‘Causality’ in systems with collective behaviour

Fatimah Abdul Razak[1,2] Henrik Jeldtoft Jensen[2]

[1]School of Mathematical Sciences
Faculty of Science & Technology
Universiti Kebangsaan Malaysia (National University of Malaysia)
fatima84@ukm.my

[2]Complexity and Networks Group
& Department of Mathematics
Imperial College London

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Complex systems and 'causality'

More often than not when dealing with complex systems one is more concerned with the non directional correlation.

However, rampant interactions and communications in complex system inevitably leads to some form of directionality. Related terms include 'causality', direction, information transfer and dependence over time.

The usage of 'causality' measures on complex systems is already on the rise especially in the advent of 'big' data. Unfortunately these measures are NOT completely understood, especially in relation to complex systems.

Most of basic testings of these 'causality' measures are done on dynamical systems or simple time series. Schreiber (2000); Lungarella et al. (2007); Pompe and Runge (2011).

Outline

> *"If we can measure degrees of causality.....We can then observe how much a change in one aspect of the universe will bring out changes in others. "*
>
> *".. I was forced to consider the theory of information, and above all, that partial information which our knowledge of one part of the system gives us the rest of it."*
>
> \- Norbert Wiener, *I Am A Mathematician* (1956)

Prediction based outlook of ‘causality’ in the Wiener-Granger framework: Variable A ‘causes’ variable B if the ability to predict B is improved by incorporating information about A in the prediction of B.

The most common is **Granger causality** (G-causality)

- was introduced by Granger (1969) in the context of autoregressive (AR) models thus essentially linear.
- depends on how good the model fits to data.

Granger has always been clear that G-causality is not absolute causality and admitted that the optimal predictor may very well be nonlinear. The nonlinear **Transfer Entropy** was first introduced by Schreiber (2000) as a measure to determine directionality using the Markov property. It is

- conceptually as an extension of G-causality.
- based on information theoretic (Shannon) entropy.
- a nonlinear measure that is theoretically model agnostic.

Define random variables X and Y with discrete probability distributions $p_X(x), x \in \mathcal{X}$ and $p_Y(y), y \in \mathcal{Y}$. Let X^τ be the variable X that is shifted by τ, so that the values of $X^\tau(t) = X(n-\tau)$ where $X(n)$ is the value of X at time step n and similarly for Y.

The **lag specific Transfer Entropy** of Y to X at causal lag τ, $T_{YX}^{(\tau)}$ is

$$T_{YX}^{(\tau)} = \sum_{x \in \mathcal{X}} \sum_{x' \in \mathcal{X}} \sum_{y \in \mathcal{Y}} p_{XX^1Y^\tau}(x, x', y) \log \frac{p_{X|X^1Y^\tau}(x|x', y)}{p_{X|X^1}(x|x')} \qquad (1)$$

where $0 \, log \, 0 = 0$. $T_{YX}^{(\tau)}$ measures whether the state of $Y(n-\tau)$ influences the current changes in X. This coincides with the view that 'causality' is a situation where the state of one variable (the source) influences the changes in another variable (the target) in the future.

What does these measures quantify on complex systems?

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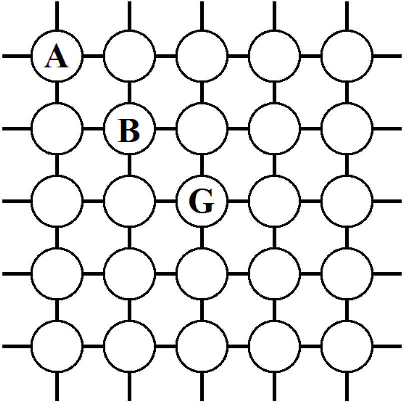
One of the defining features: collective behaviour.

- Complex system whose main characteristic consist in essential collective behaviour takes into account instances when the whole system is interdependent.
- Perhaps the simplest way to investigate this behaviour is by testing on the Ising model with its critical temperature long range interactions (diverging correlation lengths).
- The lag specific Transfer Entropy is tested on the simple form of collective behaviour near criticality of the Ising model.

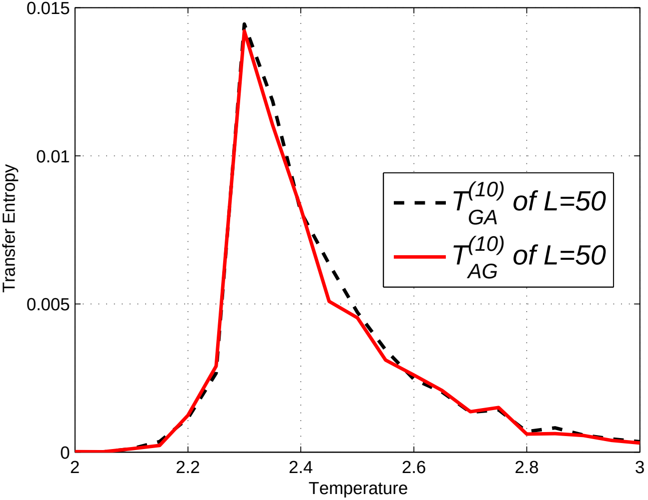
The Ising Model
$L = 5, N = 25$



- The Metropolis Monte Carlo (MMC) algorithm is used to simulate the 2D Ising model with periodic boundary conditions. The algorithm proposed by Metropolis and co-workers in 1953 was designed to sample the Boltzmann distribution γ_B by artificially imposing dynamics on the simulation.
- The interaction strength is set to be $J = 1$ and the Boltzmann constant is fixed as $K_B = 1$ for all the simulations. We let the system run up to 2000 samples before sampling at every $N = L^2$ time steps. As a result of the MMC algorithm, a Markov chain (process) is formed for every site on the lattice. The state of each site at each sample will be taken as a time step n in the Markov chain $(s_X)_n$. Sample sizes are all $100,000$.

Transfer Entropy on Ising model

Collective behaviour at criticality as a type of 'causality'.



Transfer Entropy on Ising model

Collective behaviour as a type of ‘causality’.

Transfer Entropy treats collective behaviour as a type of ‘causality’ in the Wiener-Granger framework.

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This must be taken into account when estimating Transfer Entropy on data sets from systems with collective behaviour.

Is collective behaviour different from individual (coupled) interactions?

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If it is, then is it possible to differentiate these individual interactions from collective behaviour?

The amendment: making A and B dependent on G

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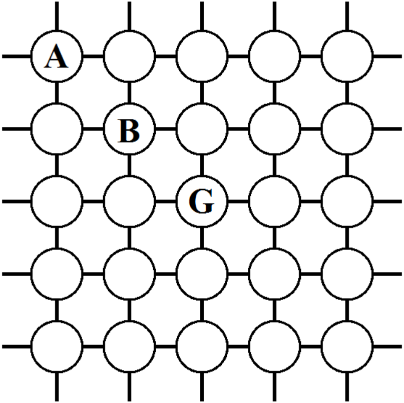


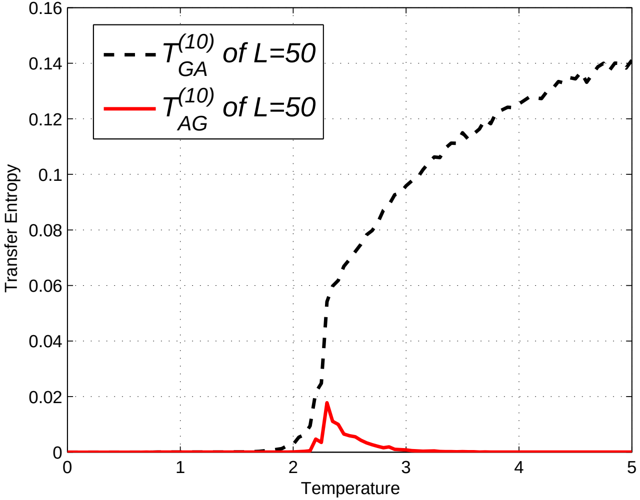
An amended Ising model

replicating 'causality'

- Generated using the standard MMC algorithm albeit special rules apply whenever site A or B is chosen for flipping consideration.
- At each step in the MMC algorithm, a site chosen at random to be considered for flipping with a certain probability γ_B except when A or B is selected where an extra condition needs to be fulfilled first before it can be allowed to change.
- If $(s_G)_{n-t_G} = 1$, A (or B) can be considered for flipping with probability γ_B as usual, however if $(s_G)_{n-t_G} = -1$, no change is allowed. Thus only one state of G ($s_G = 1$ in this case) allows sites A and B to be considered for flipping.
- Therefore, although A (and B) have their own dynamics, their changes still depend on the state of G at a certain 'causal' lag.

Transfer Entropy on amended Ising model with $t_G = 10$

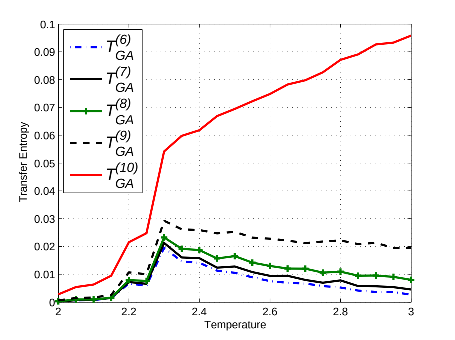
Transfer Entropy detects direction amidst collective behaviour at exact lag



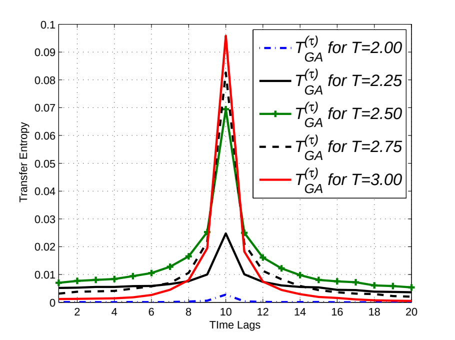
Transfer Entropy on amended Ising model with $t_G = 10$

Transfer Entropy detects the exact 'causal' lag and approaches zero at different rates depending on time distance from the exact lag. Figures clearly shows 'causal' lag detected by Transfer Entropy by clear difference in magnitude.

$T_{GA}^{(\tau)}$ versus T for different time lags τ



$T_{GA}^{(\tau)}$ versus τ for different temperatures T

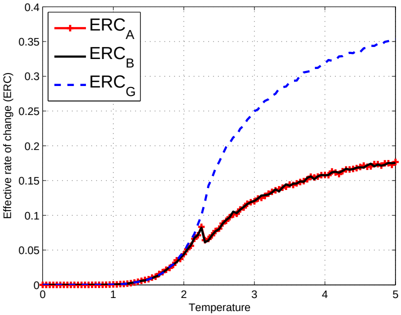


Changes on the amended Ising model with $t_G = 10$

Effect of the amendment on transition probabilities of A and B.

The Effective Rate of Change

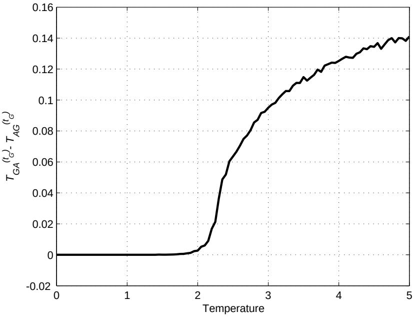
For any sites X, the effective rate of change is $ERC_X = P(X_n \neq X_{n-1})$. ERC_A and ERC_B are identical since both sites are conditioned on site G. These two variables clearly reflects how the external influence (the condition) changes the transition probability of A and B. ERC_G is identical to the ERC of all the other sites on the lattice (save A and B).



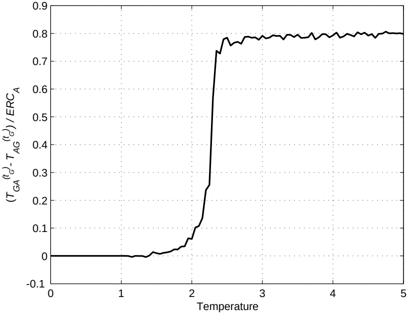
Extracting directionality: Phase transition like behaviour

Comparing both directions of Transfer Entropy values and dividing by ERC on amended Ising model with $t_G = 10$ with $L = 50$.

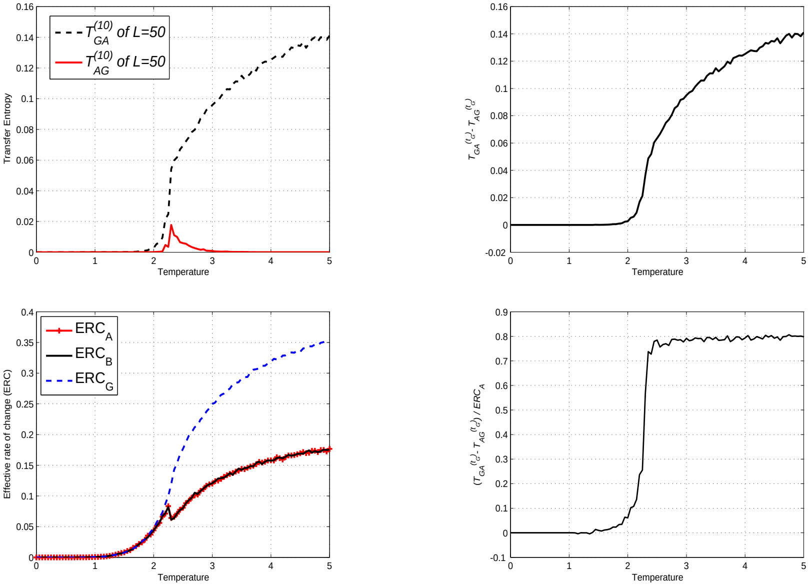
$$T_{GA}^{(t_G)} - T_{AG}^{(t_G)}$$



$$\frac{T_{GA}^{(t_G)} - T_{AG}^{(t_G)}}{ERC_A}$$



The amended Ising model with $t_G = 10$ and $L = 50$.



Discussion

- Distinguishing collective behaviour and individual influences. The effect of different influences must be identified in order to understand the data obtained from complex systems.
- Propose that definition of 'causality' in the Wiener-Granger sense includes collective behaviour.
- The ability to differentiate collective and individual (coupled) behaviour is key in understanding complex systems.
- Phase transition like behaviour of Transfer Entropy when divided with the ERC.

Take home message

What you need to remember if nothing else

- Transfer Entropy measures collective behaviour as a type of ‘causality’ in (amended) Ising model.
- Transfer Entropy manages to identify the implanted directionality (individual influence) amidst collective influence in the Ising model.
- Moreover, it also succeeds at identifying the exact causal lag.

When using Transfer Entropy and possibly any type of ‘causality’ measure in the Wiener-Granger sense on systems with collective behaviour, one needs to take into consideration the collective and individual (coupled) behaviour.

Future Research

Understanding 'causality' on complex systems

Among the issues we propose to further investigate using this novel method of the amended Ising model:

- many conflicting influences (for eg internal and external influences)
- distinguishing interactions of multiple sources and targets
- indirect 'causality': the underlying 'cause' of an apparent source-target relationship. Eg thunder and lightning.
- Phase transition like behaviour of Transfer Entropy when divided with the ERC on complex systems.

paper: Transfer Entropy in the Ising model (arxiv:1309.0305)

fatimahabdulrazak.wordpress.com

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