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Geometry and Photography: A Connection

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ABSTRACT

Geometry revolves around the properties, measurement, and relationships of points, lines, angles, surfaces and solids; while photography is the art or practice of taking and processing photographs. In this research, geometry in photography refers to the geometrical elements that can be used and identified in photographic works. The main aim of the research is to refute the notion of Mathematics and art as two unrelated fields by discussing geometrical concepts present in photography, which include lines, curves, angles, shapes, the golden ratio, and transformations. Definitions and examples in the form of photographs and graphical representations were provided. In addition, the mathematical software, MATLAB was used to computationally depict the rigid motions of transformations, namely reflection, rotation, translation and resizing. Furthermore, geometrical analyses of photographs were conducted to identify the geometrical elements that exist within the photographs. In this research, it has been found that many concepts in geometry were indeed applied in photography, which subsequently showed that Mathematics and art were undoubtedly related to one another.

Keywords: geometry, golden ratio, photography, photographs, MATLAB

INTRODUCTION

Geometry is known as the branch of Mathematics that explores the properties, measurement, and relationships of points, lines, angles, surfaces and solids (Bass, 2007). On the other hand, photography is defined as the art or practice of taking and processing photographs (Soanes and Stevenson, 2010). A photograph is a picture made using a camera, in which an image is focused onto film or other light-sensitive material and then made visible and permanent by chemical treatment, or stored digitally (Soanes and Stevenson, 2010). In this research, geometry in photography is referred to the geometrical elements, such as shapes, lines, angles, proportions, and transformations that can be used and identified in photography and photographic works (photographs), respectively.

To begin with, lines and curves are essential in photography as they build outlines and boundaries, which give form to the subject and overall image. Angles significantly dictate the way we perceive a subject while shapes provide an immediate visual stimulant. Moreover, emphasis is put on a subject in a photograph when it undergoes a transformation. Using MATLAB, transformations were clearly visualised. Other than that, the photographs that were analysed in this research were categorised differently according to pre-existing types of photography, namely landscape photography, wedding photography, sports photography, street photography, and portrait photography.

In the next section, the preliminaries to this research are provided.

PRELIMINARIES

The following are some definitions and descriptions of the geometrical concepts discussed in this research.

Firstly, the golden ratio is $1:1.618$ (Hoddinott and Bauer, 2012), which is derived from the computation of ratios of lengths of a straight line. Two quantities are in the golden ratio if their ratio is the same as the ratio of their sum to the larger of the two quantities. Mathematically, the derivation of the golden ratio, ϕ is written as follows:

$$\phi = \frac{a+b}{a} = \frac{a}{b} = \frac{1+\sqrt{5}}{2} \approx \frac{1.618}{1} \text{ or } 1:1.618.$$

In photography, the golden ratio is used for composition. Here, the photographer creates a photograph that is not only balanced but also aesthetically pleasing. It is utilised by arranging the elements in the photograph according to a golden ratio grid as shown in Figure 1.

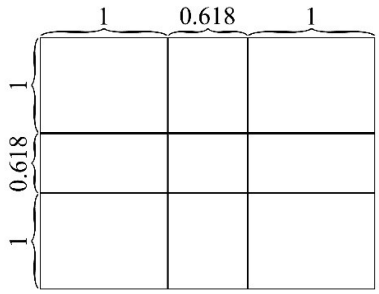


Figure 1: The golden ratio grid

The two horizontal lines that run across the frame are where the horizon should be placed in the photograph. This is to discourage placing the horizon in the centre of the photograph, which would make it seem like the horizon divides the photograph in half. The four points of intersection of the horizontal and vertical lines are where the subject should be placed, as these are the strongest focal points in a composition, which are beneficial in elevating a photograph's aesthetic value (Hoddinott and Bauer, 2012).

Next, lines are described as length without breadth; whereas curves are similar to lines in that they are infinite without width, but are not required to be straight. Angles are a result of two rays sharing an endpoint while shapes are the form of an object or its external boundary, outline, or external surface (Bass, 2007). On the other hand, the definition of transformations is given as follows.

Definition 1 (Geltner and Peterson, 1995) Transformations
A transformation T is a one-to-one correspondence between two sets of points R and S.

The concept of transformations encompasses reflection, rotation, translation, and resizing. The following are the definitions of those concepts.

Definition 2 (Geltner and Peterson, 1995) Reflection
A reflection with respect to line k is a transformation $T: R \to S$ in which every point R_1 in R corresponds to a point S_1 in S such that R_1 is symmetric to S_1 with respect to k.

Definition 3 (Geltner and Peterson, 1995) Rotation

A rotation about the origin is a transformation $T: R \to S$ in which every point in R with coordinates (x, y) corresponds to a point in S with coordinates $(ax - by, ay + bx)$ and $a^2 + b^2 = 1$.

Definition 4 (Geltner and Peterson, 1995) Translation
A translation is a transformation $T: R \to S$ in which every point in R with coordinates (x, y) corresponds to a point in S with coordinates $(x + a, y + b)$.

When an object undergoes resizing, the resulting image is either enlarged or reduced.

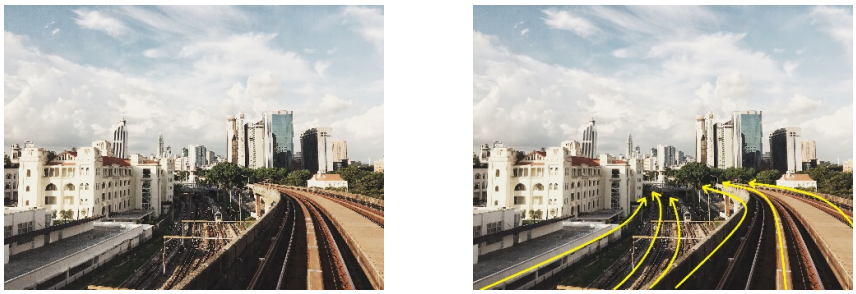
Besides that, there are also some techniques in photography that are discussed in this research, for instance, the technique of composition. Composition is about deciding on what to include and exclude from the photograph, how particular subjects are placed within the photograph, and how the photographer wants to draw in the viewer. Photographs should be strategically composed to ensure balance between elements in the frame (Hoddinott and Bauer, 2012).

Furthermore, the technique of creating frames within a frame is the practice of placing an object in between other objects to create a natural frame within the frame of the photograph and is used to keep the attention focused inward to the principal part of the picture or the main subject (Hoddinott and Bauer, 2012).

In the next section, some concepts in geometry that have been used and identified in photography, which are lines, curves, angles and shapes, are discussed.

SOME CONCEPTS IN GEOMETRY

Lines and curves can be used as lead-in lines, where the viewer's eye is lead through the frame using lines to emphasise the main subject of the photograph (Hoddinott and Bauer, 2012). This is shown in Figure 2, where the lines of the railroad lead into the frame, which draws the viewer's attention towards the cityscape. Here, Figure 2(a) shows a cityscape photograph; whereas Figure 2(b) is the same photograph overlaid with arrows to show the lead-in lines.



(a) (b)

Figure 2: A cityscape photograph a) Original photograph b) Original photograph overlaid with arrows

Besides that, angles also play an important role in photography, where different camera angles depict the subject in different ways. Figure 3 shows different angle shots, where in Figure 3(a), a high-angle shot depicts the subject as weak or vulnerable. When capturing a photograph from a high angle and a far distance, a bird's-eye view is obtained as seen in the photograph by Steve McCurry (McCurry, 2015) in Figure 3(b). However, a low-angle shot may give the subject a sense of dominance or threat, such as the photograph by Marvel (Marvel, 2014) in Figure 3(c) while in Figure 3(d), a worm's-eye view shot is obtained when it is taken from ground-level.

(a) (b) (c) (d)

Figure 3: Different angle shots a) A high-angle shot b) A bird's-eye view shot c) A low-angle shot d) A worm's-eye view shot

Lastly, an eye-level shot is a shot taken at the same level as the subject. This creates a neutral view of the subject, where the angle does not build any sense of superiority or inferiority.

Additionally, shapes are used in photography in two ways, which are composing with shapes and creating frames within a frame using shapes. Composing with shapes is the technique of arranging multiple subjects in a photograph so that they generate shapes, whether they act as vertices or edges. This is so that the photograph achieves a sense of balance, as shown in Figure 4, which is a photograph of athletes composed in a triangle. Figure 4(a) is the original photograph by Bill Frakes (Sports Illustrated, 2012) while Figure 4(b) is the original photograph overlaid with a triangle to show the triangular composition.

(a) (b)

Figure 4: A photograph of athletes a) Original photograph b) Original photograph overlaid with a triangle

Finally, by placing the subject in an implied shape (the frame), the viewer's attention is automatically fixated on the subject because a frame within the frame is created. Not only that, this technique also helps define the third-dimensionality of the image. Figure 5 demonstrates this technique, where Figure 5(a) is a photograph by Steve McCurry (McCurry, 2015) of a woman being framed by a tunnel. Figure 5(b) is the same photograph that is overlaid with rectangles to show the frames within the frame.

(a) (b)

Figure 5: A photograph of a woman a) Original photograph b) Original photograph overlaid with rectangles

In the next section, the concept of transformations is discussed.

TRANSFORMATIONS

Transformations are used in photography to give the photograph more impact and also to highlight the subject of interest. One type of transformation is reflection, where corresponding points on different sides of a line of reflection are equidistant from one another. An example of the utilisation of reflection in photography is shown in a sunset photograph by Søren (Søren, 2015) in Figure 6.

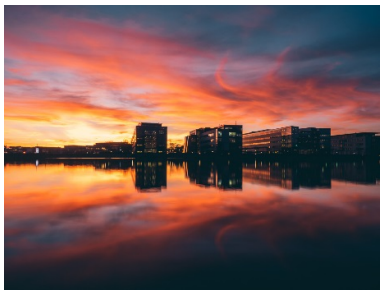


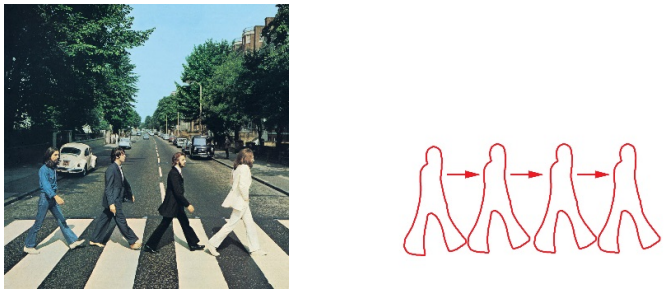
Figure 6: A sunset and its reflection

A rotation is when an object's image is moved with respect to a point of rotation. Figure 7(a) shows a photograph by Søren (Søren, 2015) that depicts rotation along with its graphical representation in Figure 7(b).

(a) (b)

Figure 7: Multiple subjects depict rotation a) Original photograph b) Graphical representation of rotation

Next, the concept of translation, in which a figure is moved onto another by sliding it without reflecting or rotating it, is displayed in Figure 8. Figure 8(a) is the iconic photograph of The Beatles crossing Abbey Road by Iain Macmillan (Macmillan, 1969) depicting translation. Figure 8(b) is the graphical representation of translation of the photograph.



(a) (b)

Figure 8: The Beatles crossing Abbey Road a) Original photograph b) Graphical representation of translation

Lastly, resizing is when a similar image is produced but is either larger or smaller than the original object. Figure 9 is an example of resizing in photography, where it shows a man running in a photograph by Steve McCurry (McCurry, 2015). The shadow of the running man acts as the enlarged image.

Figure 9: A running man and his shadow

In this research, MATLAB was used to computationally visualise the concepts of reflection, rotation, translation, and resizing by using the following code:

```
I = imread('favicon.jpg');
M = flipdim(I,1)
J = imrotate(I, -90);
T = [1 0 0; 0 1 0; 125 125 1];
tform = maketform('affine', T);
[N,xdata,ydata] = imtransform(I, tform);
L1 = imresize(I, 0.5)
L2 = imresize(I, 2)

subplot (2,3,1), imshow(I), axis on, axis ([0 1300 0 1300]),
title('original image, I')
subplot (2,3,2), imshow(M), axis on, axis ([0 1300 0 1300]),
title('reflected image, M')
subplot (2,3,3), imshow(J), axis on, axis ([0 1300 0 1300]),
title('rotated image, J')
```

```
subplot (2,3,4), imshow(N,'XData',xdata,'YData',ydata), axis on, axis
([0 1300 0 1300]), title('translated image, N')
subplot (2,3,5), imshow(L1), axis on, axis ([0 1300 0 1300]),
title('minimised image, L1')
subplot (2,3,6), imshow(L2), axis on, axis ([0 1300 0 1300]),
title('maximised image, L2')
```

Figure 10 shows the output of the code, which displays the original image *I*, the reflected image *M*, the rotated image *J*, the translated image *N*, the minimised image *L*1, and the maximised image *L*2.

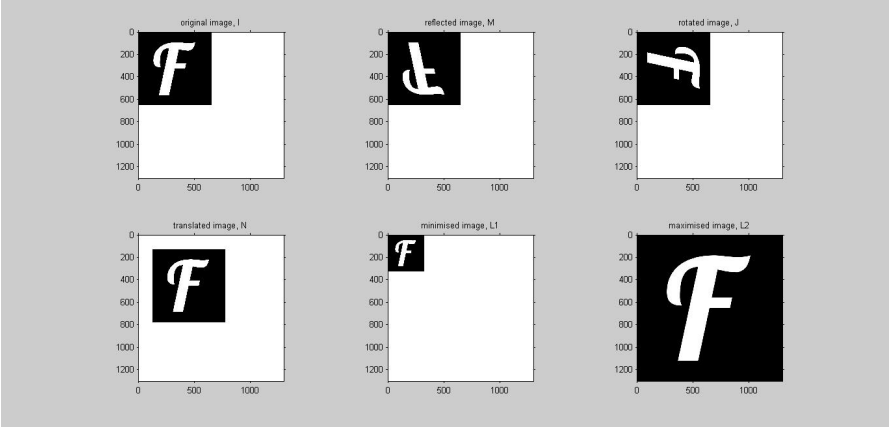


Figure 10: Output of the code in MATLAB

Geometrical analyses of some photographs from different types of photography are discussed in the next section.

GEOMETRICAL ANALYSES OF PHOTOGRAPHS

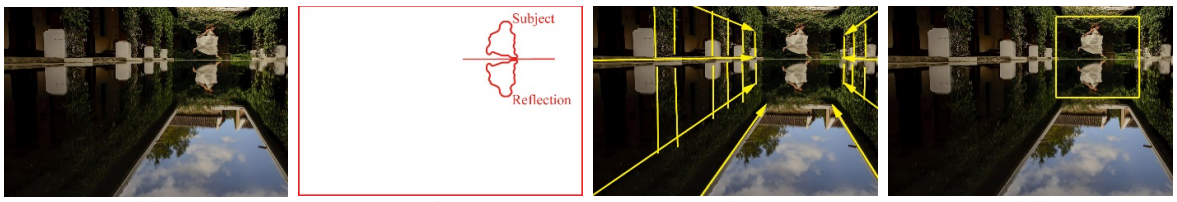
In this research, geometrical analyses of some photographs were done to extract, describe and explain the geometrical elements that exist or that are utilised in the photographs. The analyses were done according to the types of photography that include landscape photography, wedding photography, sports photography, street photography, and portrait photography.

In landscape photography, a photograph of a mountain range in Figure 11(a) by Steve McCurry (McCurry, 2015) was found to be composed according to the golden ratio as shown in Figure 11(b). Other than that, the concept of shapes was also visible in Figure 11(c). The recurring triangles generated a lead-in line as shown by the arrow in Figure 11(d).

(a) (b) (c) (d)

Figure 11: A landscape photograph a) Original photograph b) Original photograph overlaid with the golden ratio grid c) Original photograph overlaid with triangles d) Original photograph overlaid with an arrow

Meanwhile, in wedding photography, a photograph of a bride by Daniel Diaz (Diaz, 2015) also contained various geometrical elements, which is shown in Figure 12. Here, Figure 12(a) is the original photograph and Figure 12(b) is the graphical representation of reflection in the photograph. In Figure 12(c), the original photograph was overlaid with arrows to show the lead-in lines while in Figure 12(d), it was overlaid with a rectangle to show the frame within a frame.

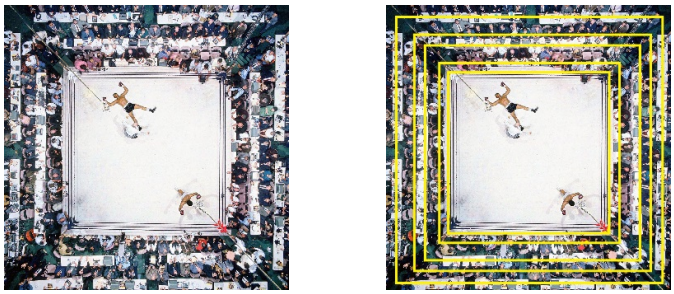


(a) (b) (c) (d)

Figure 12: A wedding photograph a) Original photograph b) Graphical representation of reflection c) Original photograph overlaid with arrows d) Original photograph overlaid with a rectangle

Note that the photograph seems off balance due to the horizon being too close to the top of the photograph. To retain balance, the photograph should be composed according to the golden ratio, where the horizon is placed on one of two horizontal lines of the golden ratio grid.

Aside from that, the concepts of a bird's-eye view shot and creating frames within a frame using shapes could also be seen in a sports photograph by Neil Leifer (Sports Illustrated, 2012), as shown in Figure 13. Figure 13(a) displays the original photograph, which was shot at a bird's-eye view. Figure 13(b) displays the original photograph overlaid with squares to show the frames within a frame.



(a) (b)

Figure 13: A sports photograph a) Original photograph b) Original photograph overlaid with squares

Figure 14 shows an example of a street photograph that also displays geometrical elements. The photo of the Gypsy men in Figure 14(a), taken by Josef Koudelka (Koudelka, 1975), displayed the techniques of creating frames within a frame using shapes and composing using shapes, shown by an overlaid rectangle and triangle in Figure 14(b) and Figure 14(c), respectively. The golden ratio was also identified, which is shown by an overlaid golden ratio grid in Figure 14(d).

(a) (b) (c) (d)

Figure 14: A street photograph a) Original photograph b) Original photograph overlaid with a rectangle c) Original photograph overlaid with a triangle d) Original photograph overlaid with the golden ratio grid

Lastly, the following portrait photograph by Steve McCurry (McCurry, 2015), where Figure 15(a) is the original photograph, showcases a frame within the frame and lead-in lines, which is observed in Figure 15(b) by the overlaid semicircle and arrows, respectively. The photograph also has a composition that adheres to the golden ratio, shown in Figure 15(c).

(a) (b) (c)

Figure 15: A portrait photograph a) Original photograph b) Original photograph overlaid with a semicircle and arrows c) Original photograph overlaid with the golden ratio grid

CONCLUSION

In conclusion, this research has shown that there are underlying geometrical concepts that are applied in photography. First of all, the golden ratio is utilised for composition; whereas lines and curves are used as lead-in lines. Different angles of the camera depict the subject in different ways and shapes can be used for composition or to generate frames within a frame. Transformations create a more impactful photograph, where they were envisaged through the use of MATLAB. All of these geometrical concepts were present in photographs as shown in the geometrical analyses of photographs. In the future, researchers may want to elaborate more on the utilisation of lines and curves, discuss the consequences of different angles of light source, find more photographic examples of the utilisation of other shapes and provide some examples where multiplication of transformations is involved. Future researchers may also deploy more computational analyses and representations of the geometrical concepts through other Mathematical software besides MATLAB.

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