

An Application of Soft Matrices in Group Decision Making Problems (Aplikasi Matrik Lembut dalam Masalah Pembuatan Keputusan secara Berkumpulan)

Samsiah Abdul Razak & Daud Mohamad

*Faculty of Computer and Mathematical Sciences, University Teknologi MARA
40450 Shah Alam, Selangor Darul Ehsan, MALAYSIA.*

[.amsha_abdrazak@yahoo.com](mailto:amsha_abdrazak@yahoo.com) , daud@tmsk.uitm.edu.my

ABSTRACT

Since the introduction of soft set by Molodstov in 1999, this concept has been applied in many areas of applications such as smoothness of function, operation research, Riemann integration, Perron integration and so on. The matrix approach has been introduced in soft set and some of its properties have been discussed. However, at present the application of this soft matrix is formulated to decision making problems with maximum of two decision makers only. In this paper we propose a group decision making method using soft matrices that generalizes to n decision maker where $n \leq 2$. This new method is an extension of Cagman and Enginoglu and utilizes soft max-min decision making method in solving problems. A numerical example of a house selection is given to show the effectiveness of the new method.

Keywords: Soft set, Soft Matrices, Soft max-min decision making

ABSTRAK

Sejak diperkenalkan set lembut oleh Molodstov pada tahun 1999, konsep ini telah dilaksanakan dalam pelbagai bidang aplikasi seperti fungsi kelancaran, kajian operasi, kamiran Riemann, kamiran Perron dan sebagainya. Pendekatan matrik telah diperkenalkan pada set lembut dan beberapa sifatnya telah dibincangkan. Namun demikian, pelaksanaan matrik lembut diformulasikan dalam menyelesaikan masalah yang melibatkan maksimum dua orang pembuat keputusan sahaja. Dalam makalah ini kami mencadangkan kaedah membuat keputusan berkumpulan dengan menggunakan perisian matrik lembut yang secara umumnya melibatkan n pembuat keputusan dimana $n \leq 2$. Kaedah baru ini merupakan lanjutan daripada Cagman dan Enginoglu dan kaedah perisian maks-min lembut ini digunakan dalam menyelesaikan masalah. Untuk menunjukkan keberkesanan kaedah baru ini, satu contoh berangka dalam pemilihan rumah diberikan.

Katakunci: Set lembut, Matrik lembut, Penyelesaian maks-min lembut

INTRODUCTION

Soft set theory was first introduced by Russian researcher Molodstov in 1999. He defined soft sets theory to deal with the complexities of modeling uncertain data, such as problems in economics, engineering, and environment that cannot be successfully solved by using classical methods, for example theory of probability, theory of fuzzy set and the interval mathematic because of various types of uncertainties present in these problems. All these theories can be considered as mathematical tools for dealing with uncertainties but all these theories have their inherent difficulties as pointed out in (Molodstov, 1999). Soft set theory was successfully applied to many different fields including

the smoothness of function, game theory, operation research, Riemann and Perron integration, probability theory and measurement theory (Molodstov, 1999). Maji and Roy (2002) applied soft set theory in decision making based on the concept of knowledge reduction in rough set theory. Later Maji *et al.* (2003) studied on this concept and introduced some basic operations of that theory like AND, OR, union and intersection of two soft sets and also defined equality of two soft set, subset and supersets, complement and null soft sets with some properties. However Yang (2008) argued that some properties defined by Maji *et al.* (2003) are incorrect by giving counterexamples. Irfan *et al.* (2009) defined some new operations of soft sets and they also commented on Maji *et al.* (2003) paper that properties on complement between two soft set is incorrect and discuss it with examples. Yang *et al.* (2009) introduced the combination between fuzzy set and soft set on the interval-valued. At present, the development of soft sets theory is progressing rapidly. Even though many approaches have been applied using soft set theory, the used this method is limited to one decision maker and generally soft set theory is combined with rough set or fuzzy soft set in solving problems. So far most of the proposed method do not cater group decision makers but only utilize single decision maker. Cagman and Enginoglu, 2010, already introduced soft matrices and effectively solve problem that contain uncertainty without rough set or fuzzy soft set. However, Cagman and Enginoglu approach only focusing on two decision makers in their application to solve decision making problems. In this paper Cagman and Enginoglu method is generalized, so it can solve decision making problems that involved more than two decision makers. Soft max-min decision making method is used to solve that problem in this case study.

SOFT SET THEORY

We give the definition of soft set according to Molodstov (1999).

Definition 2.1: Let U be an initial universe set and E be a set of all parameters. Let $P(U)$ denotes the power set of U and $A \subseteq E$. A pair (F, A) is called a soft set over U , where F is a mapping given by $F: A \rightarrow P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U . For $e \in A$, $F(e)$ may be considered as the set of e -approximate elements of the soft set (F, A) .

Soft Matrices

The following definitions are taken from Cagman and Enginoglu (2010) in introducing soft decision making method.

Definition 2.1.1: Let U be an initial universe set, $P(U)$ denotes the power set of U and E be a set of all parameters and $A \subseteq E$. A soft set (f_A, E) on the universe is defined by the set of ordered pairs, $(f_A, E) = \{(e, f_A(e)): e \in E, f_A(e) \in P(U)\}$ where $f_A: E \rightarrow P(U)$ such that $f_A(e) = \emptyset$ if $e \notin A$. Here, f_A is called an approximate function of the soft set (f_A, E) . The set $\{f_A(e)\}$ is called e -approximate value set or e -approximate set which consist of related objects of the parameter $e \in E$.

Definition 2.1.2: Let (f_A, E) be a soft set over U . Then subset of $U \times E$ is uniquely defined by $T_A: \{(u, e): e \in A, u \in f_A(e)\}$ which is called a relation form of (f_A, E) . The characteristics function of T_A is written by:

$$X_{T_A}: U \times E \rightarrow \{0, 1\}, \quad X_{T_A}(u, e) = \begin{cases} 1, & (u, e) \in T_A \\ 0, & (u, e) \notin T_A \end{cases}$$

If $U = \{u_1, u_2, \dots, u_m\}$, $E = \{e_1, e_2, \dots, e_n\}$ and $A \subseteq E$, then T_A the can be presented by a table as in the following form:

R_A	e_1	e_2	\dots	e_n
u_1	$X_{T_A}(u_1, e_1)$	$X_{T_A}(u_1, e_2)$	\dots	$X_{T_A}(u_1, e_n)$
u_2	$X_{T_A}(u_2, e_1)$	$X_{T_A}(u_2, e_2)$	\dots	$X_{T_A}(u_2, e_n)$
\vdots	\vdots	\vdots	\ddots	\vdots
u_m	$X_{T_A}(u_m, e_1)$	$X_{T_A}(u_m, e_2)$	\dots	$X_{T_A}(u_m, e_n)$

$$\text{If } a_{ij} = X_{T_A}(u_i, e_j), \text{ we can defined matrix: } [a_{ij}]_{m \times n} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

which is called an $m \times n$ soft matrix of the soft set (f_A, E) over U .

Product of Soft Matrices

Definition 2.2.1: Let $[r_{ij}], [s_{ik}] \in SM_{m \times n}$. The And – product between $[r_{ij}]$ and $[s_{ik}]$ is defined by $\wedge: SM_{m \times n} \times SM_{m \times n} \rightarrow SM_{m \times n^2}$, $[r_{ij}] \wedge [s_{ik}] = [t_{ip}]$ where $[t_{ip}] = \min\{r_{ij}, s_{ik}\}$ such that $p = (n-1) + k$

METHODOLOGY

Methods of Soft Matrix Theory by Cagman and Enginoglu (2010)

Cagman and Enginoglu (2010) constructed a soft max-min decision making method by using *And-product*. They introduced the soft max-min decision function as follows:

Definition 3.1.1: Let $[r_{ip}] \in SM_{m \times n^2}$, $I_k = \{p: \exists i, r_{ip} \neq 0, (k-1)n, p \leq kn\}$ for all $k \in I = \{1, 2, \dots, n\}$. Then max-min decision function, denoted Mm , is defined as $Mm: SM_{m \times n^2} \rightarrow SM_{m \times 1}$, $Mm[r_{ip}] = [\max_{k \in I} \{t_k\}]$ where,

$$t_k = \begin{cases} \min_{p \in I_k} \{r_{ip}\}, & \text{if } I_k \neq \emptyset, \\ 0, & \text{if } I_k = \emptyset, \end{cases}$$

The one column soft matrix $Mm[r_{ip}]$ is called max-min decision soft matrix.

Definition 3.1.2: Let $U = \{u_1, u_2, u_3, \dots, u_m\}$ be an initial universe and $Mm[r_{ip}] = [s_{i1}]$. Then a subset of U can be obtained by using $[s_{i1}]$ as the following expression, $Opt[s_{i1}](U) = \{u_i: u_i \in U, d_{i1} = 1\}$ is called an optimum set of U .

Now, soft max-min decision making (*SMmDM*) method is constructed by using definition 3.1.1 and 3.1.2. The algorithm for calculating the *SMmDM* method are summarized as follows:

- Step 1: From the given parameters, choose the feasible subsets of the set of parameters,
- Step 2: Use Matrix form to construct the soft matrix for each set of parameters,
- Step 3: Find the convenient product for the soft matrices,

Step 4: Find a max-min decision soft matrix,

Step 5: Find an optimum set of U , $opt_{Mm}(U) = \{u_1, u_2, \dots, u_n\}$.

The Generalization of Soft Matrices

Some modification to the method of Cagman and Enginoglu (2010) is made to solve decision making problems using soft matrices method that involve more than two decision makers. Hence a generalized soft matrices in decision making is proposed. We illustrate our method by solving a house selection problem involving three decision makers, d_1, d_2 and d_3 .

ASSOCIATIVE LAW FOR SOFT MATRICES

Associative law is an axiom that a state changing the group using an operator does not change the final result but the sequence of the group also does not change. So to in soft matrices by using soft max-min decision making method. Assume that in our case study, we have three decisions d_1, d_2 and d_3 . The soft max-min decision making and *And-product* are used. We have, $(d_1 \wedge d_2) \wedge d_3 = d_1 \wedge (d_2 \wedge d_3)$, and in general $(d_1 \wedge d_2) \wedge d_3 \wedge \dots \wedge d_{n-1} \wedge d_n = d_1 \wedge (d_2 \wedge d_3) \wedge \dots \wedge d_{n-1} \wedge d_n = d_1 \wedge d_2 \wedge d_3 \wedge \dots \wedge (d_{n-1} \wedge d_n)$.

By incorporating the method, the modified algorithm is given as follows:

Step 1: From the given parameters, choose the feasible subsets of the set of parameters,

$$E = \{e_1, e_2, \dots, e_n\}$$

Step 2: Use matrix form to construct the soft matrix for each set of parameters,

$$[A_{ij}]_{m \times n} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

Step 3: Find the *And-product* of the combination soft matrices, $[A_{ij}]$ and B_{ik} .

$$[A_{ij}] \wedge [B_{ik}] = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \wedge \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{bmatrix}$$

Step 4: Find the minimum of *And-product* between $[A_{ij}]$ and B_{ik} , i.e.

$$d_{ir} = \begin{bmatrix} d_{11} & d_{12} & \dots & d_{1n} \\ d_{21} & d_{22} & \dots & d_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1} & d_{m2} & \dots & d_{mn} \end{bmatrix},$$

where $d_{ir} = \min_{r=i,2,\dots,n} [A_{ij} \wedge B_{ik}]$.

Step 5: Find the *And – product* between $[d_{ir}]$ and $[C_{il}]$,

$$[d_{ir}] \wedge [C_{il}] = \begin{bmatrix} d_{11} & d_{12} & \dots & d_{1n} \\ d_{21} & d_{22} & \dots & d_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1} & d_{m2} & \dots & d_{mn} \end{bmatrix} \wedge \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \dots & c_{mn} \end{bmatrix}, \quad [t_{ip}] = \begin{bmatrix} t_{11} & t_{12} & \dots & t_{1n} \\ t_{21} & t_{22} & \dots & t_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ t_{m1} & t_{m2} & \dots & t_{mn} \end{bmatrix}$$

where $t_{ip} = \min_{p=1,2,\dots,n} [d_{ir} \wedge C_{il}]$.

Step 6: Find the max-min decision soft matrix, $Mm([A_{ij}] \wedge [B_{ik}]) \wedge [C_{il}] = [u_1 \ u_2 \ \dots \ u_n]^T$.

Step 7: Find an optimum set of $U.opt_{Mm}(U) = \{u_1, u_2, \dots, u_n\}$.

Case Study: Criteria of House Selection

Assume that Mr. Y wants to buy a house for his family, but he also needs opinion from his wife and his daughter to choose the relevant criteria before deciding to buy the house. The selection process involves three decision makers, Mr. Y, Mrs. Y and their daughter. Suppose that there are six houses to be selected.

There are three main criteria and nine sub criteria involved in the selection process. The first criteria are neighborhood with sub criteria aesthetics (e_1) and safety (e_2). Second main criteria is property with sub criteria exterior(e_3), interior(e_4), and system(e_5) and the third main criteria is community with sub criteria school(e_6), government(e_7), social(e_8) and entertainment(e_9).

Based on these criterions, Mr. Y, Mrs. Y and their daughter can choose their own criteria. In this research can solve the problem using soft max-min decision making follow by *And – product* and according to associative law. Microsoft Excel will be used for all computation involves to find the best choice of house selection.

House Selection Problem

A soft (f_A, E) describes the attractiveness of the house that Mr. Y family is going to buy. On this decision making its involved three decision makers, Mr. Y, Mrs. Y and their daughter denoted by R, S and T respectively.

Assume that U = Set of six houses under consideration, and E = Set of parameters (set of criteria in house selection). Suppose that there are six houses in the universe U given by $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ and $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9\}$.

Step 1: Mr. Y, Mrs. Y and their daughter choose their own sets of parameters. Let the choice respectively as,

$$R = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}, S = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}, T = \{e_1, e_2, e_4, e_5, e_6, e_7, e_8\}.$$

Menemui Matematik Vol. 34 (1) 2012

Step 5: Using *And-product*, the product of soft matrices between $[u_{ip}]$ and $[T_{il}]$ written row by row is given as follows:

$$R_1 = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 = R_3 = R_4 = R_5 = R_6 = \{0, 0, 0, \dots, 0\}$ (81 zeros), where r_i indicates the elements in row i .

Step 6: The max-min soft matrix is $[c_{ip}] = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

and $Mm([r_{ij}] \wedge [s_{ik}] \wedge [t_{il}]) = [1 \ 0 \ 0 \ 0 \ 0 \ 0]^T$

Step 7: The optimum set of U is $Opt_{Mm}([R_{ij}] \wedge [S_{ik}] \wedge [T_{il}]) (U) = \{u_i\}$, where u_i is an optimum set of U . Hence Mr. Y family will buy the house h_1 .

CONCLUSION

In this paper, we generalized the approach introduced by Cagman and Enginoglu (2010), which now can be applied to more than two decision makers. Since the soft matrix theory satisfy the associative law, we present an applications of soft matrix theory to group decision making problem that have more than two decision makers by using soft max-min decision making method (2010).

We provided an example that demonstrated that this generalization method. House selection problem is an example, where in this problem consider three decision makers, Mr. Y, Mrs. Y and their daughter. They can give their own opinion in selection criteria for the house selection. This new proposal also proves that it is very meaningful since in real world, many important decisions are made by an expert group, instead of only a single decision maker.

REFERENCES

- Cagman, N., & Enginoglu, S. (2010). Soft matrix theory and its decision making, *Journal of Computers and Mathematics with Applications*, 59, 3308-3314.
- Irfan, M., Feng, F., Liu, X., Min, W.K., & Shabir, M. (2009). On Some New Operations in Soft Sets Theory. *Journal of Computers and Mathematics with Applications*, 57, 1547-1553.
- Maji, P.K., & Roy, A.R. (2002). An Applications of Soft Sets in A Decision Making Problem. *Journal of Computers and Mathematics with Applications*, 44, 1077-1083.
- Maji, P.K., Biswas, R.A., & Roy, R. (2003). Soft Set Theory. *Journal of Computers and Mathematics with Applications*, 45, 555-562.
- Molodstov, D. (1999). Soft Set Theory-First Result. *Journal of Computers and Mathematics with Applications*, 37, 19-31.
- Yang, C.F. (2008). A note on "Soft Set Theory". *Journal of Computers and Mathematics with Applications*, 56, 1899-1900.
- Yang, X., Lin, T.Y., Yang, J., Li, Y., & Yu, D. (2009). Combination of Interval-valued Fuzzy Set and Soft Set. *Journal of Computers and Mathematics with Applications*, 58, 521-527.