental Test for Discord Induced Quantum Advantage

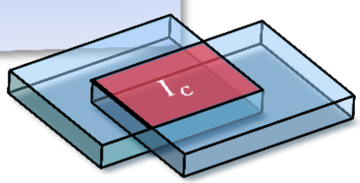
1

Generate some discorded bipartite state ρ by LOCC and encode a variable 'X' onto ρ.



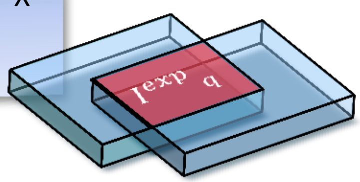
2

Compute I_c from theory to determine the maximum possible knowledge of X with single local measurements.
.



3

Experiment attempt to extract as much information about 'X' as possible by coherent interaction. Call this $I^{exp}{}_q$



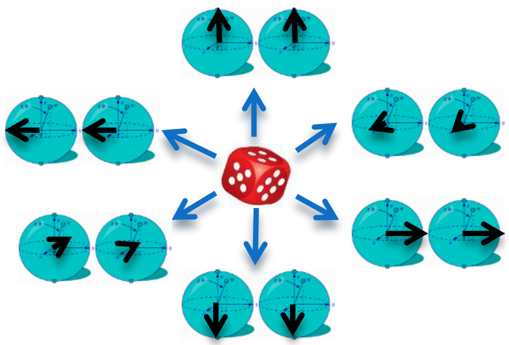
4

Evaluate the amount of information we can extract beyond the incoherent limit:

$$\Delta I^{exp} = I^{exp}{}_q - I_c$$

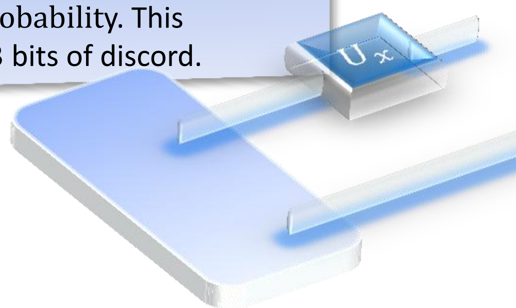
Provided $\Delta I^{exp} > 0$, we are guaranteed that ΔI^{exp} units of discord has been consumed to deliver the observed quantum advantage.

A Simple Example:

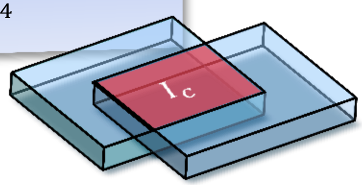


1 Generate the mixture of 3 one-time pads by LOCC, with discord = 1/3

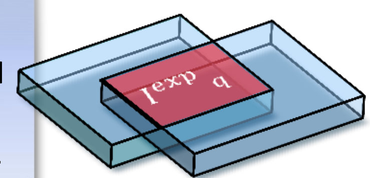
2 Encode 2 bits on one arm of ρ by application of one of the four unitary operations $\{I, X, Z, XZ\}$ *with* equal probability. This consumed 1/3 bits of discord.



2 The maximum possible knowledge of X we can extract with single local measurements is $\frac{5}{4} - \log 3 \approx 0.08$.



3 Experimentally extract information about 'X' application of a CNOT gate prior to measurement. Call this $I^{exp}{}_q$. In the limit of zero errors, $I^{exp}{}_q = I_c + \frac{1}{3}$

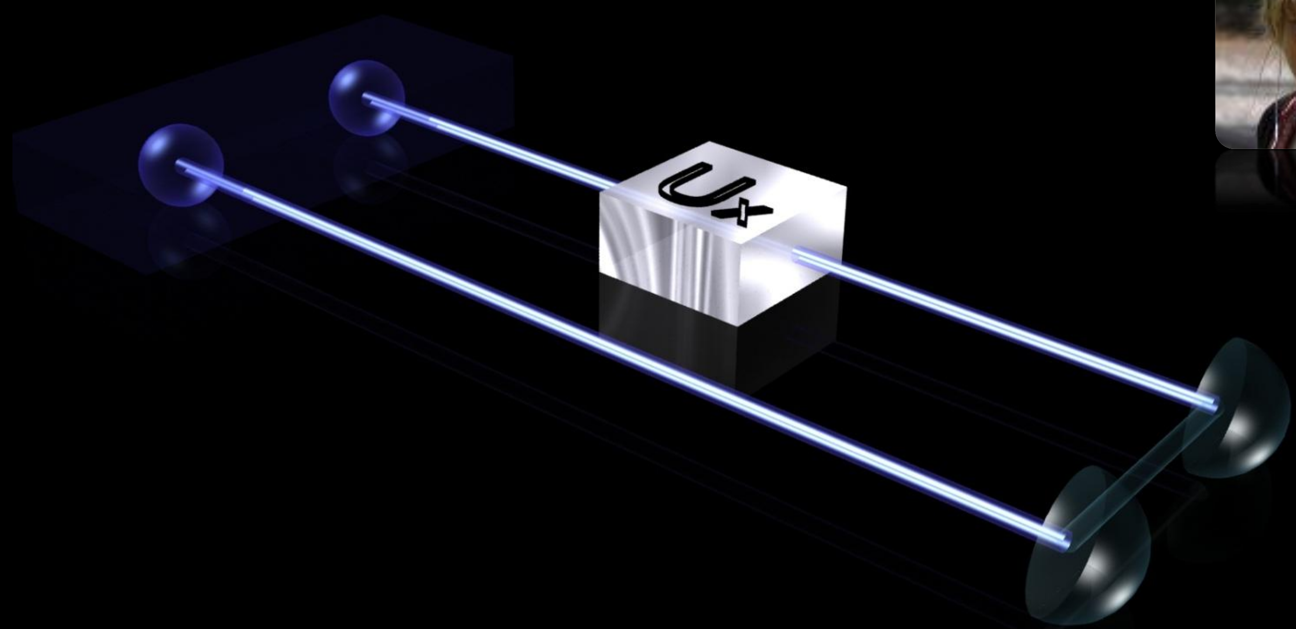


4 Evaluate the amount of information we can extract beyond the incoherent limit:

$$\Delta I^{exp} = I^{exp}{}_q - I_c$$

Provided $\Delta I^{exp} > 0.08$, we are guaranteed that ΔI^{exp}-0.08 units of discord has been consumed to deliver the observed quantum advantage.

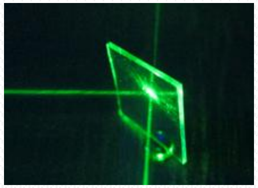
Experimental Test in Continuous Variables



The Case for Continuous Variables

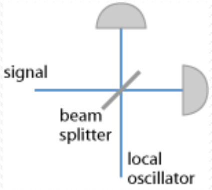
For Continuous Variables:

Coherent interactions should be easy to synthesize, and deterministic.



Beam splitters are a standard optical element, and can deterministically interact two optical modes.

Measurements should introduce as little error as possible.



Homodyne and Heterodyne measurements can be made deterministically with relative accuracy and reliability.

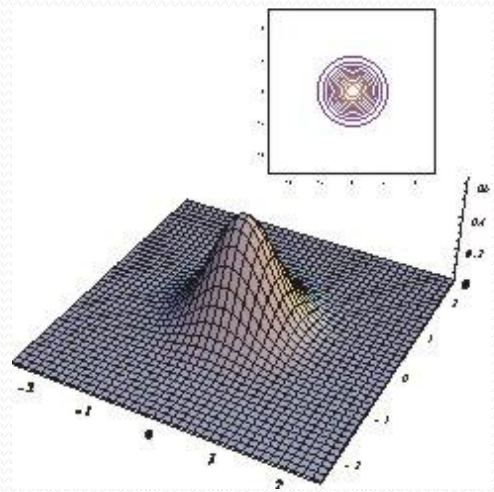
It should be reasonable easy to adjust experimental parameters to test for..

- Resources with varying discord.
- Encodings that consume varying discord.

Varying both quantities involve only adjusting the phase and amplitude of Gaussian beams, which is a standard procedure in linear quantum optics experiments.

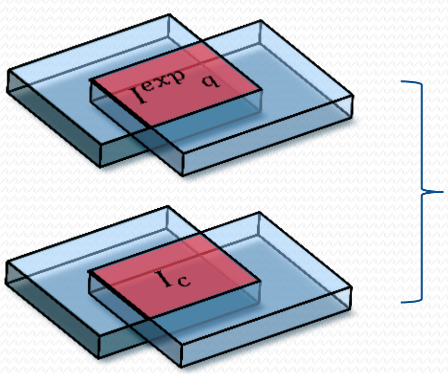
Discord in Continuous Variables

1998: Gaussian Discord jointly proposed by
Giorda and Paris (PRL, 105, 020503)
Adesso and Datta (PRL, 104, 030501)



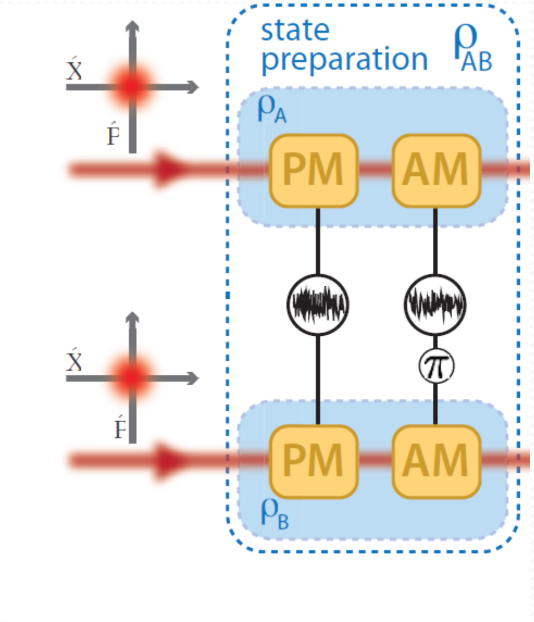
$$J(A|B) = S(A) - \min_{\Pi} S(A|B_{\Pi})$$

Class of feasible measurements is defined by set of Gaussian measurements



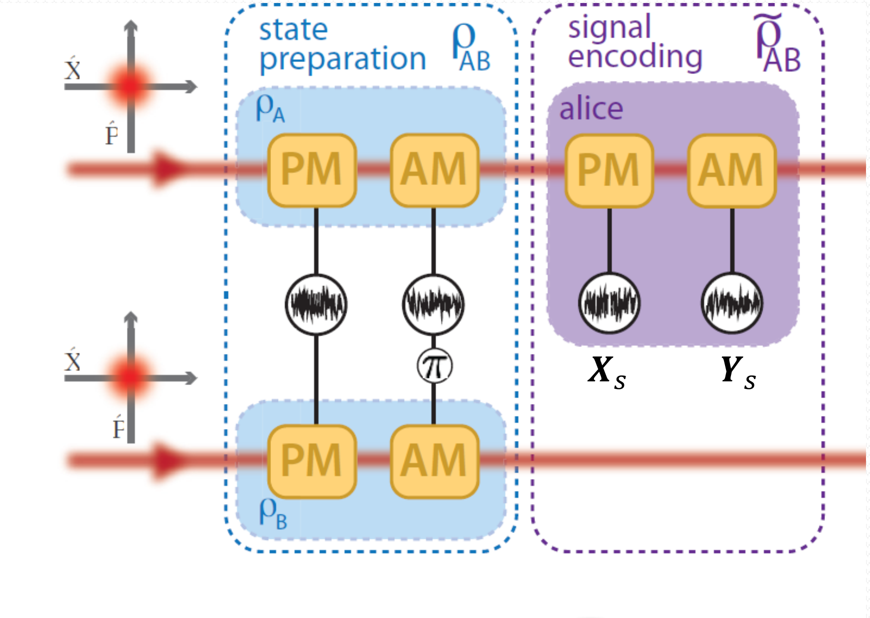
$\Delta I = I_q - I_c$ now characterizes the advantage of having coherent interactions when otherwise limited to Gaussian measurements,

Experimental Implementation



1 Generate a discorded state on two optical beams adding correlated and anti-correlated noise in momentum and position of variance V

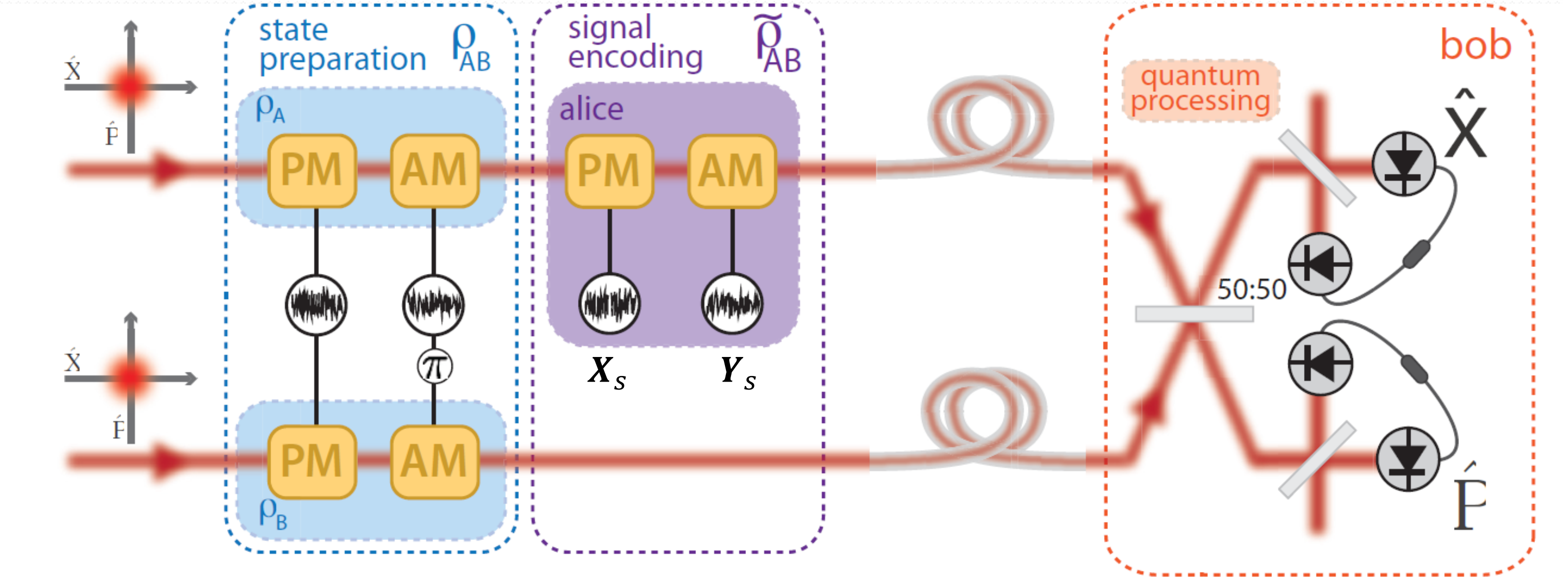
Experimental Implementation



1 Generate a discorded state on two optical beams adding correlated and anti-correlated noise in momentum and position of variance V

2 Encode classical Gaussian signals, (X_s, P_s) , of variance V_s by displacements in phase space. Greater V_s implies more discord consumed.

Experimental Implementation



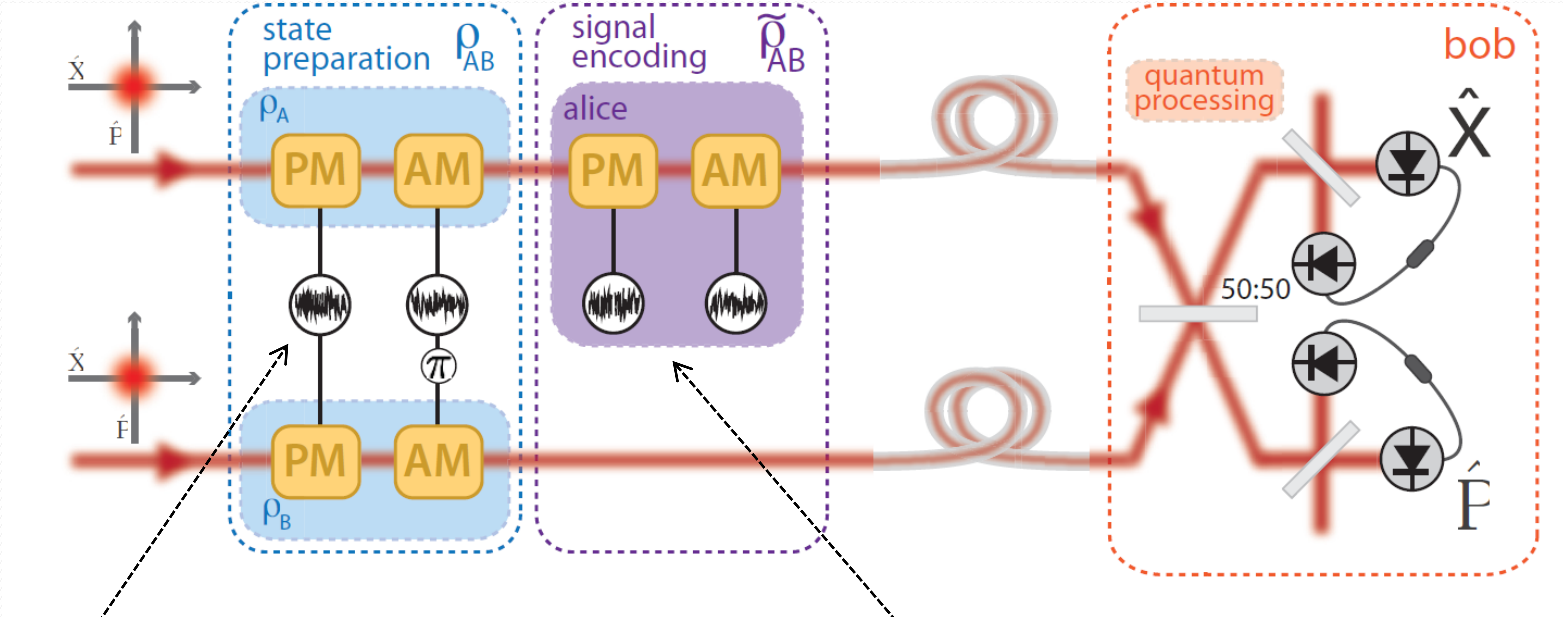
1 Generate a discorded state on two optical beams adding correlated and anti-correlated noise in momentum and position of variance V

2 Encode classical Gaussian signals, $(\boldsymbol{X_s}, \boldsymbol{P_s})$, of variance V_s by displacements in phase space. Greater V_s implies more discord consumed.

3 Extract the encoded information by coherent interaction of the two beams to obtain estimates $(\boldsymbol{X_o},\boldsymbol{Y_o})$. Compute the experimental performance

$$I^{exp}{}_q = I[(\boldsymbol{X_s},\boldsymbol{Y_s}):(\boldsymbol{X_o},\boldsymbol{Y_o})]$$

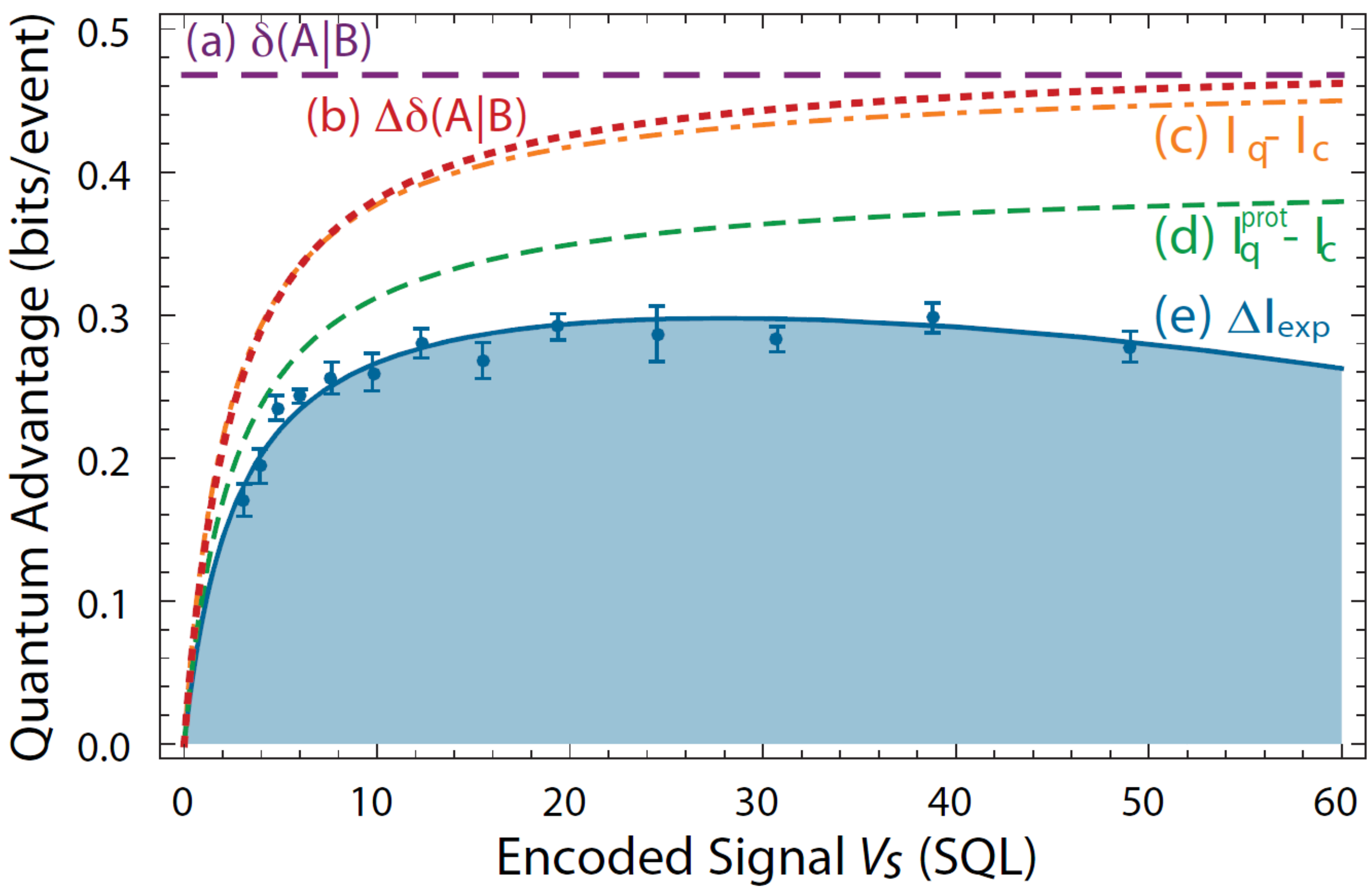
Experimental Implementation



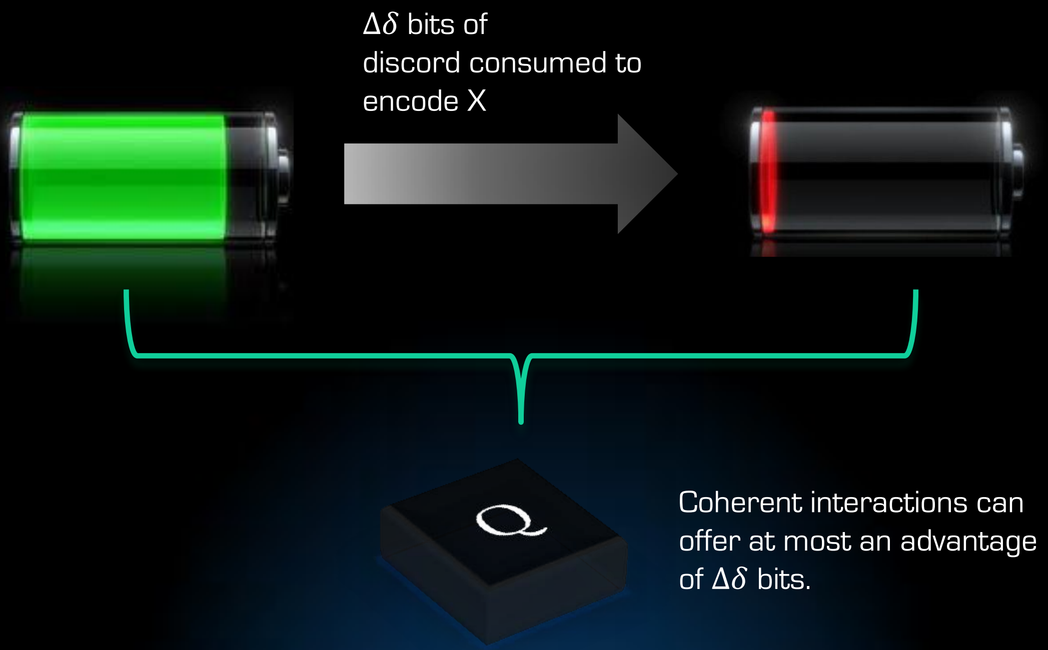
We can test resources with various values discord by adjusting the various of the correlated noise V.

We can test decoding that consumed varying amounts of discord by adjusting the variance of the encoding V_s

Experimental Results



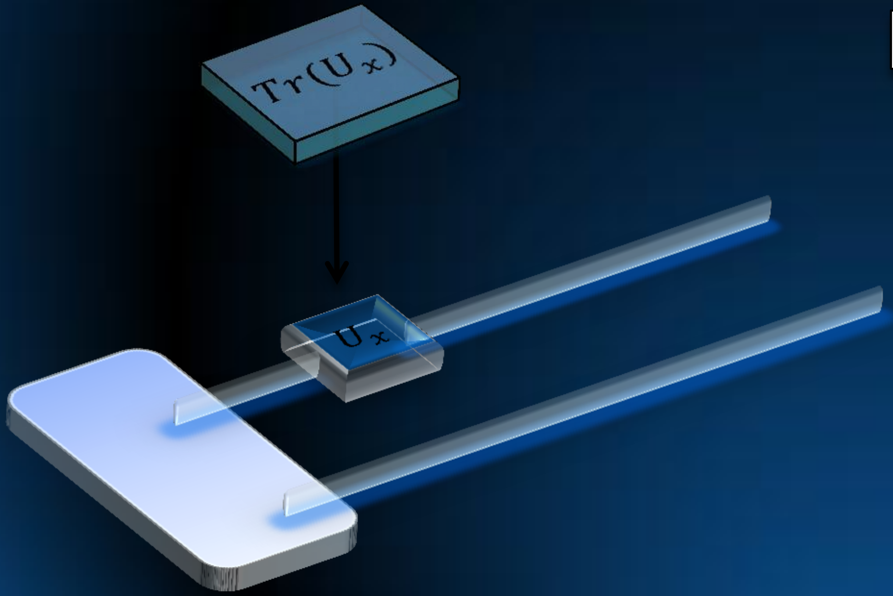
Discord as a Quantum Resource



Outlook

A Test for Quantum Computation?

Coherent 2-qubit are an essential component of quantum computers. Should Bob claim that he has a quantum computer, Alice can prepare a discorded state by LOCC and challenge Bob to extract more information than the incoherent limit.



Connection with DQC1?

DQC1 is about the computation of the trace of a unitary. We can however think about this as encoding a classical variable in the trace of a given Unitary, and computation of this trace to be a decoding process. Can we use this link to formalize the connection between discord and DQC1?