Occam's Quantum Razor

How quantum mechanics can reduce the complexity of classical Models

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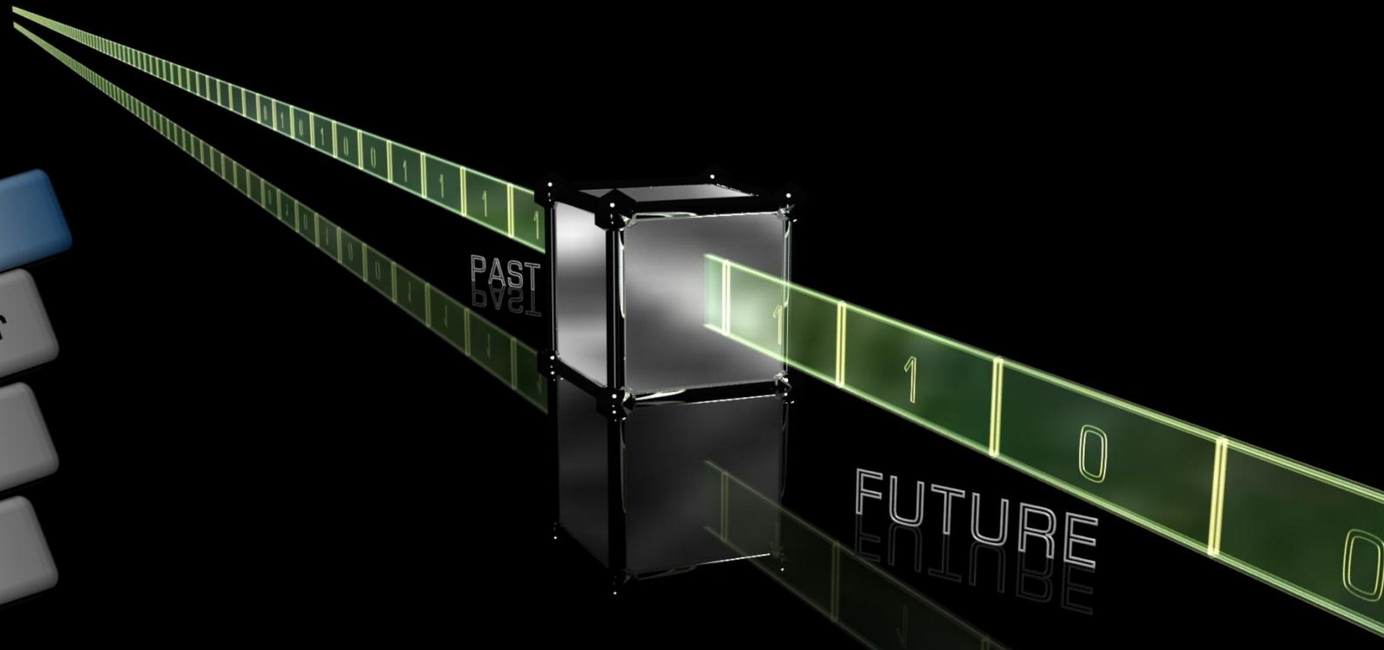
E. Rieper
V. Vedral



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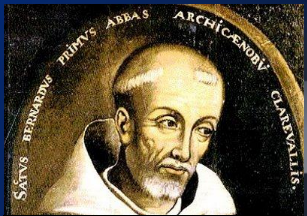
Centre for Quantum Technologies
National University of Singapore

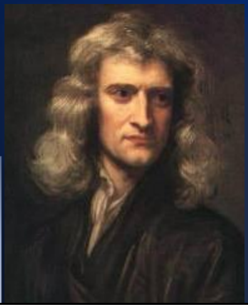
M.Gu, K.Wiesner, E.Rieper, V.Vedral, *Nature Communications, 3, 762 (arXiv:1102.1994)*

Occam's Razor



William of Ockam

"Plurality is not to be posited without necessity."

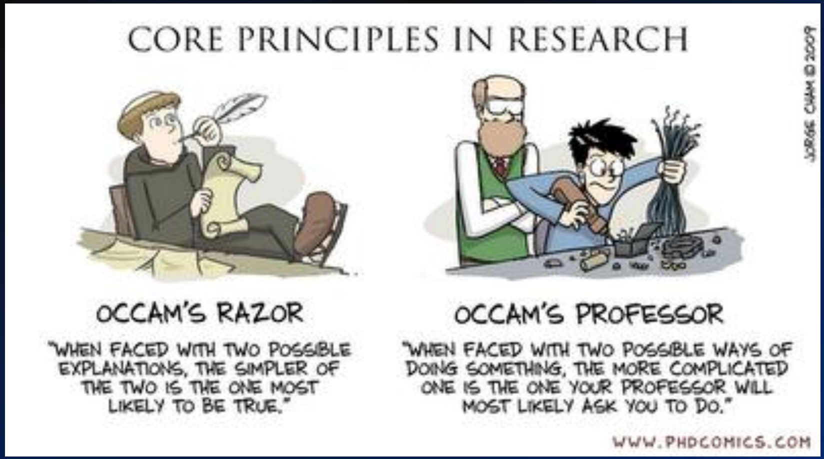


Isaac Newton

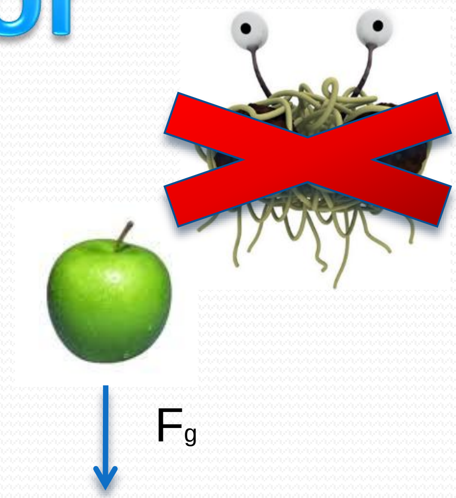
"We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances."

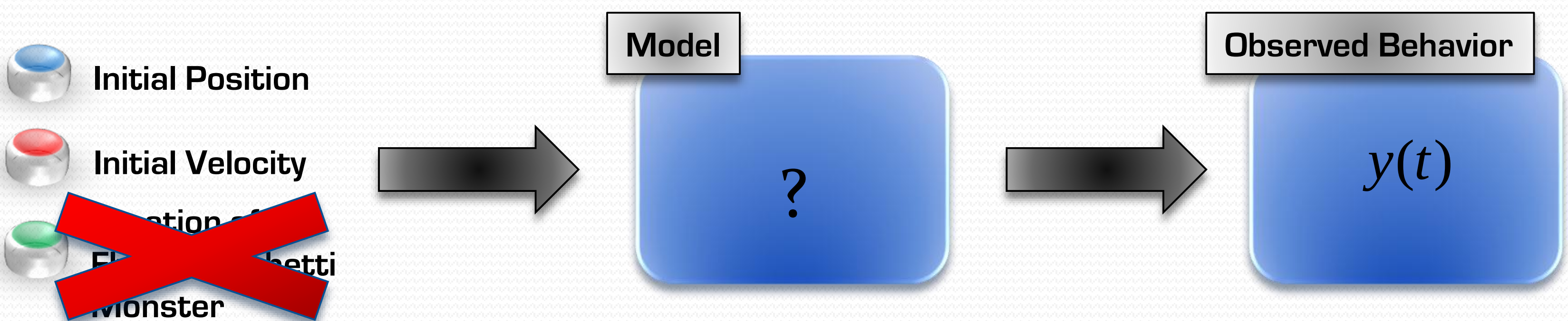
Albert Einstein

"Everything should be made as simple as possible, but not simpler."



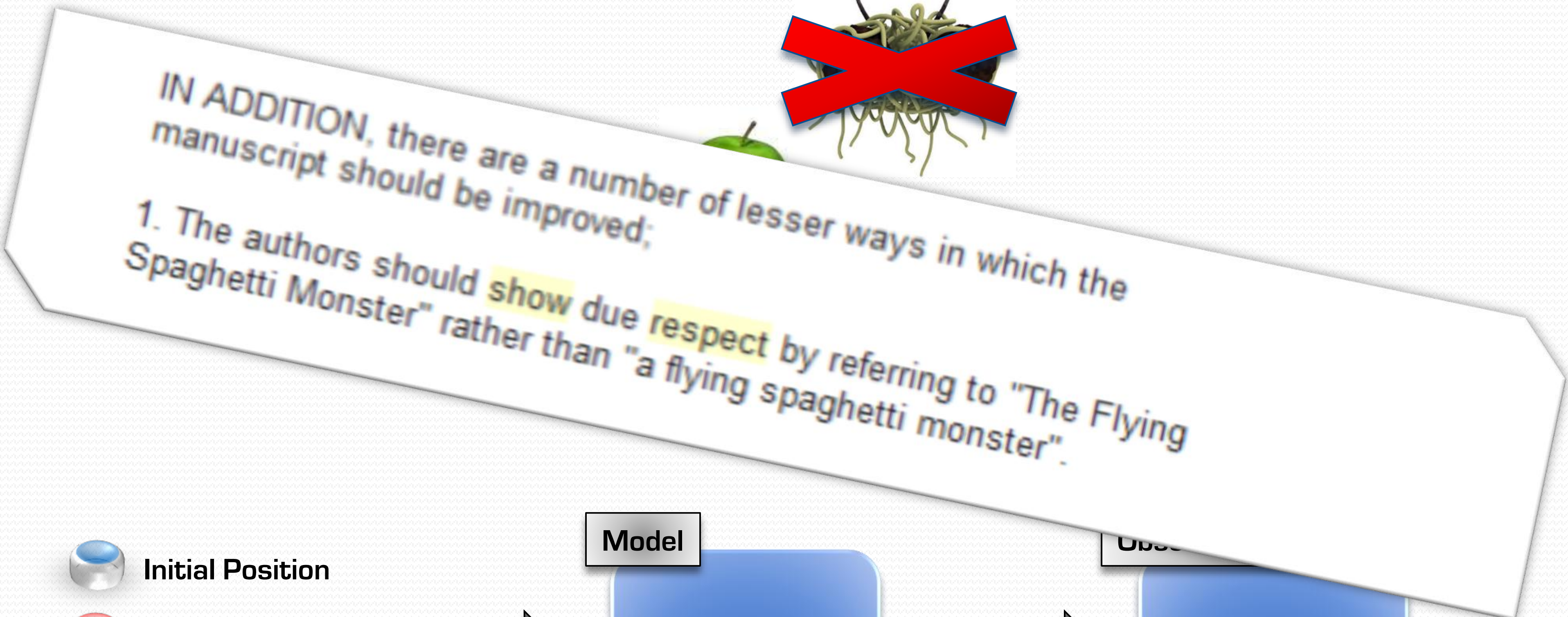
Occam's Razor



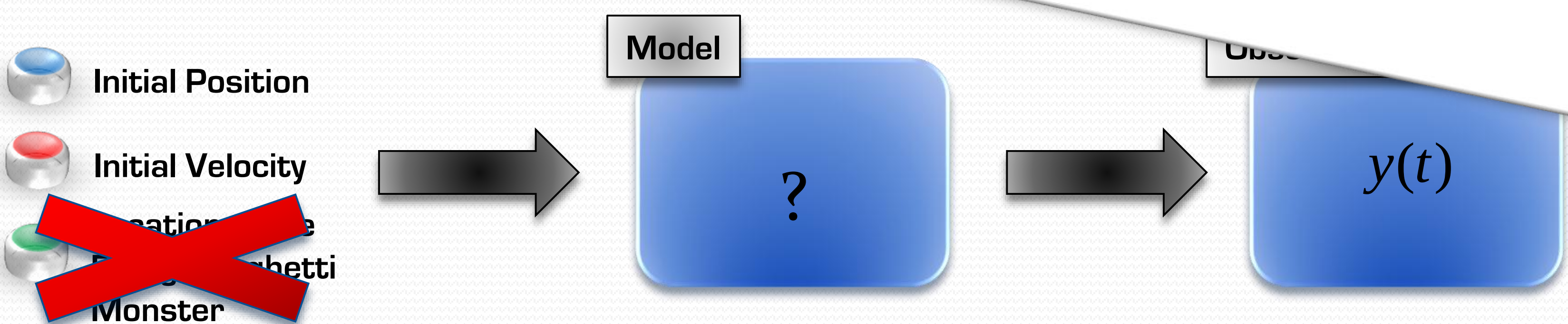


"We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances." -Newton

Occam's Razor


IN ADDITION, there are a number of lesser ways in which the manuscript should be improved;

1. The authors should show due respect by referring to "The Flying Spaghetti Monster" rather than "a flying spaghetti monster".



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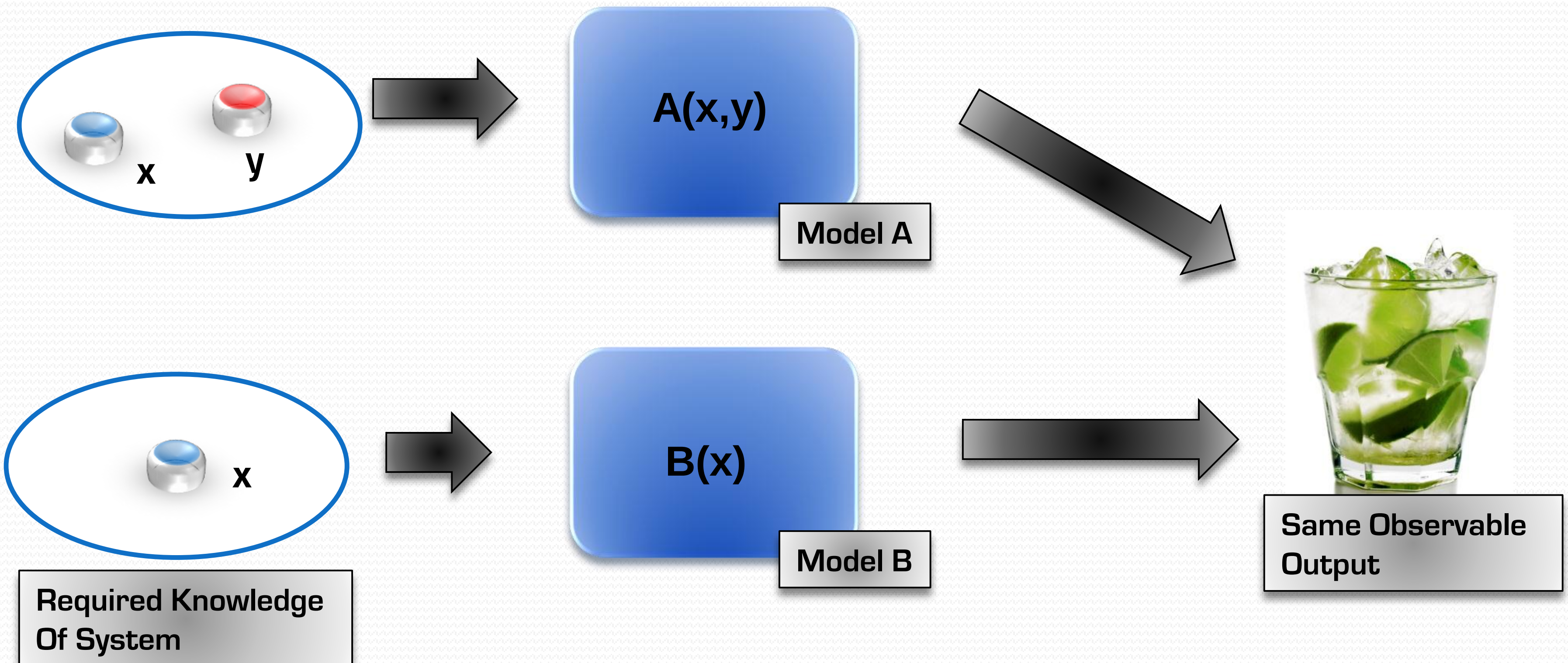
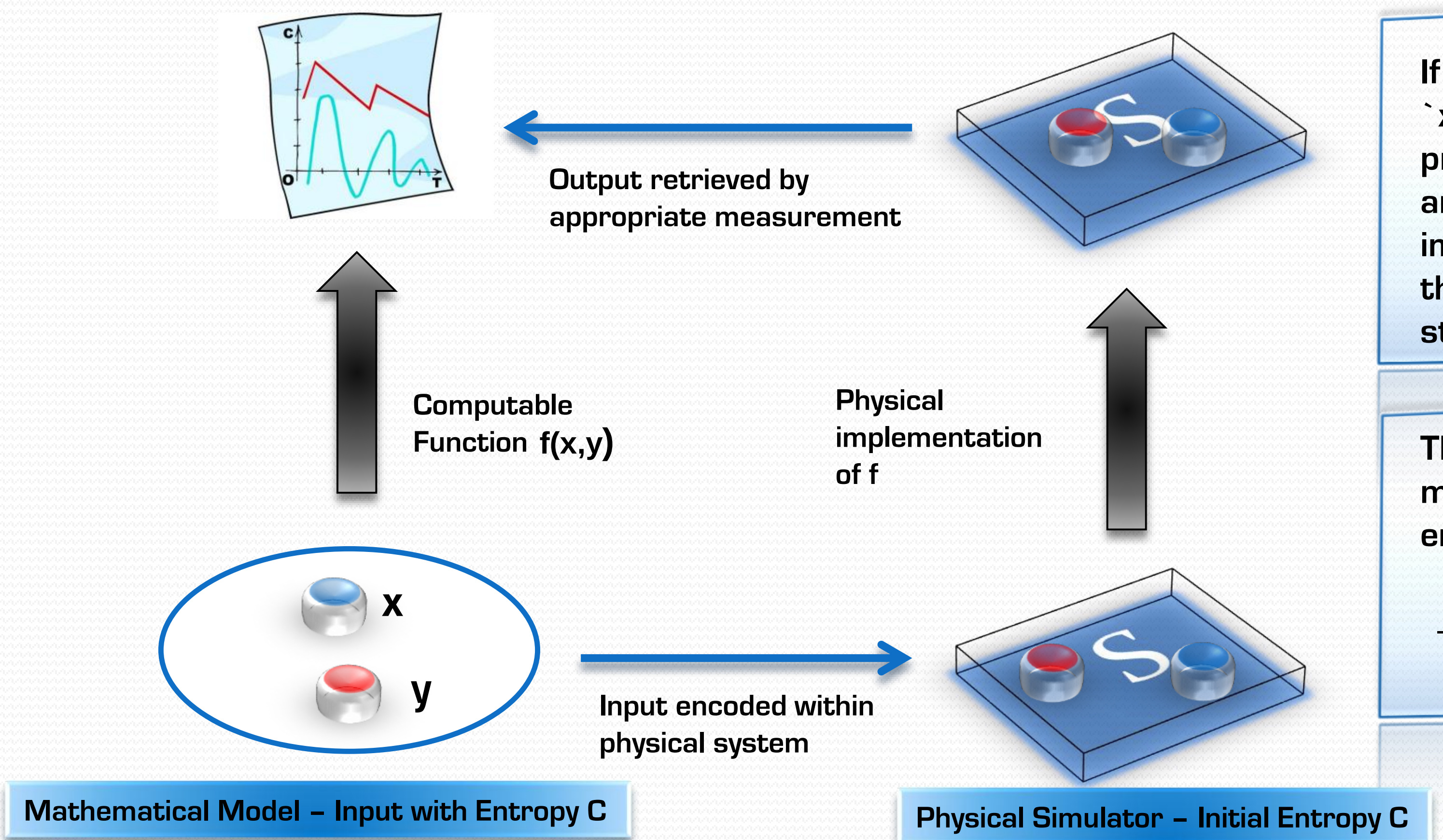
Occam's Razor



If two models make statistically identical predictions, the model that requires the least *input information* is preferred.

Occam's Razor gets Physical



If a model requires \`x' to make a prediction, then any physical implementation of the model must store 'x'

That is, the system must have at least entropy

$$-\sum p_x \log p_x$$

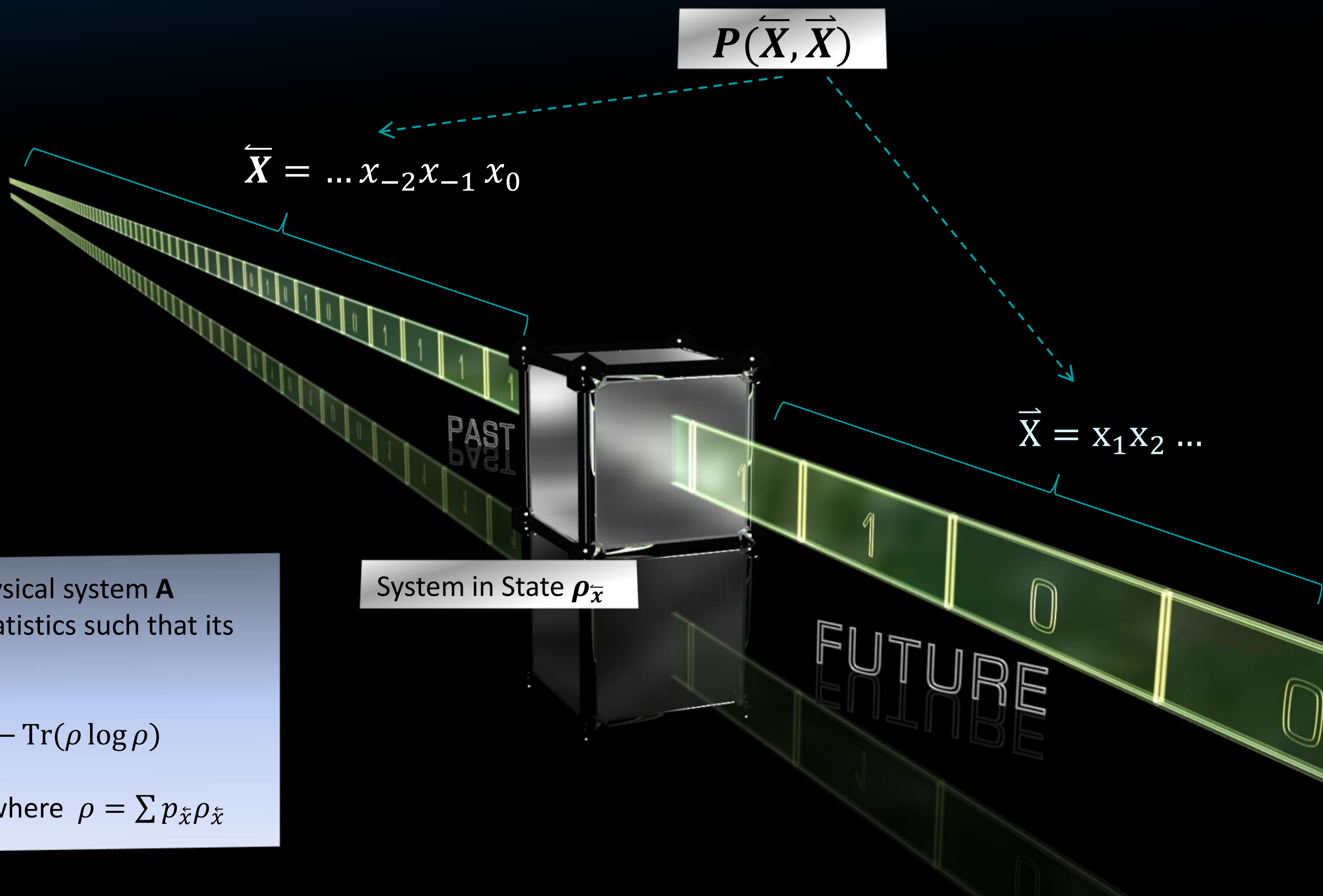
The Matrix Problem

Suppose you're a programmer for the matrix

You are tasked to program an object to simulate a particular desired behaviour.

How do you do this with the least hard drive space? i.e. system of minimal internal entropy?

Simulating Stochastic Processes



Task:

Construct a physical system **A** with desired statistics such that its entropy:

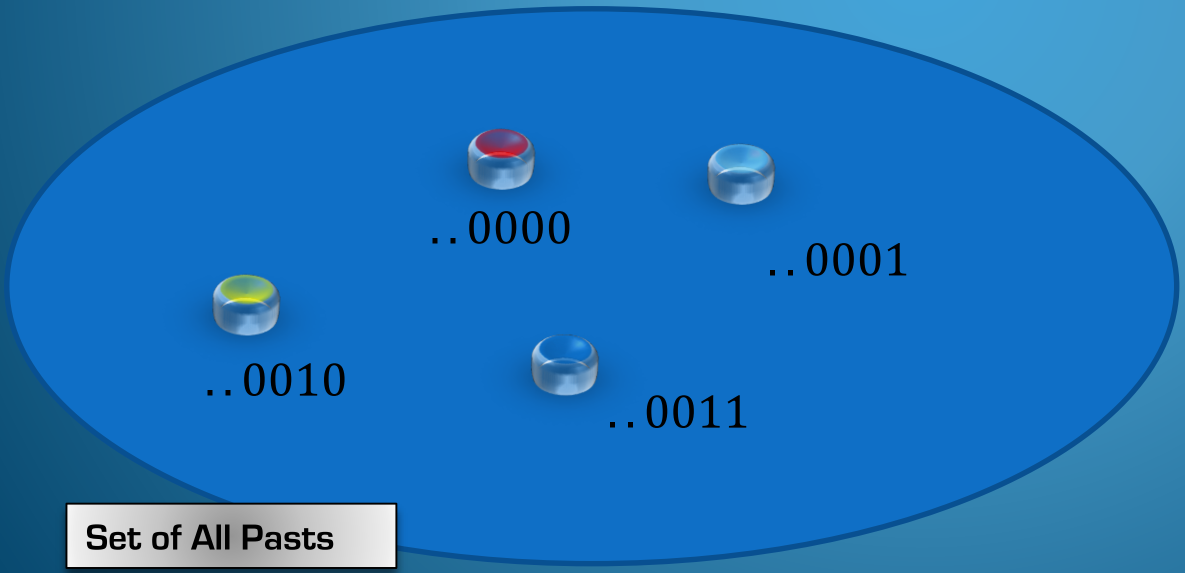
$$S(\boldsymbol{A}) = -\,\mathrm{Tr}(\rho \log \rho)$$

Is minimised, where $\rho = \sum p_{\overleftarrow{x}} \rho_{\overleftarrow{x}}$

The Brute Force Approach

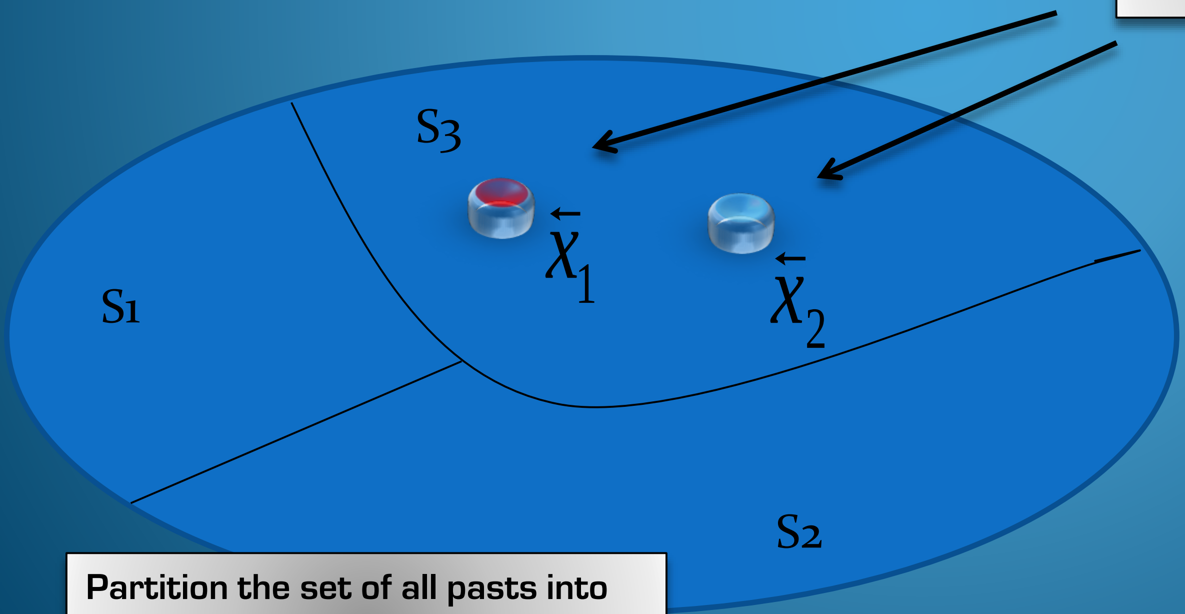
Construct a system that stores each possible past in a separate configuration.

$$\rho_{\overleftarrow{x}} = \overleftarrow{x}$$



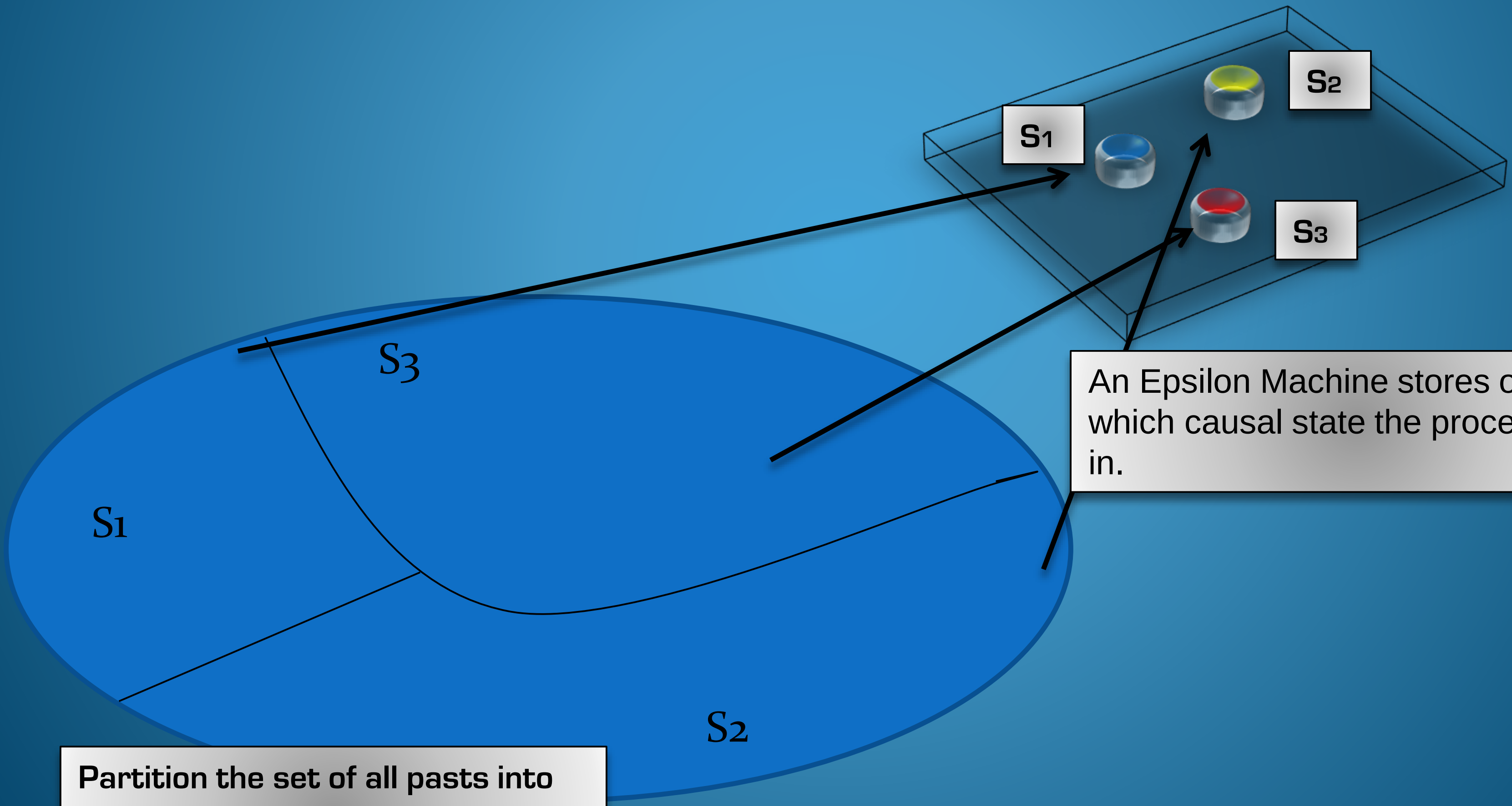
A More Refined Approach

Two different possible pasts belong to the same Causal state if they have coinciding futures.



Partition the set of all pasts into equivalence classes, referred to as Causal states.

Epsilon Machines



An Epsilon Machine stores only which causal state the process is in.

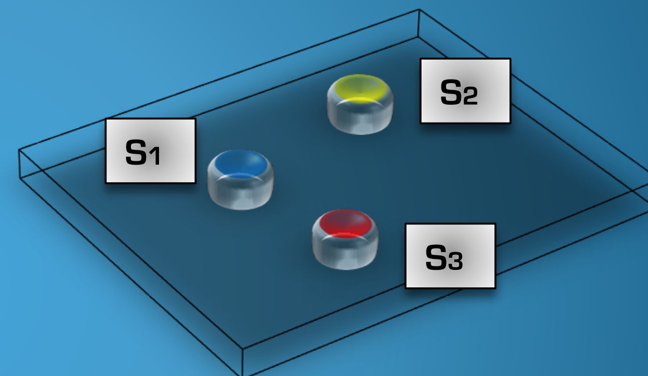
Partition the set of all pasts into equivalence classes, referred to as Causal states.

Epsilon Machines

Internal Entropy:

$$C_\mu = -\sum p_i \log p_i$$

Probability the process is in Causal State S_i



To simulate a sequence of random coin flips….

We have a process with exactly 1 Causal State

No Information about the Past is required!

No classical system can simulate a given stochastic process using less information than a Epsilon Machine

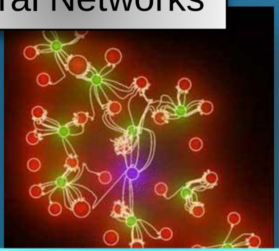
C_μ Is a **intrinsic** property of a stochastic process that measures the minimal amount of memory required to simulate the given process.

Crutchfield 1989

Phys. Rev. Lett. 63, 105–108 (1989)

Statistical Complexity

Neural Networks



IEEE Trans. Neural Networks, 10, 2, 284-302

Pseudo-random Number generators.

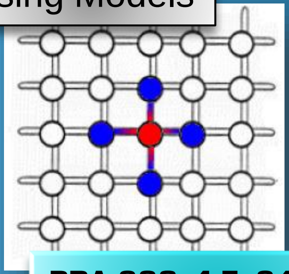
Physica A: 356, 1, 133-138

Applied to wide range of systems.

Dripping Faucets

Physica A: 257, 1-4, 385-389

Ising Models



PRA 238, 4-5, 244-252

Crutchfield 1989

No classical system can simulate a given stochastic process using less information than a Epsilon Machine

Phys. Rev. Lett. 63, 105–108 (1989)

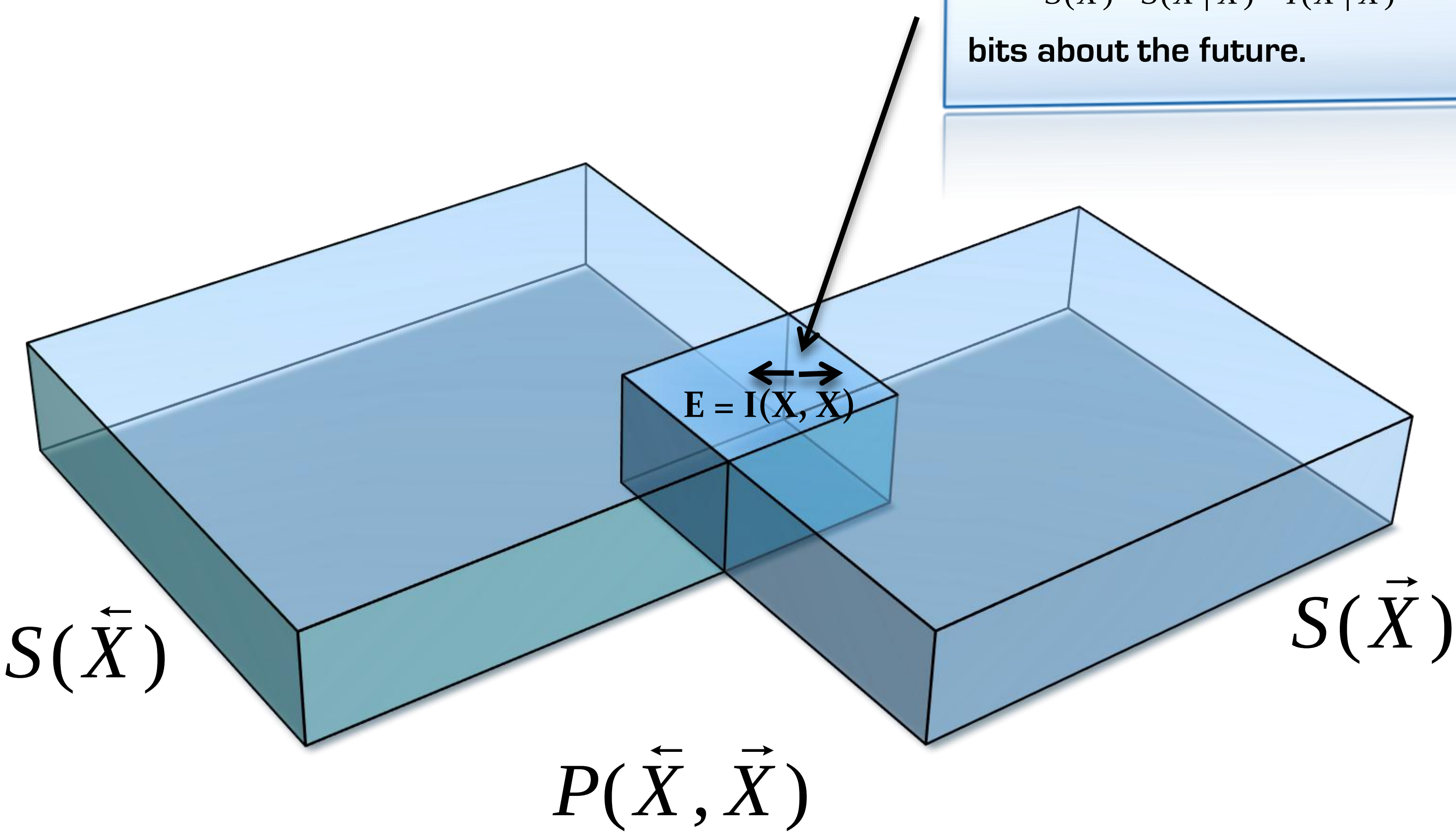
C_μ Is a intrinsic property of a stochastic process that measures the minimal amount of memory required to generate the given process.

Can Epsilon Machines be improved?

Knowledge of the past contains

$$S(\vec{X}) - S(\vec{X} \mid \overleftarrow{X}) = I(\vec{X} \mid \overleftarrow{X})$$

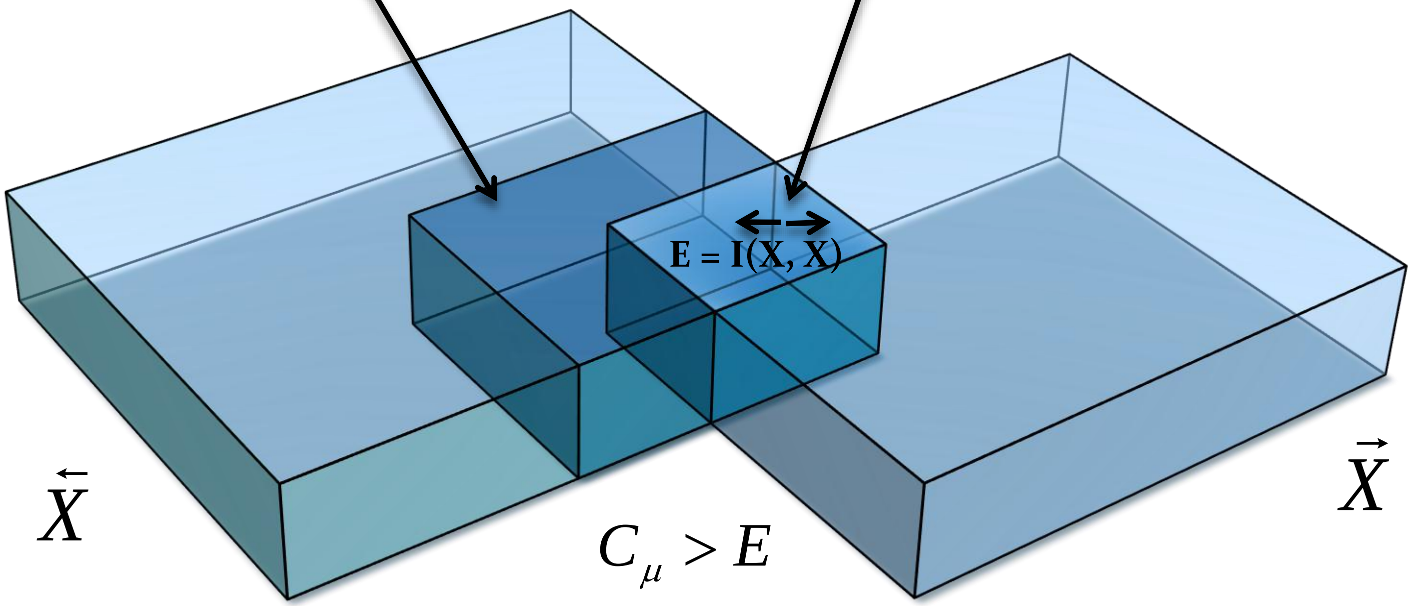
bits about the future.



Can Epsilon Machines be improved?

An epsilon machines must store C_μ bits to know to extract entire past.

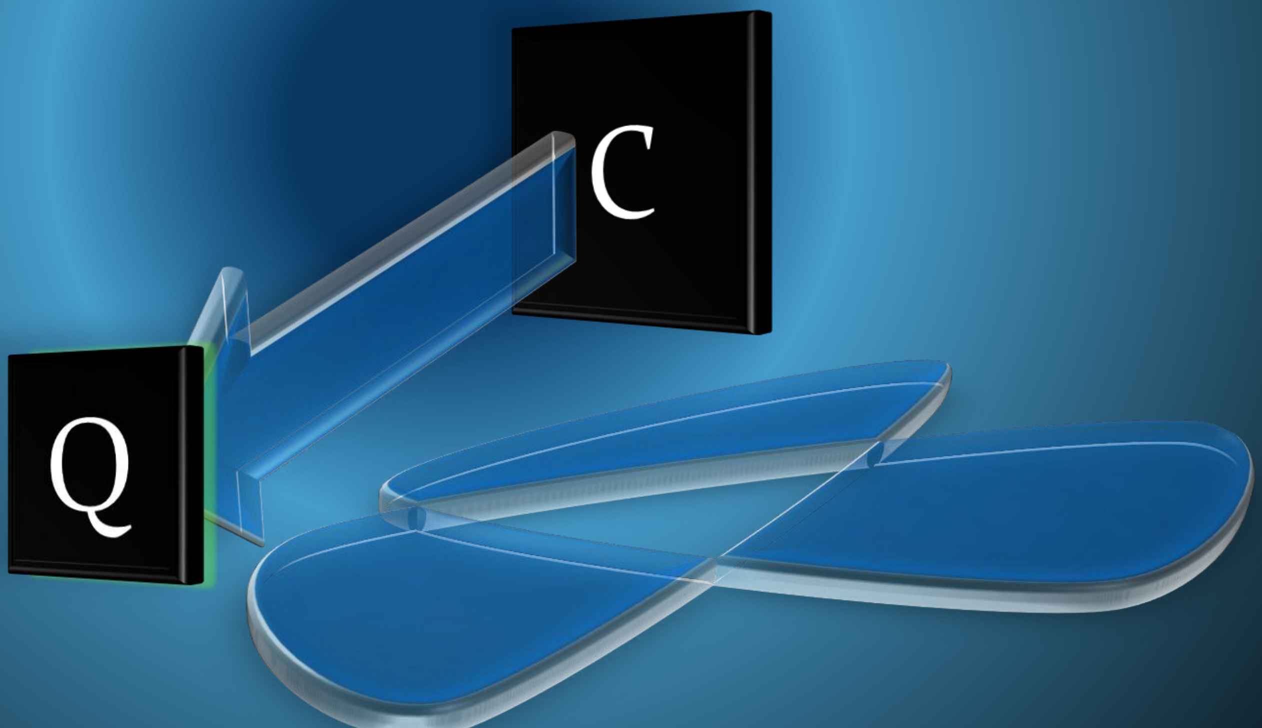
Lower bound on how many bits any simulator of The process must store.



JP **Crutchfield**, CJ Ellison, "Time's Barbed Arrow." PRL 2009 - APS

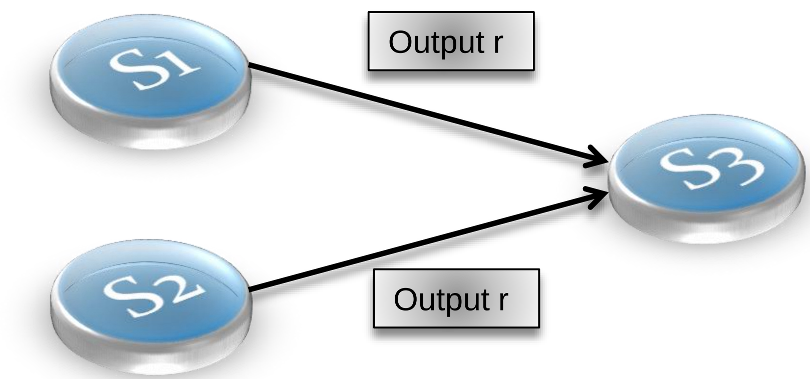
Some of the information recorded is still not relevant to the future – the Epsilon machine still wastes information!

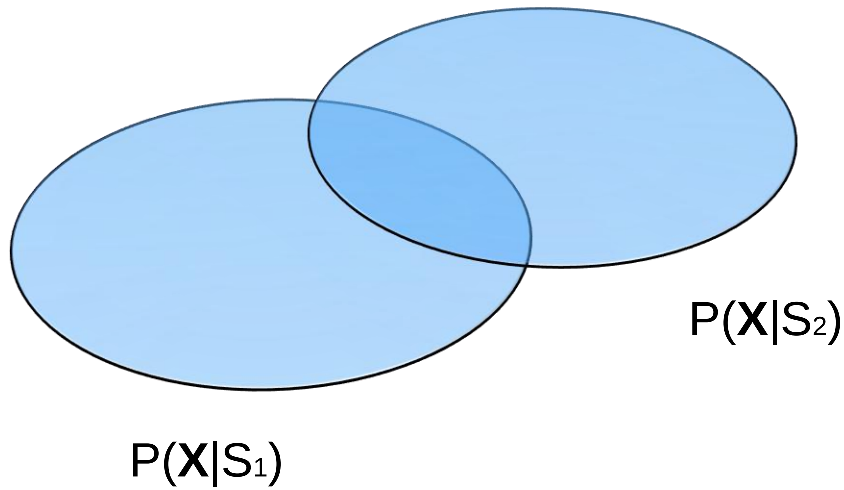
Can Quantum Razors Be Sharper?



The source of classical inefficiency

Suppose two differing causal states have finite probability to transition to an coinciding causal state after coinciding output.





The future of the two causal states are not entirely distinct.

A classical epsilon must store a property A that distinguishes S_1 and S_2. But observation of the entire future does not guarantee the ability to retrieve A.

Some of the storage used to keep track of A is wasted.

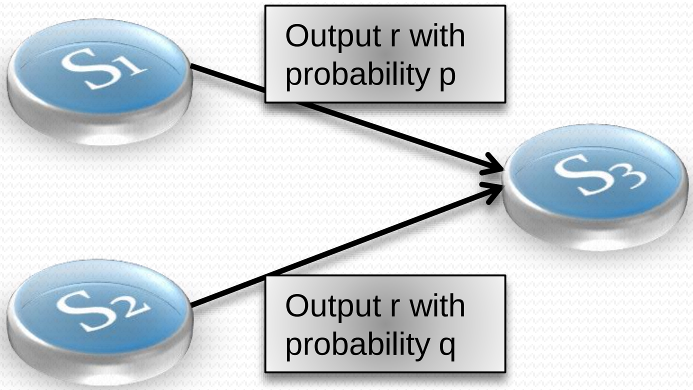
Quantum Improvement

A quantum epsilon machines does not distinguish the causal states completely.

Map each causal state Sj to a corresponding quantum state

$$|S_j\rangle = \sum_r \sum_k \sqrt{T^r{}_{j,k}}\,|r\rangle|k\rangle$$

Probability a Stochastic process in Causal state Sj will emit output `r' and transition to Sk



$$\langle S_1|S_2\rangle \geq \sqrt{pq} > 0$$

Storage of quantum causal states require less memory than their classical counterpart

Quantum Improvement

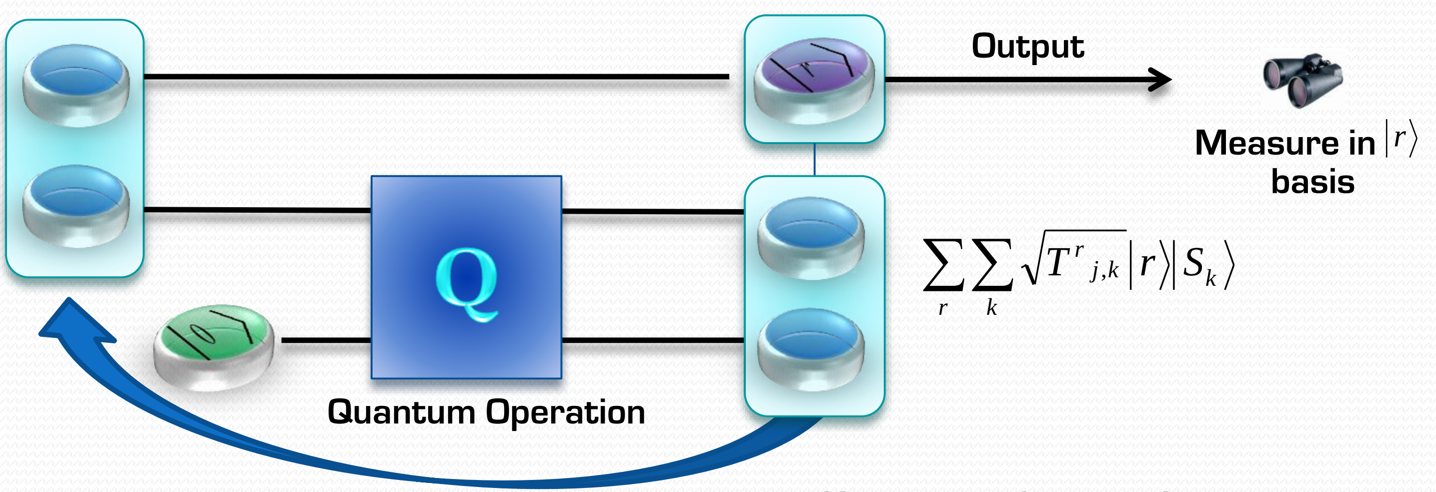
A quantum epsilon machines does not distinguish the causal states completely.

Map each causal state Sj to a corresponding quantum state

$$\left|S_j\right\rangle = \sum_r \sum_k \sqrt{T^r{}_{j,k}}\left|r\right\rangle\left|k\right\rangle$$

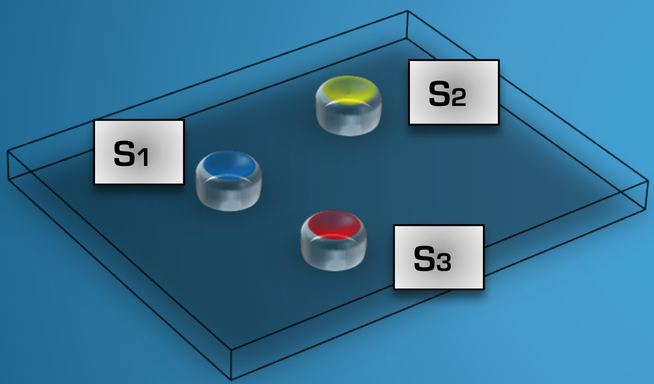
Future statistics can still be accurately reproduced:

Simulator initialized in $|S_j>$



Use as next input and repeat.

The Quantum Epsilon Machine



A classical Epsilon Machine allocates enough storage to distinguish every causal state

Quantum systems can go beyond this by compressing the information further… distinguishing the causal states only to the degree that they affect the future.

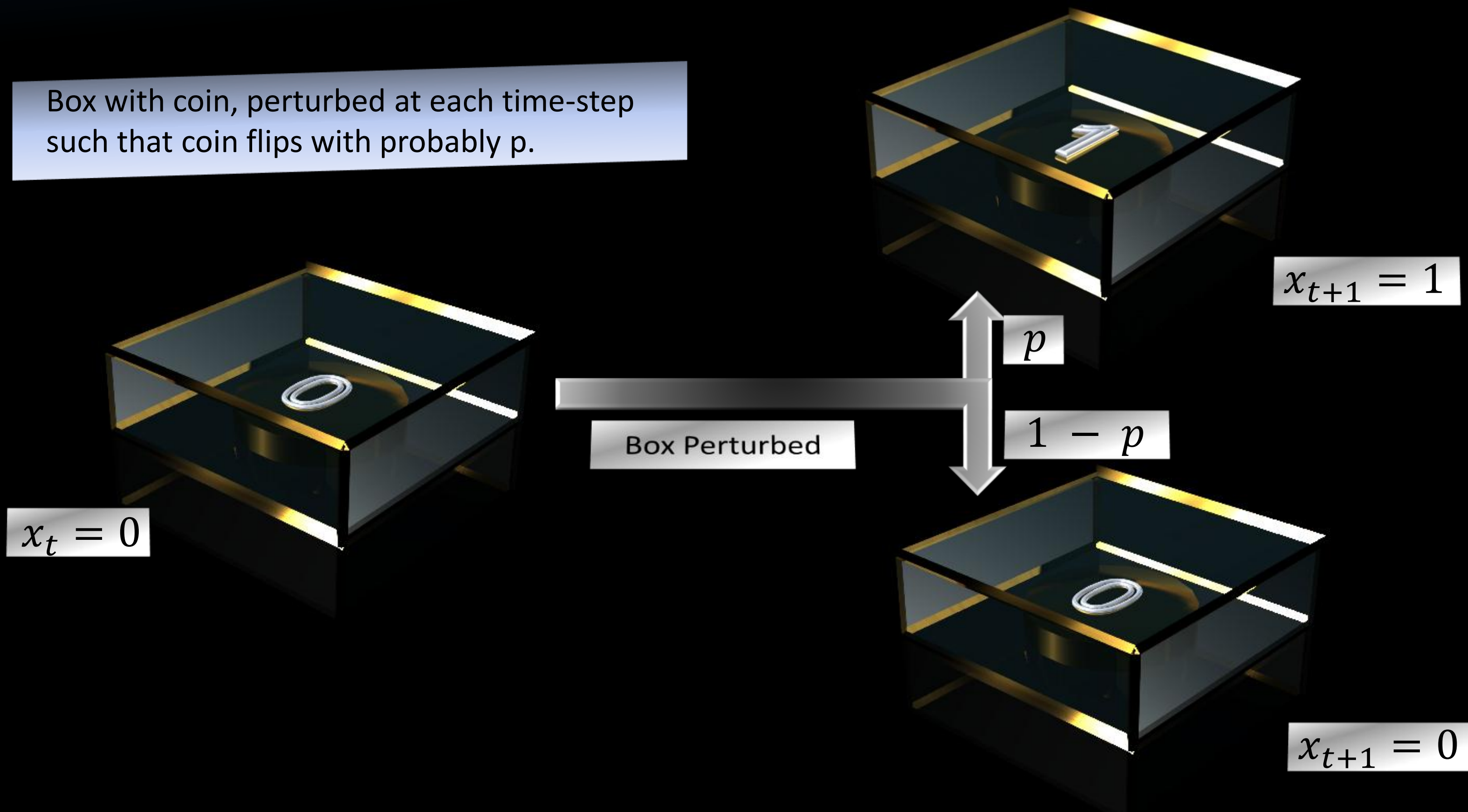
THEOREM:

Provided the best Classical simulator for a stochastic process erases some information, the Quantum razor is sharper

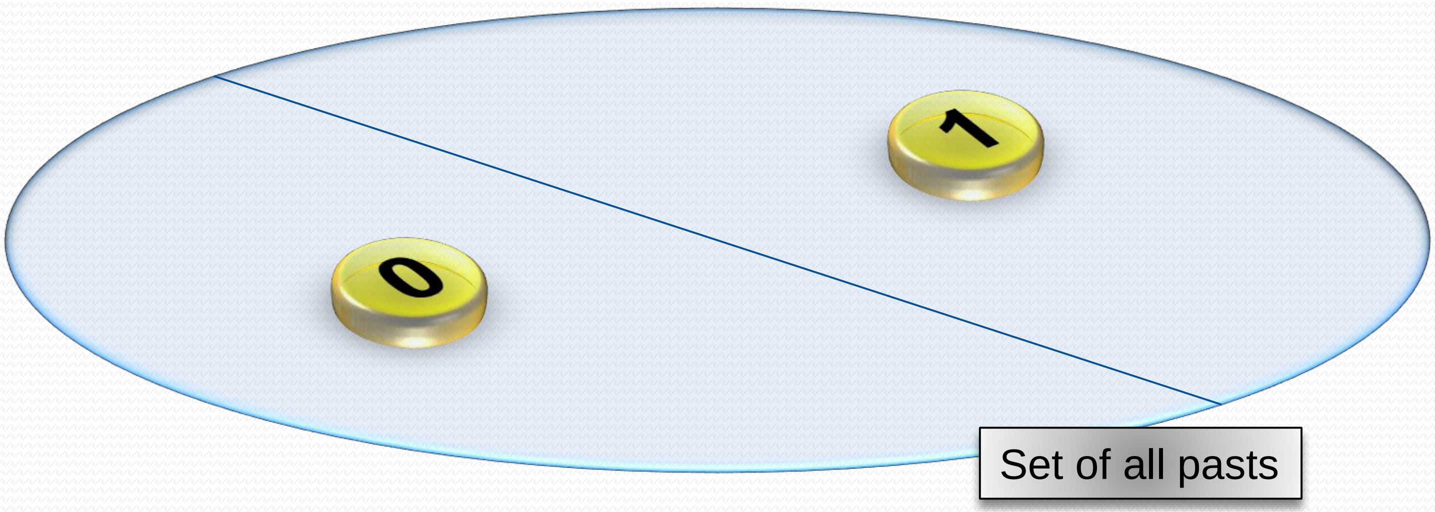
M.Gu, K.Wiesner, E.Rieper, V.Vedral, Nature Communications, 3, 762

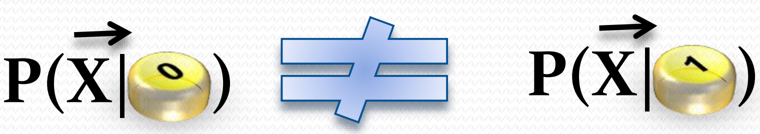
Example: The Perturbed Coin

Box with coin, perturbed at each time-step such that coin flips with probably p.



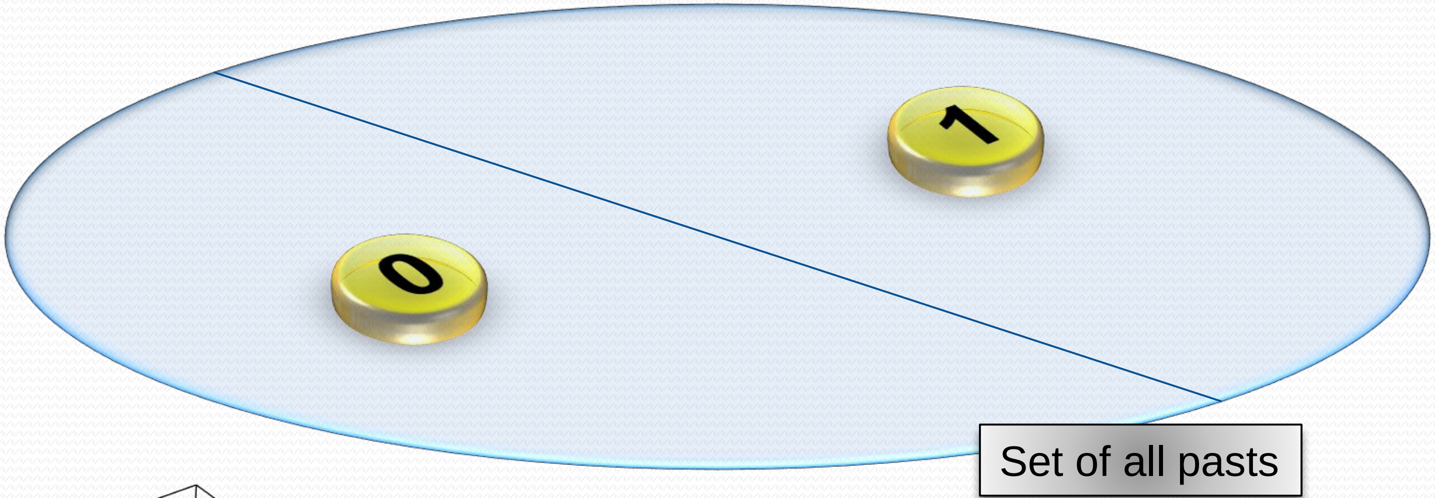
Classical Epsilon Machine

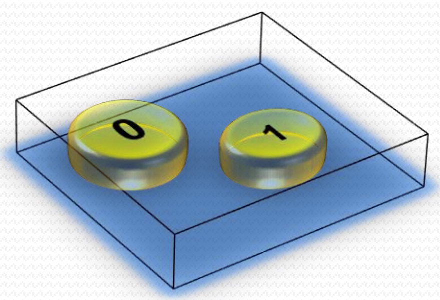




We cannot discard information about the state of the coin.

Classical Epsilon Machine





Optimal classical system for generating $\mathbf{P}(\overleftarrow{\mathbf{X}}, \overrightarrow{\mathbf{X}})$
has internal entropy 1, for any $p \neq 0.5$

However this model isn't very efficient….

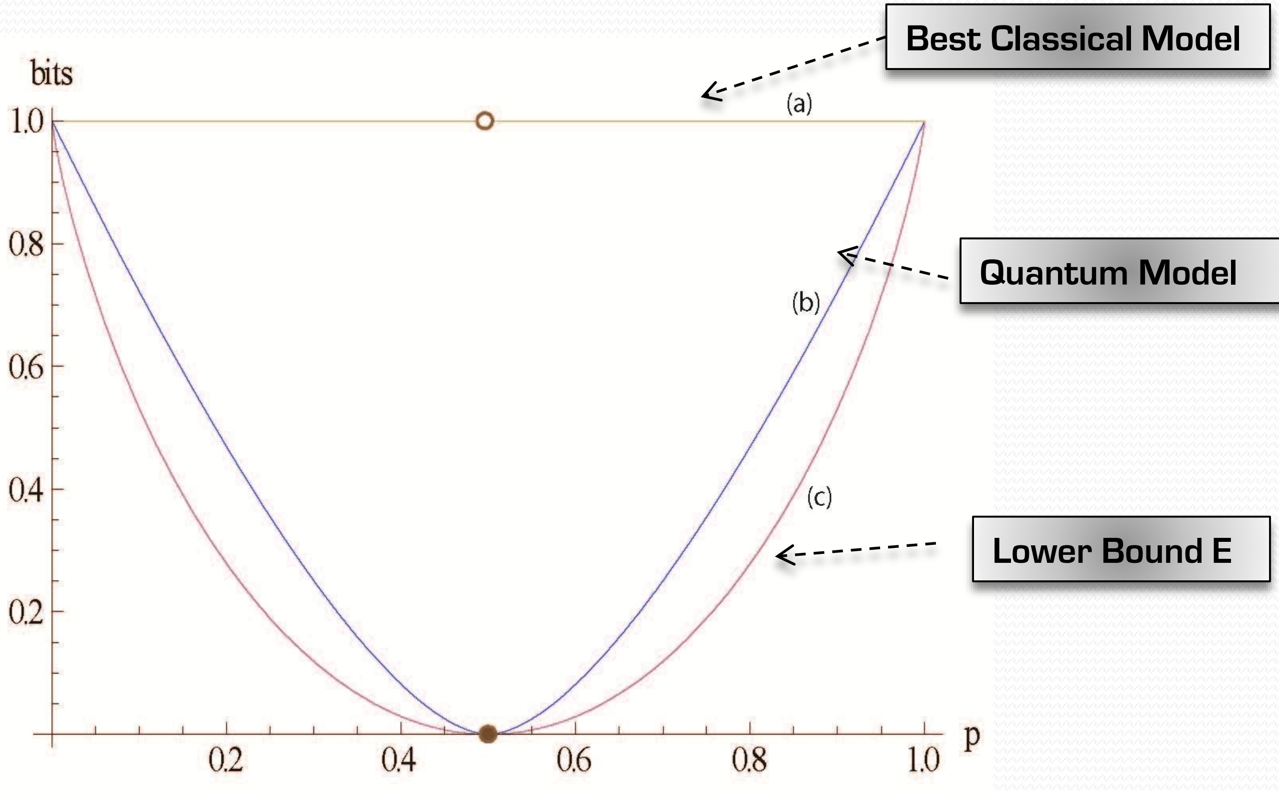
$P \rightarrow 0.5$: $\mathbf{E} \rightarrow \mathbf{0}$

Quantum Does Better

The Quantum Refinement

Encode 0 as $|S_0\rangle = \sqrt{1-p}|0\rangle + \sqrt{p}|1\rangle$

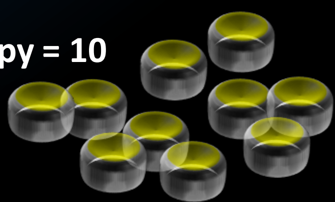
Encode 1 as $|S_1\rangle = \sqrt{p}|0\rangle + \sqrt{1-p}|1\rangle.$



Perturbed Coin Lattice

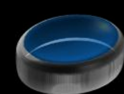
Example: K = 10, p = 0.4

Entropy = 10

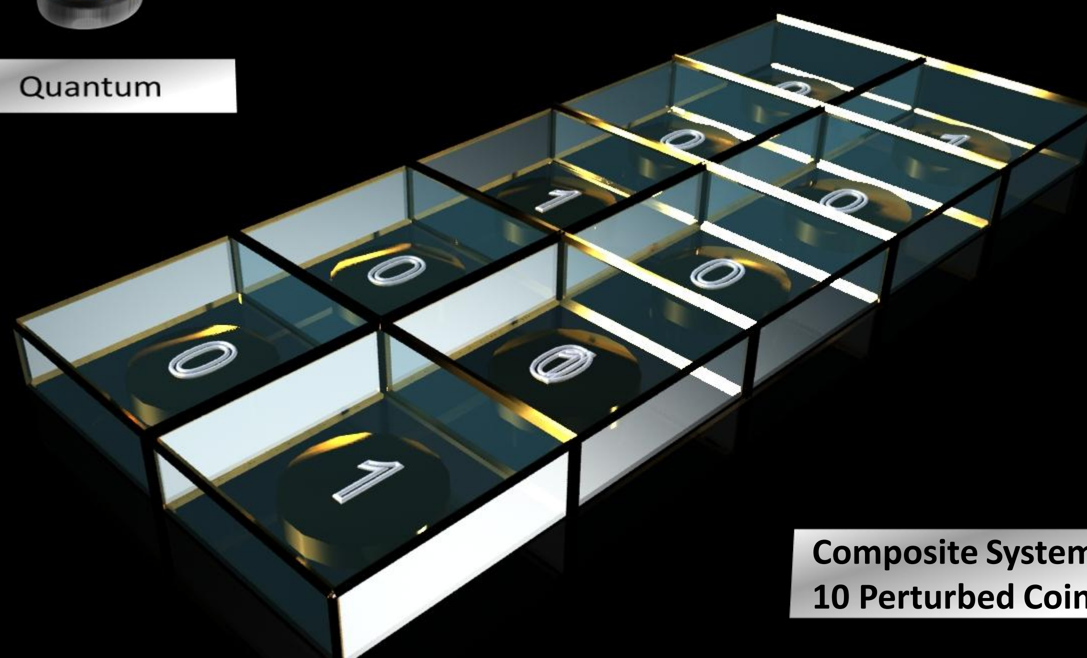


Classical

Entropy < 1

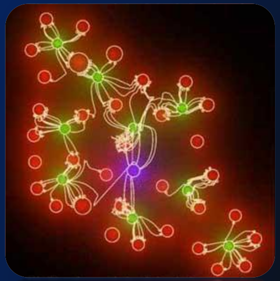


Quantum



Composite System with 10 Perturbed Coins

Outlook

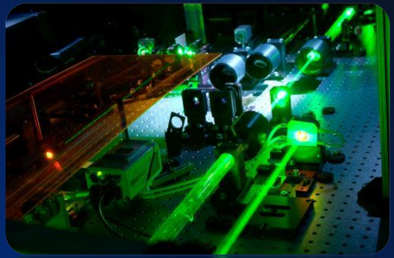


A Question of Complexity

Many organisms and devices operate based on the ability to predict and thus react to the environment around them. The fact that it is possible to make identical predictions with less memory by exploiting quantum dynamics implies that such systems need not be as complex as one originally thought.

Minimizing Irreversibility and Information Erasure

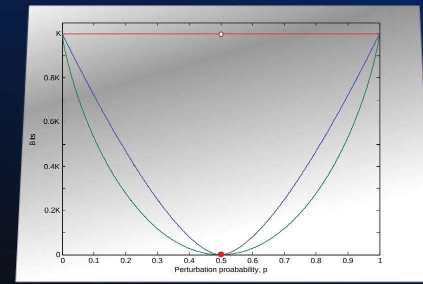
Landauer's erasure principle states that it costs energy to erase information. If we want to minimise unnecessary information erasure to generated some desired behaviour, quantum dynamics is advantageous.



Experimental Realization

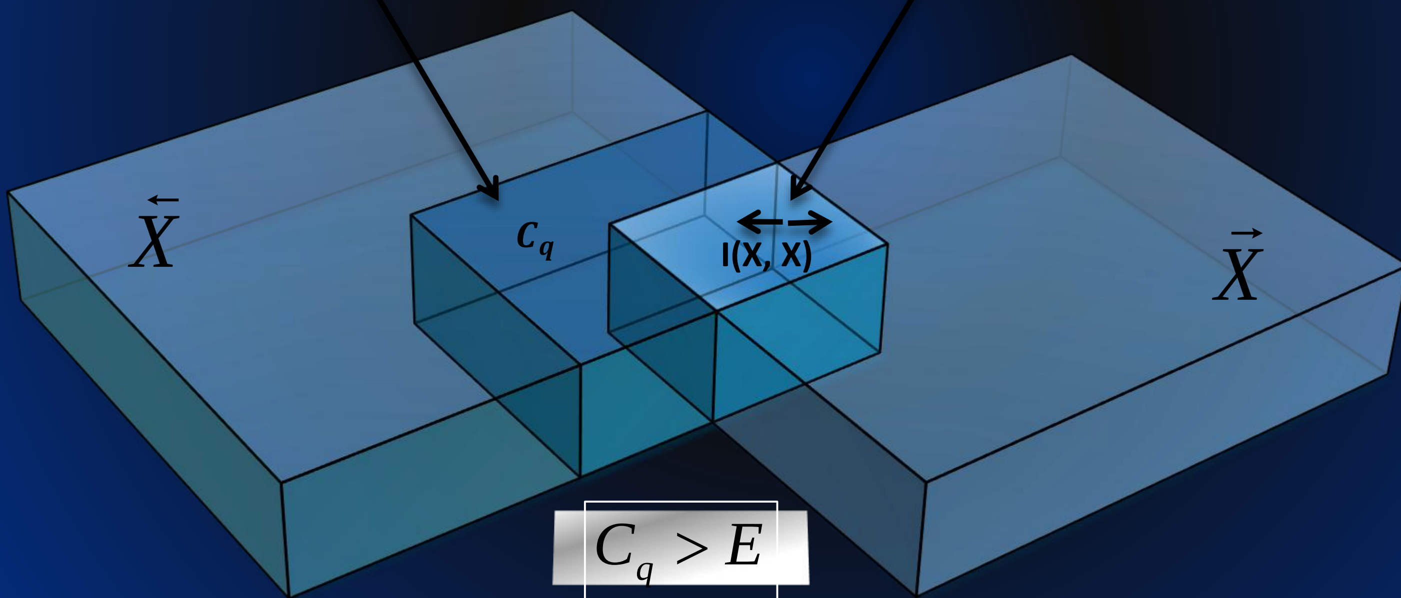
Can we realize a quantum epsilon machine in experiment? Joint work with Griffith university has began in constructing a quantum simulator for the perturbed coin.

Mysteries....



Even quantum simulators seem to store unnecessary information

The amount of useful information the past contains about the future.

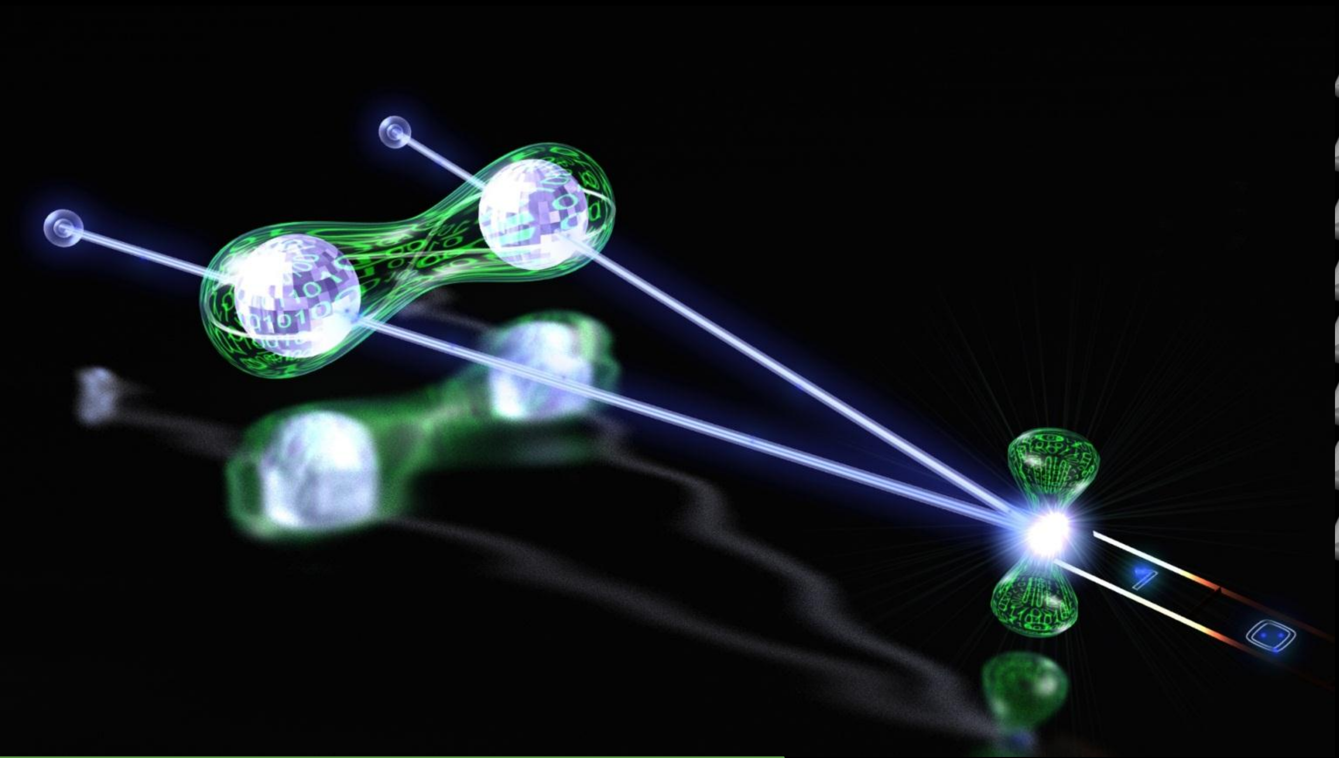


$$C_q > E$$

? Is this an emergent source is irreversibility in quantum mechanics?

? Can we formulate even more powerful theories of information that surpass quantum mechanics?

Operational Significance of Discord and its Consumption



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