Menemui Matematik (Discovering Mathematics)
Vol. 32, No. 2: 43– 50 (2010)

Constructing Multiple Development Index by Multi Criteria Methods

Nurul Husna, I.[1] and Maznah, M.K[2]

[1 & 2] *Quantitative Science Building, UUM College of Arts and Sciences,*
Universiti Utara Malaysia, UUM Sintok, 06010, Kedah, Malaysia
Tel: (O) 04-9286900/6925, (M) 012 5594849, (F) 04 928 6906

ABSTRACT

An index is simply a single number which is calculated from a set of numbers or quantities. It can be developed by various methods. This paper discusses the usage of two multi criteria methods, namely the CRITIC and TOPSIS methods to construct multiple development index for districts in Peninsular Malaysia based on State and District Data Bank of Malaysia for year 2005. The first method was utilized to determine the weights of the criteria selected, while the second method composed the values in criteria together with the weights to end up as the index value for each district selected. Due to limited data available, only three development dimensions were considered, education, health and public safety. Four indices were successfully constructed, the three basic individual dimensions and one multiple development index which is the combination of the three basic indices. Even though the resulted indices cannot represent the exact level of developments, the indices can give us some estimated evaluation as a whole. The paper highlighted the top five and the bottom five districts for every dimension, and also from the multiple development perspective.

Keywords: **development, districts, CRITIC, index, TOPSIS**

INTRODUCTION

The United Nation Development Program (UNDP) has been constructing the Human Development Index (HDI) annually for all countries in the world since 1990 (UNDP 1990). Even though the index has been criticized even since, it has given the world some kind of indicator related to the development of human in general. This paper attempts to create alike index for districts in Peninsular Malaysia by utilizing two multi criteria methods. A technique that was used for weight assignment to the criteria is the Criteria Importance Through Inter-criteria Correlation method (CRITIC) (Diakoulaki *et al.* 1995), while the indices were constructed by the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method (Hwang & Yoon 1981).

The construction of development indices for districts in Peninsular Malaysia is a measurement tool to evaluate the level of development in every district in Peninsular Malaysia (Abdul Aziz Jemain 2005). Based on the availability of the data, three major dimensions of development are chosen namely education, public safety and health and the criteria for each dimension are as what available from the Malaysia State/District Data Bank (Jabatan Perangkaan Malaysia 2008) for year 2005. Once the index for each dimension was constructed, the multiple development index is ready to be build by composing the three basic indices. Although the selected criteria are not exactly representing the development dimension, but the criteria can be considered as the estimated measures to the real ones.

In order to develop the indices, there are three steps that one has to follow. The first step is assigning weight to each of the criteria, followed by normalizing and aggregating the data. As a result, each district would have its own aggregated value or the index for each dimension considered. Based on the resulted values, the ranking of the 83 districts could be determined, and this paper discusses the top five and bottom five districts for each dimension. The whole result for the multiple index is in the appendix.

THE CONSTRUCTION OF MULTIPLE DEVELOPMENT INDEX

The construction of index involves three basic steps. Let assume $d_1, ..., d_m$ represent the m districts in Peninsular Malaysia. While $c_1, ..., c_n$ is the n criteria for the pth development dimension where $p = 1, ..., l$. The ith district development for dimension p based on criteria j, c_j^p, is marked as x_{ij}^p, while w_j^p is the weight for the criteria where $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$ Table 1 gives the representation of information about the districts according to the dimensions of the development.

Table 1: Data Representation for Dimension *p*

Districts	The criteria for dimension p					
	c_1^p	c_2^p	....	c_j^p	....	c_n^p
d_1	x_{11}^p	x_{i2}^p	....	x_{1j}^p	....	x_{1n}^p
:	:	:	....	:	....	:
d_i	x_{i1}^p	x_{i2}^p	....	x_{ij}^p	....	x_{in}^p
:	:	:	:	:	:	:
d_m	x_{m1}^p	x_{m1}^p	....	x_{mj}^p	....	x_{mn}^p
Weight	w_1^p	w_1^p	....	w_j^p	....	w_n^p

Criteria Weight: The CRITIC Method

Based on the data from Malaysia Data Bank for year 2005, there are eight criteria under the education dimension, three criteria under health dimension and seven criteria under public safety dimension. In order to find the criteria weight or the relative importance of the criteria (Choo *et al*. 1999) using CRITIC method, firstly the linear correlation coefficient, r_{jk} between the jth criterion and the kth criterion where $j \neq k$ is computed. The value C_j with respect to the decision situation defined by the rest of criteria can be formulated as

$$C_j = \sum_{j=1}^{n} (1 - r_{jk}) \quad [1]$$

Then, according to the method, the amount of T_j, emitted by the jth criterion can be determined by composing the measures which quantify the two notions through the following multiplicative aggregation formula,

$$T_j = \sigma_J C_j \quad [2]$$

where σ_j is the standard deviation for the jth criterion. The higher the T_j value, the larger the amount of information transmitted by the corresponding criterion and the higher its relative importance for the decision making process. By normalizing the values to unity, the objective weight for criteria j is

$$w_j = \frac{T_j}{\sum_{j=1}^{n} T_j} \quad [3]$$

Data Normalization

The districts development data which were collected from the Malaysia Data Bank 2005 is the raw data in different units and directions. The data can be categorized into two groups: 1) cost or loss data and 2) benefit or profit data. For example in this paper, the lower the road crash is better while the higher the number of government assisted school is preferable. Thus to overcome this, the districts development data for every criterion have to be standardized. The cost and benefit data will be standardized according to the following formula. For cost data,

$$z_{ij} = 1 - \frac{x_{ij}}{\hat{x}_j} \quad [4]$$

and for profit data,

$$z_{ij} = \frac{x_{ij}}{\hat{x}_j} \quad [5]$$

where is $\hat{x}_j$ the maximum value for x_{ij} which represents the districts performance i, $i = 1, 2, ..., m$ for criteria j, $j = 1, 2, ..., n$.

Aggregation Phase: The TOPSIS Method

After the criteria under each development dimension have their own weights, and the data has been normalized, the next task is to aggregate these two quantities. It would be carried out using the TOPSIS method which was developed by Hwang and Yoon (1981) The basic principle of this method is based on the concept that the chosen district should have the shortest distance from the positive ideal solution, and the farthest from the negative ideal solution (Maznah M.K 2008, Triantaphyllou 2000). The ideal solution is a hypothetical solution for which all criteria values correspond to the maximum criteria values in the database comprising the satisfying solutions. It is vise versa for the negative ideal solution. Thus, TOPSIS gives a solution that is not only closest to the hypothetically best, it is also the farthest from the hypothetically worst. The main procedure of the TOPSIS method for the selection of the best district from all districts is described as follows.

Step 1: Obtain the normalized matrix, v_{ij}. This is done by the multiplication of each normalized element of the jth column with its weight w_j (from the CRITIC method). Hence the elements of the weighted normalized matrix Z_{ij} are expressed as:

$$v_{ij} = w_j Z_{ij} \quad [6]$$

Step 2: Obtain the positive ideal (best) solution and negative ideal (worst) solution. It can be expressed as

i. Positive Ideal Solution and
ii. Negative Ideal Solution as follows.

$A^* = \{\nu_1^*, ..., \nu_m^*\}$, where $\nu_j^* = \{\max(\nu_{ij})\ if\ j \in J, \min(\nu_{ij})\ if\ j \in J'\ for\ i = 1, ..., n\}$

$A^\circ = \{\upsilon_1^o, ..., \upsilon_m^o\}$, where $\nu_j^o = \{\max(\nu_{ij})\ if\ j \in J', \min(\nu_{ij})\ if\ j \in J\ for\ i = 1, ..., n\}$ [8]

where J is the set of benefit criteria, and J' is the set of the loss criteria.

Step 3: Obtain the separation measures. The separation of each district from the ideal one is given by the Euclidean distance in the following equations.

i. The separation from the positive ideal district is

$$S_i^* = \left[\sum_{j=1}^{n} (\upsilon_j^* - \upsilon_{ijk})^2\right]^{1/2} \qquad [9]$$

ii. The separation from the negative ideal district is

$$S_i^o = \left[\sum_{j=1}^{n} (\upsilon_j^o - \upsilon_{ijk})^2\right]^{1/2} \qquad [10]$$

Step 4: The relative closeness of a particular district to the ideal solution, D_i^*, can be expressed as

$$D_i^* = S_i^* / (S_i^* + S_i^o) \qquad [11]$$

According to the value of D_i^*, a set of scores or index is generated indicating the most preferred and least preferred feasible solutions.

The Data

From Malaysia Data Bank 2005, there are three main dimensions of development which are education, health and safety. The evaluation criteria under each dimension are presented in Table 2.

Table 2: The criteria

Criteria	Description
Education: E1	Transition rate of pupil enrolment from primary to secondary school
Education: E2	Transition rate of pupil enrolment from lower to upper school
Education: E3	Number of pupils per teacher for primary school
Education: E4	Number of pupils per teacher for secondary school
Education: E5	Number of government assisted primary school
Education: E6	Number of government assisted secondary school
Education: E7	Number of pupils in government assisted primary school
Education: E8	Number of pupils in government assisted secondary school
Health: H1	Number of government and private hospitals
Health: H1	Number of beds at government and private hospitals
Health: H1	Number of new planning acceptors
Public Safety: PS1	Number of juvenile offenders
Public Safety: PS2	Number of road crash
Public Safety: PS3	Number of deaths
Public Safety: PS4	Number of serious injuries
Public Safety: PS5	Number of minor injuries
Public Safety: PS6	Number of breakouts
Public Safety: PS7	Expected loss (RM)

Based on Table 2, there are 8 criteria under education, 3 criteria under health dimension, and 7 criteria for public safety for each of the 83 districts in Peninsular Malaysia.

THE RESULTS

Criteria Weights

Table 3 summarizes the standard deviations for the criteria under each dimension and the resulted weights are presented in Table 4.

Table 3: Standard deviation

Education							
E1	E2	E3	E4	E5	E6	E7	E8
23.447	19.262	5.965	5.551	38.641	16.056	31025.012	20394.584

Health			Public Safety						
H1	H2	H3	PS1	PS2	PS3	PS4	PS5	PS6	PS7
6.423	796.402	593.241	73.748	7436.233	62.852	82.464	351.336	374.267	17.478

Table 4: Criteria weights for education, health and public safety criteria

Education							
E1	E2	E3	E4	E5	E6	E7	E8
0.001	0.001	0.000	0.000	0.001	0.000	0.608	0.389

Health			Public Safety						
H1	H2	H3	PS1	PS2	PS3	PS4	PS5	PS6	PS7
0.004	0.439	0.557	0.10	0.880	0.006	0.012	0.056	0.034	0.003

Referring to Table 3 and Table 4, the criteria weights are highly depend on the standard deviation values. For education dimension for example, E7 has the highest weight, while H3 and PS2 are criteria with the highest weight for health and public safety dimension respectively.

The Top and Bottom Five Districts

After the index values were computed using the TOPSIS method, all the selected districts are ranked according to the values. Those districts with higher values were ranked at higher positions as compared to districts with lower values. Table 5 and 6 portray the top five and the bottom five districts for each development dimension considered.

Table 5: The top five districts for 3 basic development dimensions

Rank	Education		Health		Public Safety	
No	District	Score	District	Score	District	Score
1	K.L	0.971	J.Bahru	0.789	Bera	0.988
2	J.Bahru	0.926	K.L	0.694	Jeli	0.994
3	Petaling	0.853	Kinta	0.577	Jelebu	0.994
4	Kinta	0.586	Kelang	0.521	B.Baharu	0.994
5	H.Langat	0.540	K.Setar	0.511	Pendang	0.994

Table 6: The bottom five districts for 3 basic development dimensions

Rank	Education		Health		Public Safety	
No	District	Score	District	Score	District	Score
1	G.Musang	0.000	Rembau	0.004	K.L	0.051
2	C.Highlands	0.007	C.Highlands	0.020	Petaling	0.098
3	Jelebu	0.018	K.Kangsar	0.029	J. Bahru	0.444
4	B.Baharu	0.023	Mid. Perak	0.030	H. Langat	0.678
5	Jeli	0.027	B.Baharu	0.037	Kelang	0.682

Based on the Table 5 and 6, Kuala Lumpur is at the top ranking with respect to education, second in health development and the lowest ranking in public safety dimension. Johor Bahru followed about the same pattern, second in education, first in health but third lowest in public safety dimension. Even though Kelang is ranked at fourth position in health dimension, it is at the fifth position from the bottom in public safety dimension. District of H. Langat is at the fifth position in education, but it is positioned at the fourth position in public safety dimension. It can concluded that those districts which are highly developed in education are at the lower position in development of public safety.

The Multiple Development Index

After the development indices with respect to individual dimensions were constructed, the multiple development index was generated by total sum of all the relative closeness, D_i^* of the three development dimension for year 2005. If the score is higher, this shows that a better development as a whole takes place in that particular districts. Table 7 shows the top and bottom five districts for year 2005 with regards to multiple development. Based on the table, J. Bharu is ranked first, followed by Kinta, K. Bahru, K. Setar and Kelang. The lowest five districts out of 83 districts considered are Rembau, C. Highlands, G. Musang, B. Baharu and Jeli.

Table 7: The top and bottom five districts from multiple development perspective

Rank	Highest		Lowest	
No.	District	Score	District	Score
1	J. Bahru	0.7198	Rembau	0.3353
2	Kinta	0.6210	C. Highlands	0.3403
3	K. Bharu	0.5913	G. Musang	0.3438
4	K. Setar	0.5789	B. Baharu	0.3511
5	Kelang	0.5722	Jeli	0.3534

DISCUSSION AND CONCLUSION

This paper discusses the construction of multiple development index for 83 districts in Peninsular Malaysia. The CRITIC method is used objectively as the method in determining the criteria weights. This method is preferable to use by those decision makers who wants to free themselves in making subjective judgment about relative importance of the criteria. Nevertheless, this method depends heavily on the standard deviations of the criteria, and this brought more weights to criteria with higher values of this measurement.

Due to limited data, there were only three development dimensions considered namely education, health and public safety. Each dimension came with debatable criteria. After each individual index was constructed, the multiple development index was ready to be developed by composing these three basic indices, and the TOPSIS method was used as aggregation method. The districts that are ranked at higher positions are better developed compared to the ones at the lower rank. The developed districts are nearer to the ideal values and farther from the non-ideal values. For year 2005, Johor Bahru is at the first ranking while Rembau is the last position among the 83 districts in Peninsular Malaysia with respect to multiple development perspective.

REFERENCES

Abdul Aziz Jemain. (2005) Indeks kemajuan antara daerah di Semenanjung Malaysia (Development Index among Districts in Peninsular Malaysia). Sains Malaysiana, 34(1), pp. 125-132.

Choo E.U., Schoner B., Wedley W.C. (1999). Interpretation of criteria weights in multicriteria decision making. *Computer and Industrial Engineering*. 37, pp.527 -541.

Diakoulaki, D., Mavrotas, G., Papayannakis, L. (1995), Determining Objective Weights in Multiple Criteria Problems: The Critic Method, *Journal of Computers Operation Research*, Volume 22, 7, pp.763-770.

Hwang C.L., Yoon K. (1981) *Multiple Attribute Decision Making*. Springer-Verlag New York.

Jabatan Perangkaan Malaysia (2008). *State / District Data Bank Malaysia.* Kuala Lumpur.

Maznah, M.K. (2008). *Penentuan wajaran kriteria dalam masalah multi kriteria*. Thesis (Ph.D) UKM, Bangi.

Triantaphyllou E. (2000). *Multi-criteria decision making methods: A comparative study*. Applied optimization. Vol. 44. Kluwer Academic Publication. Dordrecht.

UNDP (1990). *Human Development Report 1990*. Oxford University Press, New York.

Appendix 1: The multiple development index for districts in Peninsular Malaysia 2005

State	No	Districts	Index	State	No	Districts	Index
Johor	1	Batu Pahat	0.5123		42	C. Highlands	0.3403
	2	Johor Bahru	0.7198		43	Jerantut	0.3778
	3	Kluang	0.4426		44	Kuantan	0.4180
	4	Kota Tinggi	0.4641		45	Lipis	0.4786
	5	Mersing	0.3663		46	Maran	0.3938
	6	Muar	0.4775		47	Pekan	0.4013
	7	Pontian	0.4216		48	Raub	0.3773
	8	Segamat	0.4318		49	Rompin	0.3944
Kedah	9	Baling	0.4183		50	Temerloh	0.4425
	10	Bandar Baharu	0.3511	Perak	51	Btg Padang	0.4251
	11	Kota Setar	0.5789		52	Hilir Perak	0.4334
	12	Kuala Muda	0.5455		53	Hulu Perak	0.3694
	13	Kubang Pasu	0.4348		54	Kerian	0.3942
	14	Kulim	0.4712		55	Kinta	0.6210
	15	Langkawi	0.3714		56	Kuala Kangsar	0.3730
	16	Pdg Terap	0.3662		57	Larut & Matang	0.5150
	17	Pendang	0.3874		58	Manjung	0.4474
	18	Sik	0.3696		59	Perak Tengah	0.3967
	19	Yan	0.3609	Perlis	60	Perlis	0.4869
Kelantan	20	Bachok	0.4171	P.Pinang	61	Barat Daya	0.4122
	21	Gua Musang	0.3438		62	S. Perai Selatan	0.3742
	22	Jeli	0.3534		63	S. Perai Tengah	0.4638
	23	Kota Bahru	0.5913		64	S. Perai Utara	0.4393
	24	Kuala Krai	0.4120		65	Timur Laut	0.4683
	25	Machang	0.3858	Selangor	66	Gombak	0.5627
	26	Pasir Mas	0.4279		67	Hulu Langat	0.4636
	27	Pasir Puteh	0.3987		68	Hulu Selangor	0.4041
	28	Tanah Merah	0.3974		69	Kelang	0.5722
	29	Tumpat	0.4119		70	Kuala Langat	0.3852
Melaka	30	Alor Gajah	0.4048		71	Kuala Selangor	0.4957
	31	Jasin	0.3797		72	Petaling	0.4245
	32	Melaka Tengah	0.5136		73	Sabak Bernam	0.4009
N. Sembilan	33	Jelebu	0.3536		74	Sepang	0.4365
	34	Jempol	0.3886	Terengganu	75	Besut	0.4427
	35	Kuala Pilah	0.3693		76	Dungun	0.4283
	36	Port Dickson	0.3840		77	Hulu Terengganu	0.3928
	37	Rembau	0.3353		78	Kemaman	0.5032
	38	Seremban	0.5187		79	Kuala Terengganu	0.4407
	39	Tampin	0.3734		80	Marang	0.3755
Pahang	40	Bentong	0.3712		81	Setiu	0.3752
	41	Bera	0.3727	W. Persekutuan	82	W.P. K. Lumpur	0.5722
					83	W.P.Labuan	0.3584