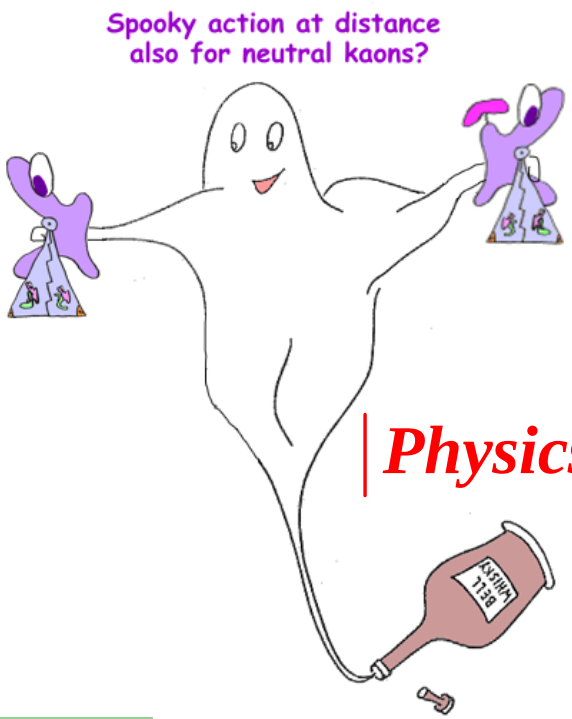
Entanglement and its Manifestations in High Energy Physics



by

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Faculty of Physics
University of Vienna
Austria

$$|Physics\rangle = \alpha\,|Particle\ Physics\rangle + \beta\,|Quantum\ Theory\rangle$$

experimental ↔ phenomenological ↔ conceptual ↔ mathematical aspects

UNIVERSITÄT WIEN

Testing qm foundations in High Energy Physics

$$\left|Physics\right\rangle = \alpha \left|Particle\ Physics\right\rangle + \beta \left|Quantum\ Theory\right\rangle$$

$$\left|\beta\right| \geq \left|\alpha\right|$$

Have done:

$$P_{QM} = \left|Quantum\ Theory\right\rangle\left\langle Quantum\ Theory\right|$$

$$P_{HEP} = \left|Particle\ Physics\right\rangle\left\langle Particle\ Physics\right|$$

Will do:

$$P_{new} = \left|New\ Physics\right\rangle\left\langle New\ Physics\right|$$

$$\left|New\ Physics\right\rangle = \frac{1}{\sqrt{2}}\left\{\left|Particle\ Physics\right\rangle + \left|Quantum\ Theory\right\rangle\right\}$$

Testing QM in High Energy Physics

- **Part I: Bell inequalities 1:**

  A symmetry violation in particle physics related to nonlocality ?!

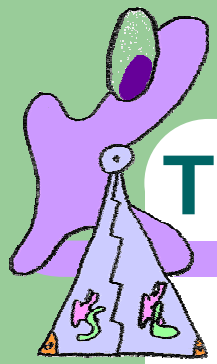
- **Part II: The quantum mechanics of neutral kaons (*Exercises)***

- **Part III: Bell inequalities 2/ How to describe the decay property?**

  "Erasing the Past and Impacting the Future" by Aharonov & Zubairy

- **Part IV: The Kaonic Quantum Eraser/ Decoherence & Measures of Entanglement**

  or *Quantum Information* (Sets of entanglement measures of multipartite qudit systems)

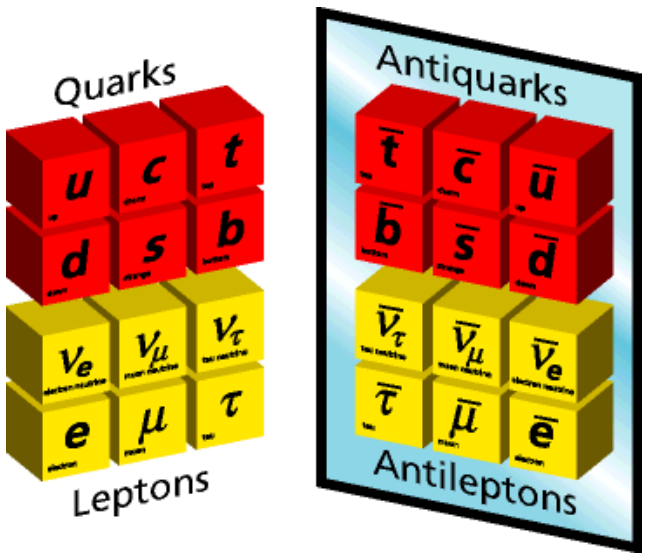


Testing QM in High Energy Physics

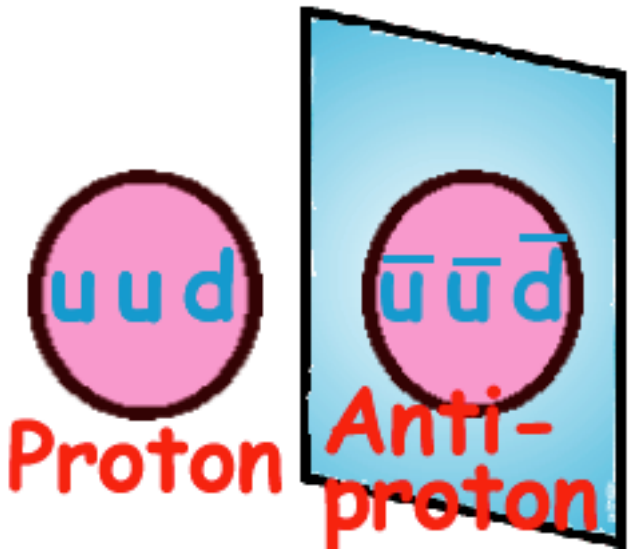
- **Part I: Bell inequalities 1:**

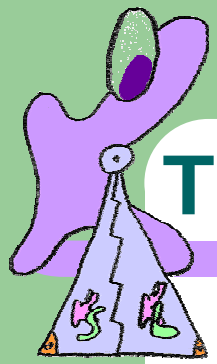
A symmetry violation in particle physics related to nonlocality ?!

world **antiworld**



world **antiworld**



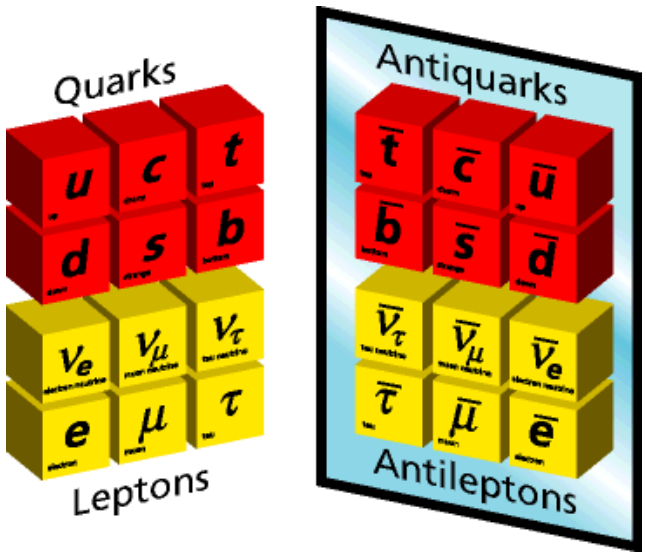


Testing QM in High Energy Physics

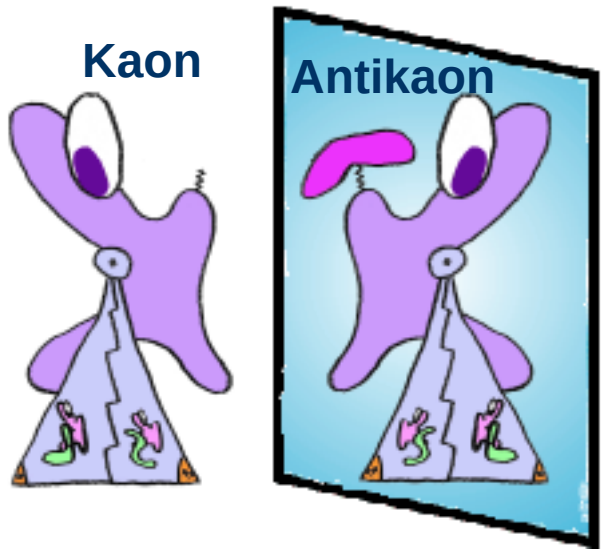
- **Part I: Bell inequalities 1:**

A symmetry violation in particle physics related to nonlocality ?!

world **antiworld**



world **antiworld**

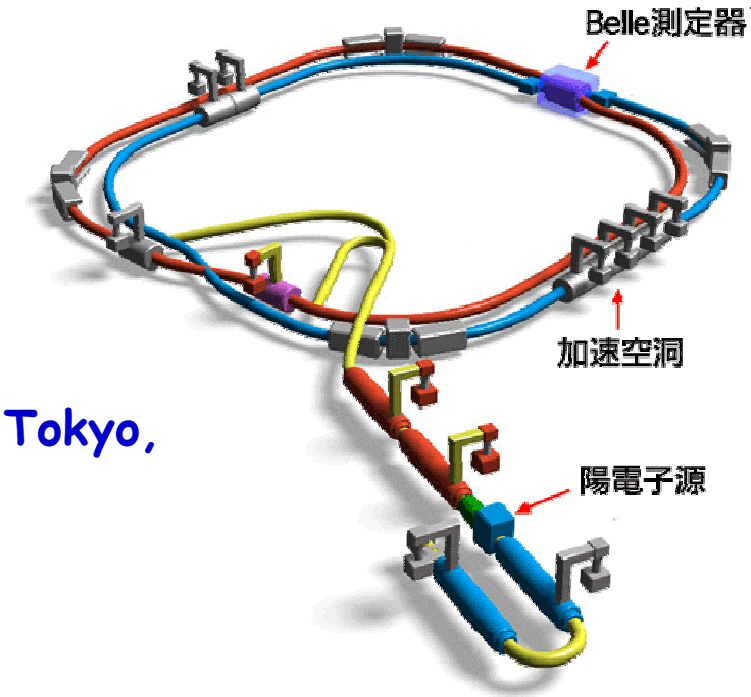


$\bar{s}d$ $s\bar{d}$

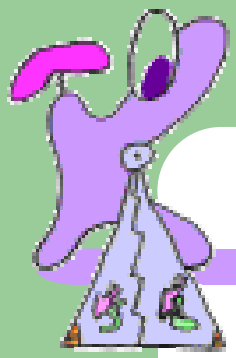
Accelerators

Accelerators

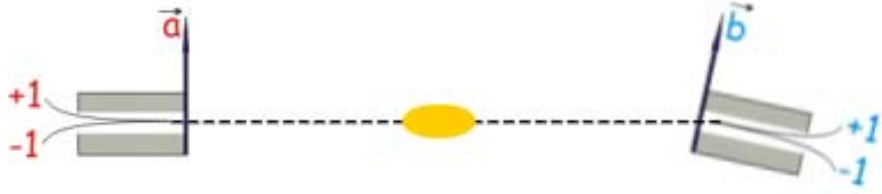
DAPHNE (next to Rome, Italy)



KEK (next to Tokyo, Japan)



The EPR scenario



Antisymmetric Bell state:

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left\{|\Uparrow\rangle_l \otimes |\Downarrow\rangle_r - |\Downarrow\rangle_l \otimes |\Uparrow\rangle_r\right\}$$ **... spin 1/2**

$$= \frac{1}{\sqrt{2}}\left\{|0\rangle_l \otimes |1\rangle_r - |1\rangle_l \otimes |0\rangle_r\right\}$$ **... qubit**

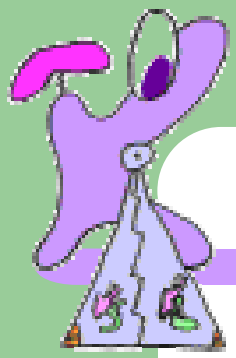
$$= \frac{1}{\sqrt{2}}\left\{|H\rangle_l \otimes |V\rangle_r - |V\rangle_l \otimes |H\rangle_r\right\}$$ **... photon**

$$= \frac{1}{\sqrt{2}}\left\{|K^0\rangle_l \otimes |\overline{K}^0\rangle_r - |\overline{K}^0\rangle_l \otimes |K^0\rangle_r\right\}$$ **... kaon**

$$= \frac{1}{\sqrt{2}}\left\{|B^0\rangle_l \otimes |\overline{B}^0\rangle_r - |\overline{B}^0\rangle_l \otimes |B^0\rangle_r\right\}$$ **... B-meson**

$$= \frac{1}{\sqrt{2}}\left\{|I\rangle_l \otimes |\Uparrow\rangle_r - |II\rangle_l \otimes |\Downarrow\rangle_r\right\}$$ **... single neutron in interferometer**

$$= \cdots$$



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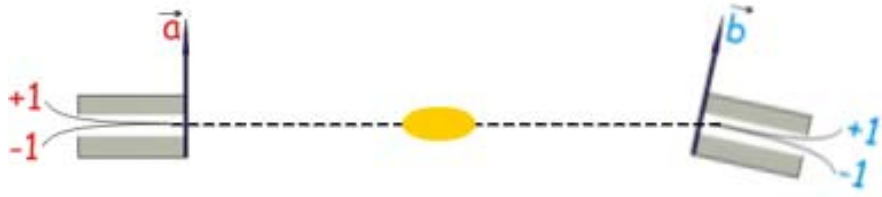

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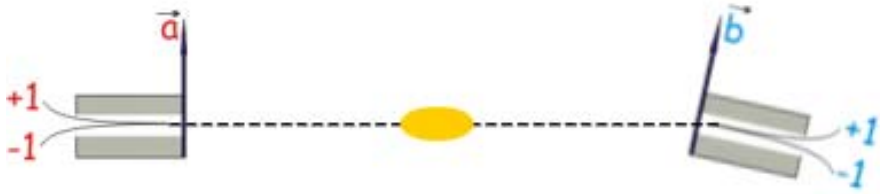
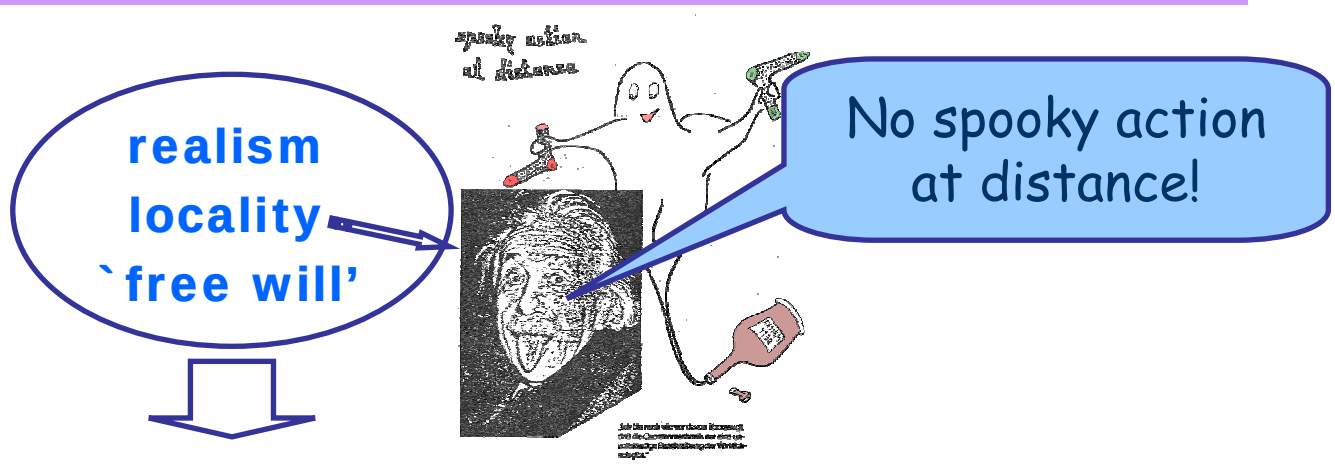


1935: Einstein-Podolsky-Rosen-PARADOX

The EPR reality criterion: "If without in any way disturbing a system, one can predict with certainty (i.e. with the probability equal to one) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

→ Quantum Theory is not complete!

What are Bell inequalities?



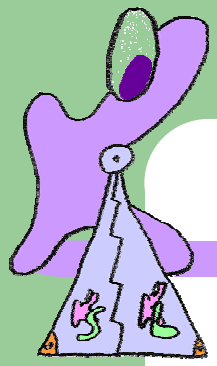
Local realistic theories: Quantum Mechanics:

$$P(a,b) \leq P(a,c) + P(c,b)$$

inequalities for probabilities → *always* satisfied!

→quantum mechanical probabilities may violate the inequalities!

Experiment has to decide!



Derivation of the BI

$A(n,\lambda)$

$B(m,\lambda)$

...values of observable

$|A|,|B| \leq 1$

QM: $A^{QM}(n) \rightarrow \vec{\sigma}\cdot\vec{n}$

$B^{QM}(m) \rightarrow \vec{\sigma}\cdot\vec{m}$

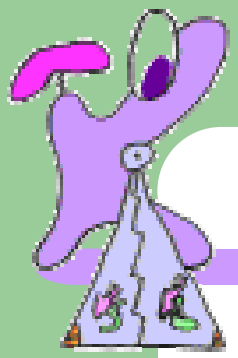
λ...hidden variable

***n,m*...quantisation directions**

Bell's locality hypothesis

$$E(n,m) = \int d\lambda\, \rho(\lambda)\, A(n,\lambda)\cdot B(m,\lambda) \qquad \textit{with} \qquad \int d\lambda\, \rho(\lambda) = 1$$

independent of **m** **n**



Derivation of the CHSH-BI

Bell's locality hypothesis

$$E(n,m) = \int d\lambda\, \rho(\lambda)\, A(n,\lambda)\cdot B(m,\lambda) \qquad with \qquad \int d\lambda\, \rho(\lambda) = 1$$

$$E(n,m) - E(n,m') = \int d\lambda\, \rho(\lambda)\{A(n,\lambda)B(m,\lambda) - A(n,\lambda)B(m',\lambda)\}$$

$$= \int d\lambda\, \rho(\lambda)\; A(n,\lambda)B(m,\lambda)\{1 \pm A(n',\lambda)B(m',\lambda)\}$$

$$- \int d\lambda\, \rho(\lambda)\; A(n,\lambda)B(m',\lambda)\{1 \pm A(n',\lambda)B(m,\lambda)\}$$

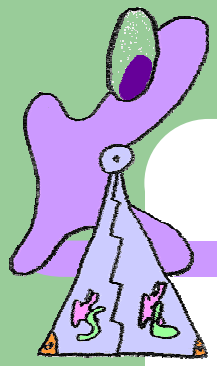
$$|E(n,m) - E(n,m')| \le \int d\lambda\, \rho(\lambda)\{1 \pm A(n',\lambda)B(m',\lambda)\} + \int d\lambda\; \rho(\lambda)\{1 \pm A(n',\lambda)B(m,\lambda)\}$$

$$= 2 \pm |E(n',m') + E(n',m)|$$

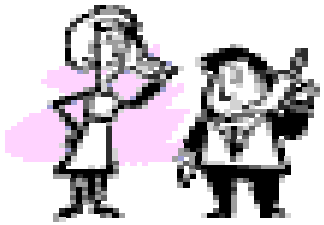
CHSH-Bell inequality:

for all local realistic theories

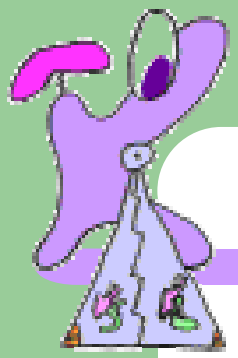
$$S(n,m,n',m') = |E(n,m) - E(n,m')| + |E(n',m) + E(n',m')| \le 2$$



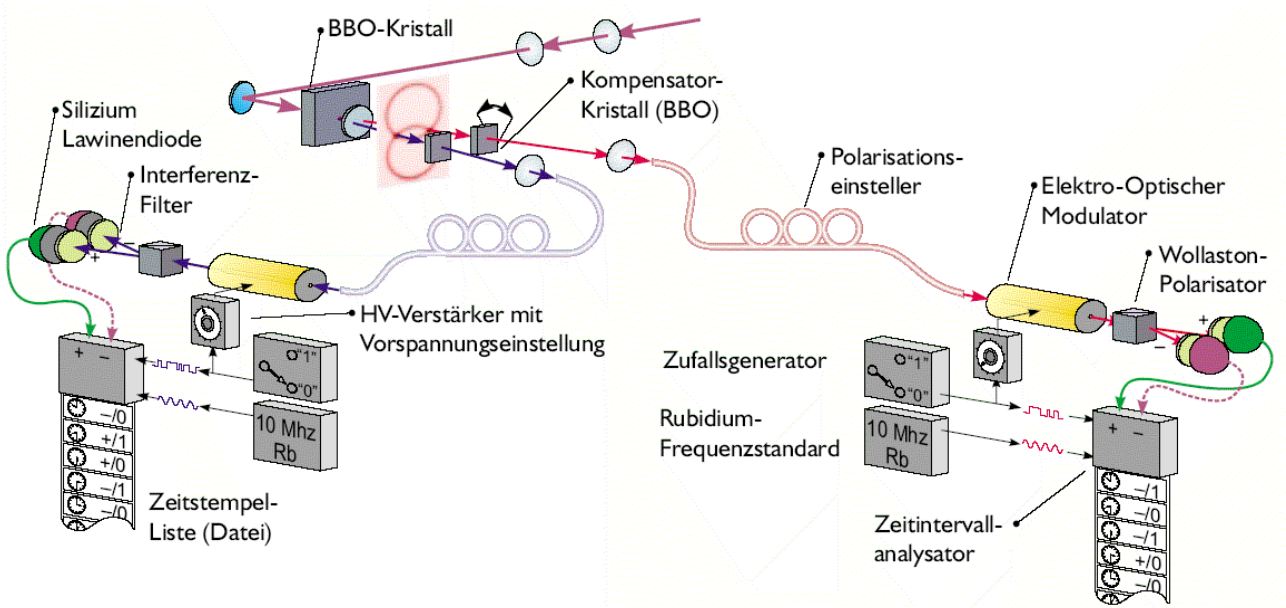
What are Bell inequalities good for?



- Has been proven: quantum cryptography protocols are save if a Bell inequality is violated!
- *Quantum Communication Complexity*: Bell inequalities are necessary and sufficient conditions for quantum protocols to beat the classical ones!
- Correlations between space-like locations exist which are stronger than any `classical' correlations
- Entanglement and non-locality are different things!



Bell inequality for photons



LOOPHOLES:

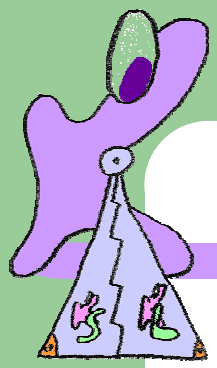
locality (~no "communication" over macroscopic distances possible within speed of light)

➢ **!!CLOSED!!**

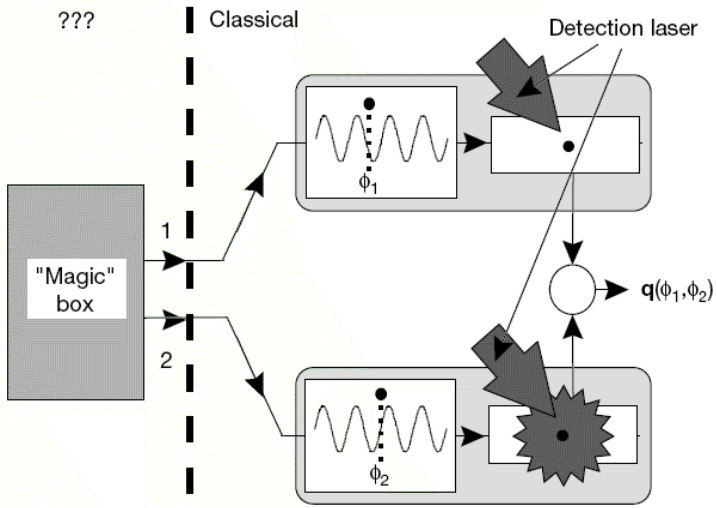
detection efficiency (~fair sampling hypothesis)

➢ **!!NOT CLOSED!!**

$\eta = 0.05\ (-0.33)$;
required $\eta > 0.83$



Bell inequality for ions



$$S_{\exp} = 2.25 \pm 0.03$$

LOOPHOLES:

locality (~no "communication" over macroscopic distances possible)

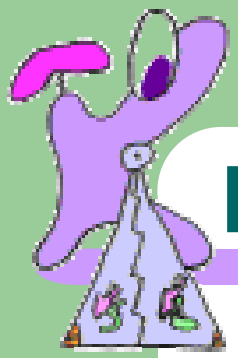
➢**!! NOT CLOSED!!**

detection efficiency (~fair sampling hypothesis)

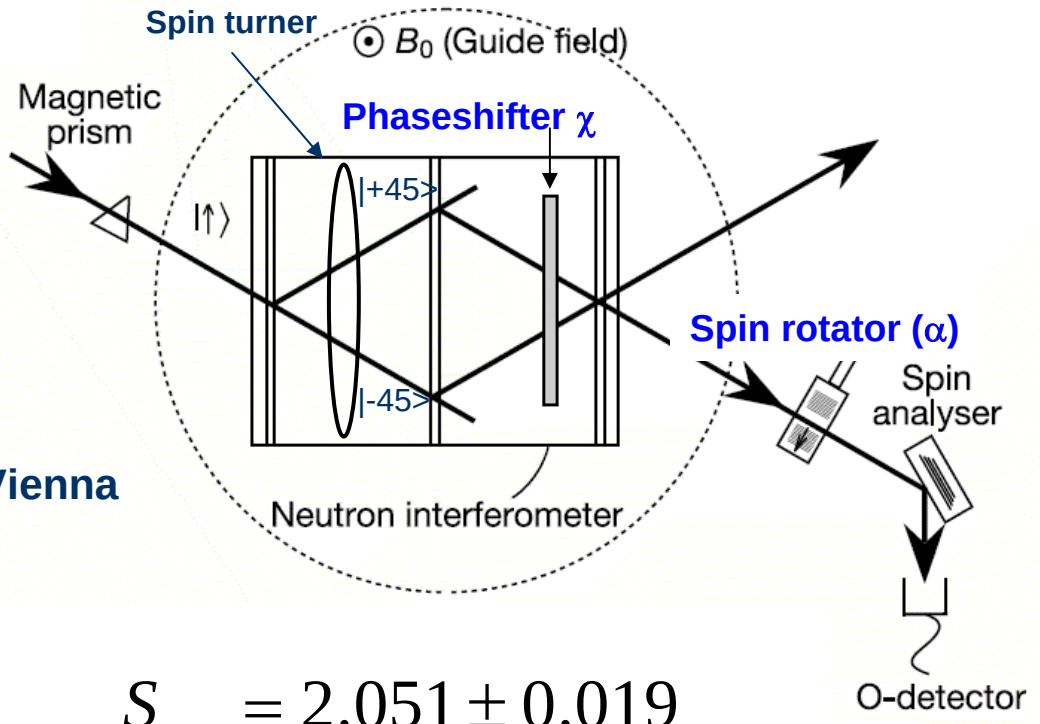
➢**!!CLOSED!!**

$d = 3\mu m$

required $d > 300km$



Bell-like inequality for neutrons

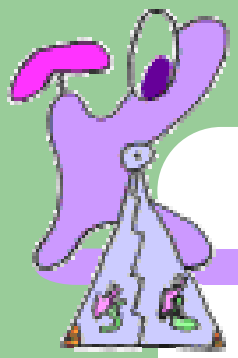


Hasegawa, Rauch,

Atomic Institute in Vienna

$$S_{\text{exp}} = 2.051 \pm 0.019$$

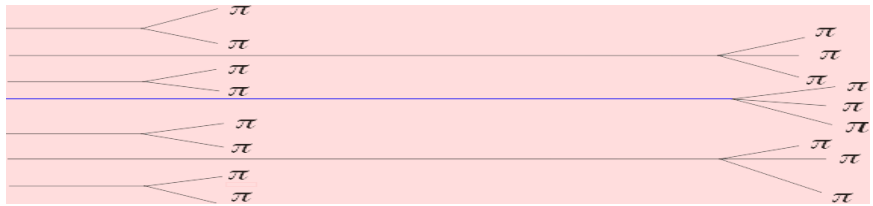
→ tests contextuality (~the value of an observable (spin) DOES depend on the co-measured observable (path))



What are neutral kaons ?

Strangeness: $S\left|K^0\right\rangle = +\left|K^0\right\rangle$

$S\left|\overline{K}^0\right\rangle = -\left|\overline{K}^0\right\rangle$



Mass-eigenstates: $\left|K_S\right\rangle, \left|K_L\right\rangle$

$$\left|K^0\right\rangle \cong \frac{1}{\sqrt{2}}\left\{\left|K_S\right\rangle + \left|K_L\right\rangle\right\}$$

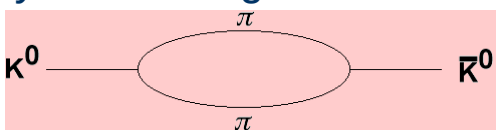
„A kaon is a kind of double slit"

Kaon in time:

short-lived state long-lived state

$$\left|K^0(t)\right\rangle \cong \frac{1}{\sqrt{2}}\left\{e^{-\frac{\Gamma_S}{2}t - im_S t}\left|K_S\right\rangle + e^{-\frac{\Gamma_L}{2}t - im_L t}\left|K_L\right\rangle\right\}$$

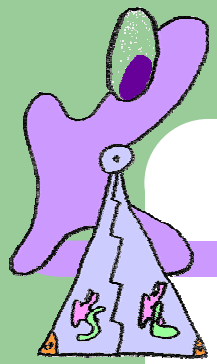
Feynman diagram



$\Gamma_S \approx 10^{10}\,\frac{1}{s}$ ... decay width of K_S

$\Gamma_L \approx 1/600\,\Gamma_S$ ... decay width of K_L

$\Delta m = m_L - m_S \approx 0.5\,\Gamma_S$ ... mass difference



Similarities/differences

Photons

$$|\psi^-\rangle \cong \{|H\rangle_l \otimes |V\rangle_r - |V\rangle_l \otimes |H\rangle_r\}$$

$$P(H,\vec{n};H,\vec{m}) = P(V,\vec{n};V,\vec{m})$$
$$= \tfrac{1}{4}(1-\cos 2\phi_{nm})$$

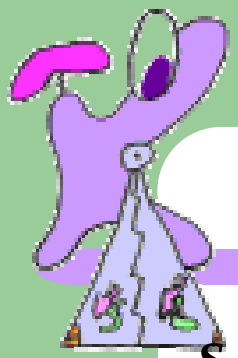
Kaons

$$|\psi^-\rangle \cong \left\{|K^0\rangle_l \otimes |\overline{K}^0\rangle_r - |\overline{K}^0\rangle_l \otimes |K^0\rangle_r\right\}$$

$$P(K^0,t_l;K^0,t_r) = P(\overline{K}^0,t_l;\overline{K}^0,t_r)$$
$$= \tfrac{1}{8}\left(e^{-\Gamma_S t_l - \Gamma_L t_r} + e^{-\Gamma_L t_l - \Gamma_S t_r}\right.$$
$$\left. - 2\cos(\Delta m \Delta t)\cdot e^{-\Gamma(t_l+t_r)}\right)$$

No decay $\Gamma_S = \Gamma_L = 0$

$$P(K^0,t_l;K^0,t_r) = P(\overline{K}^0,t_l;\overline{K}^0,t_r)$$
$$= \tfrac{1}{4}(1-\cos(\Delta m \Delta t))$$



The CHSH-inequality for the neutral kaon system

$$S_{CHSH} = \left| E_{\overline{K}^0\overline{K}^0}(t_a;t_b) - E_{\overline{K}^0\overline{K}^0}(t_a;t_c) \right| + \left| E_{\overline{K}^0\overline{K}^0}(t_d;t_b) + E_{\overline{K}^0\overline{K}^0}(t_d;t_c) \right| \leq 2$$

for all local realistic theories

QM:

$$E_{kaon}(t_a;t_b) \cong -\cos(\Delta m(t_a - t_b)) \cdot e^{-\frac{\Gamma_S}{2}(t_a+t_b)}$$

$$E_{photon}(\vec{n};\vec{m}) \cong -\cos(2\phi_{nm})$$

$S_{Photon} = 2\sqrt{2}$ Violation!

$S^{Kaon} \leq 2$ NO violation!

PROPOSITION:

The CHSH-inequality is violated iff x>2 for kaons or x>2.6 for the other mesons.

Strangeness oscillation / decay:

$$x = \frac{\Delta m}{\Gamma} \approx \frac{2\Delta m}{\Gamma_S} \approx 1$$

B-mesons: x=0.77

D-meson: x<0.03

B_d-mesons: x>20.6

Is it really not possible to distinguish between local realistic theories and quantum mechanics for neutral kaons?

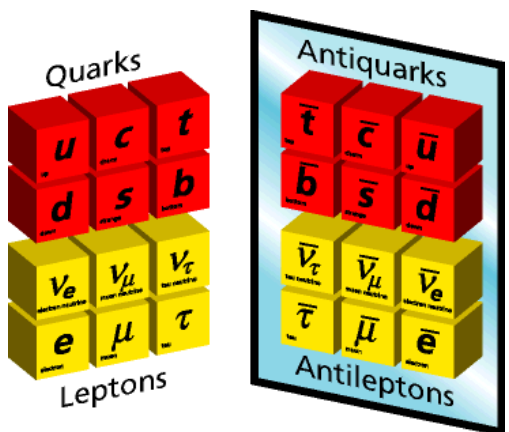
!?Entanglement related to a symmetry violation?!

1964

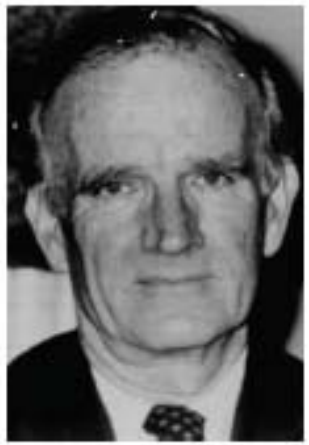
Bell inequalities

CP violation

world anti-world



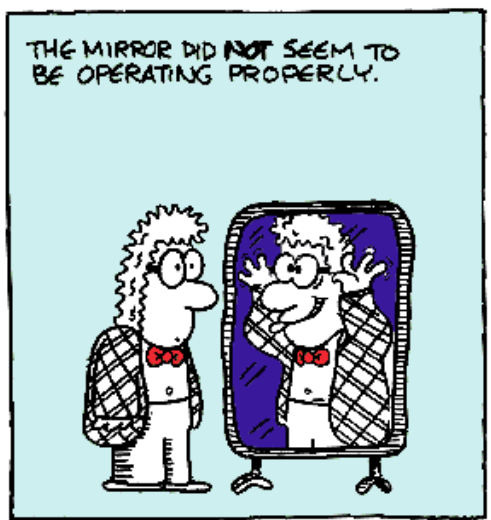
Nobelprize for CP violation

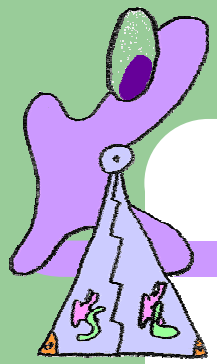


V. Fitch

J.Cronin

1980 NOBEL PRIZE





How to describe kaons?

Knowledge obtained by strong interaction

I am a kaon with S=+1

$S := 2\,(Q - I_3 - B/2)$

$$S\,|K^0\rangle = +\,|K^0\rangle$$
$$S\,|\overline{K}^0\rangle = -\,|\overline{K}^0\rangle$$

I am an anti-kaon with S=-1

$$P\,|K^0\rangle = -\,|K^0\rangle$$
$$P\,|\overline{K}^0\rangle = -\,|\overline{K}^0\rangle$$

$$C\,|K^0\rangle = e^{i\alpha}\,|\overline{K}^0\rangle$$
$$C\,|\overline{K}^0\rangle = e^{-i\alpha}\,|K^0\rangle$$

α is a unphysical phase and in conventional physics set to zero

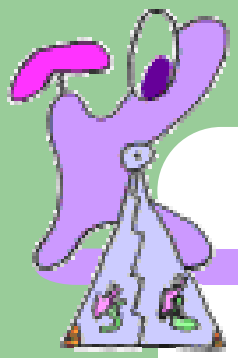
$$CP\,|K^0\rangle = -\,e^{i\alpha}\,|\overline{K}^0\rangle$$
$$CP\,|\overline{K}^0\rangle = -\,e^{-i\alpha}\,|K^0\rangle$$

$CP = +1$

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left\{|K^0\rangle - e^{i\alpha}\,|\overline{K}^0\rangle\right\}$$

$CP = -1$

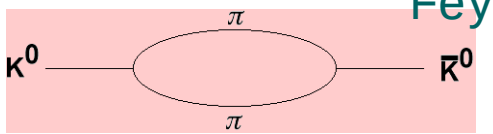
$$|K_2\rangle = \frac{1}{\sqrt{2}}\left\{|K^0\rangle + e^{i\alpha}\,|\overline{K}^0\rangle\right\}$$



CP violation

$$\left|K^0\right\rangle, \left|\overline{K}^0\right\rangle \rightarrow \underbrace{2\pi}_{CP=+1}, \underbrace{3\pi}_{CP=-1}$$

Feynman



Experiment: 1964 Christensen, Cronin, Fitch and Turlay

$$\left|K_S\right\rangle = \frac{1}{\sqrt{2(1+|\varepsilon|^2)}}\{\left|K_1\right\rangle + \varepsilon\left|K_2\right\rangle\} = \frac{1}{N}\{p\left|K^0\right\rangle - q\left|\overline{K}^0\right\rangle\}$$

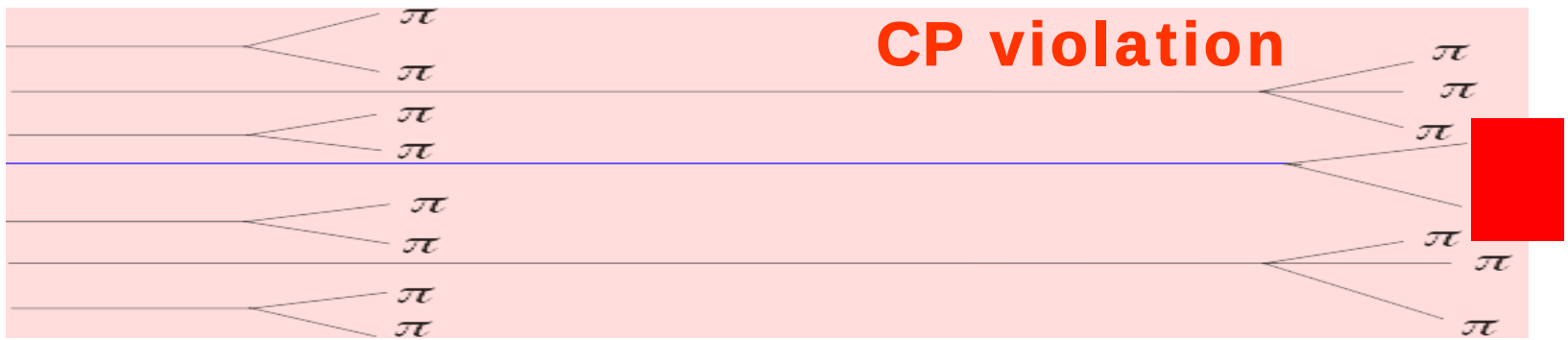
$$\left|K_L\right\rangle = \frac{1}{\sqrt{2(1+|\varepsilon|^2)}}\{\left|K_2\right\rangle + \varepsilon\left|K_1\right\rangle\} = \frac{1}{N}\{p\left|K^0\right\rangle + q\left|\overline{K}^0\right\rangle\}$$

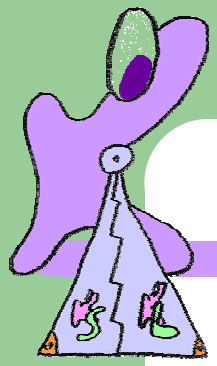
$$\varepsilon \approx 10^{-3}$$

$$\Gamma_S \approx 10^{10}\,\frac{1}{s}$$

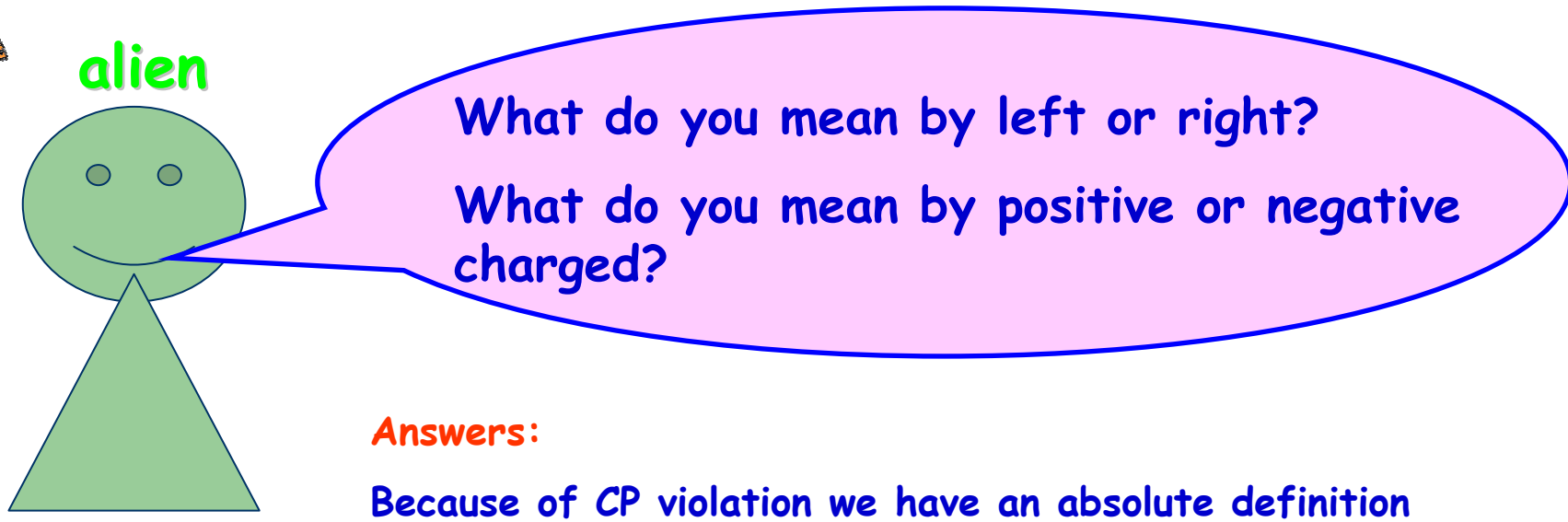
$$\Gamma_L \approx 10^{7}\,\frac{1}{s}$$

$$\Delta m \approx 0.5\,\Gamma_S$$





`Side effects´ of CP violation

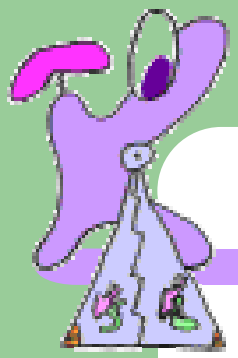


Answers:

Because of CP violation we have an absolute definition of left and right, there exists a difference between a world of matter and a world of antimatter.

Positive charge is the charge of the lepton more often produced in the semileptonic decay of the K_L.

Do NOT know: What is the origin of CP violation? Why is it that small?



BI related to CP violation?

$$P(\Uparrow \alpha, \Uparrow \beta) \leq P(\Uparrow \alpha, \Uparrow \gamma) + P(\Uparrow \gamma, \Uparrow \beta)$$

$S \equiv \sigma_3$

$CP \equiv -\sigma_1$

$\cancel{CP} \propto \sigma_2$

Uchiyama 1997

$$P(K_S, \overline{K}^0) \leq P(K_S, K_1) + P(K_1, \overline{K}^0)$$

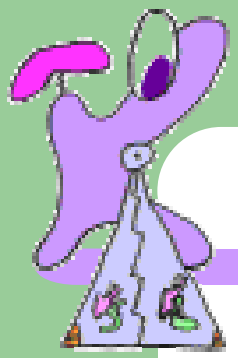
α arbitrary:

$$P^{QM}(K_S, \overline{K}^0) = \frac{1}{2N^2} |p|^2$$

$$P^{QM}(K_S, K_1) = \frac{1}{4N^2} |p \cdot e^{i\alpha} - q|^2$$

$$P^{QM}(K_1, \overline{K}^0) = \frac{1}{4}$$

$$\mathrm{Re}\left\{ e^{i\alpha} p \cdot q^* \right\} \leq |q|^2$$



Optimizing the BI

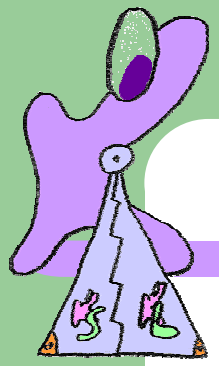
Phase choice between p and q: β

$$\mathrm{Re}\left\{e^{i\alpha}\, p\cdot q^{*}\right\} = |p|\,|q|\,\mathrm{Re}\left\{e^{i(\alpha+\beta)}\right\} \le |q|^{2}$$

Optimized: $\alpha+\beta=0$

$$|p| \le |q|$$

This BI is experimentally testable!



Experimental verification of the BI

Leptonic Charge Asymmetry:

Experiment

$$\delta = \frac{\Gamma(K_L \to \pi^- l^+ \nu_l) - \Gamma(K_L \to \pi^+ l^- \bar{\nu}_l)}{\Gamma(K_L \to \pi^- l^+ \nu_l) + \Gamma(K_L \to \pi^+ l^- \bar{\nu}_l)} = (3.27 \pm 0.12) \cdot 10^{-3}$$

Theory

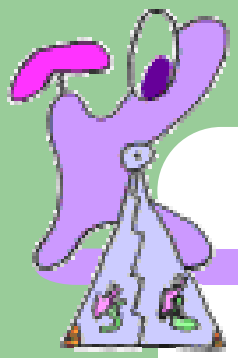
$\Delta S = \Delta Q$:

$$|K_L\rangle = \frac{p}{N}|K^0\rangle + \frac{q}{N}|\bar{K}^0\rangle \qquad \delta = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2}$$

1-particle state

$$K^0(\bar{s}d) \xrightarrow{\bar{s} \to \bar{u} + l^+ + \nu_l} \pi^- l^+ \nu_l$$

$$\bar{K}^0(s\bar{d}) \xrightarrow{s \to u + l^- + \bar{\nu}_l} \pi^+ l^- \bar{\nu}_l$$



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Bell inequality: $|p| \le |q|$ $\quad \delta = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2}$

? ! !

!? CP violation related to nonlocality !?

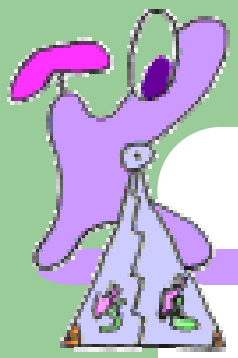
? ! ?

$\delta \le 0$

$-\delta \le 0$ $\quad \left|\overline{K}^0\right\rangle \leftrightarrow \left|K^0\right\rangle$

2-particle state

$\delta = 0$



Summary

Gedanken experiment:

$$P(K_S, \overline{K}^0) \leq P(K_S, K_1) + P(K_1, \overline{K}^0)$$

$$P(K_S, K^0) \leq P(K_S, K_1) + P(K_1, K^0)$$

Inserted q.m. probabilities into BIs:

$$\delta = 0$$

Experiment: $\delta = (3.27 \pm 0.12) \cdot 10^{-3}$

Disadvantage:
Probabilities cannot directly be measured!

Literature:

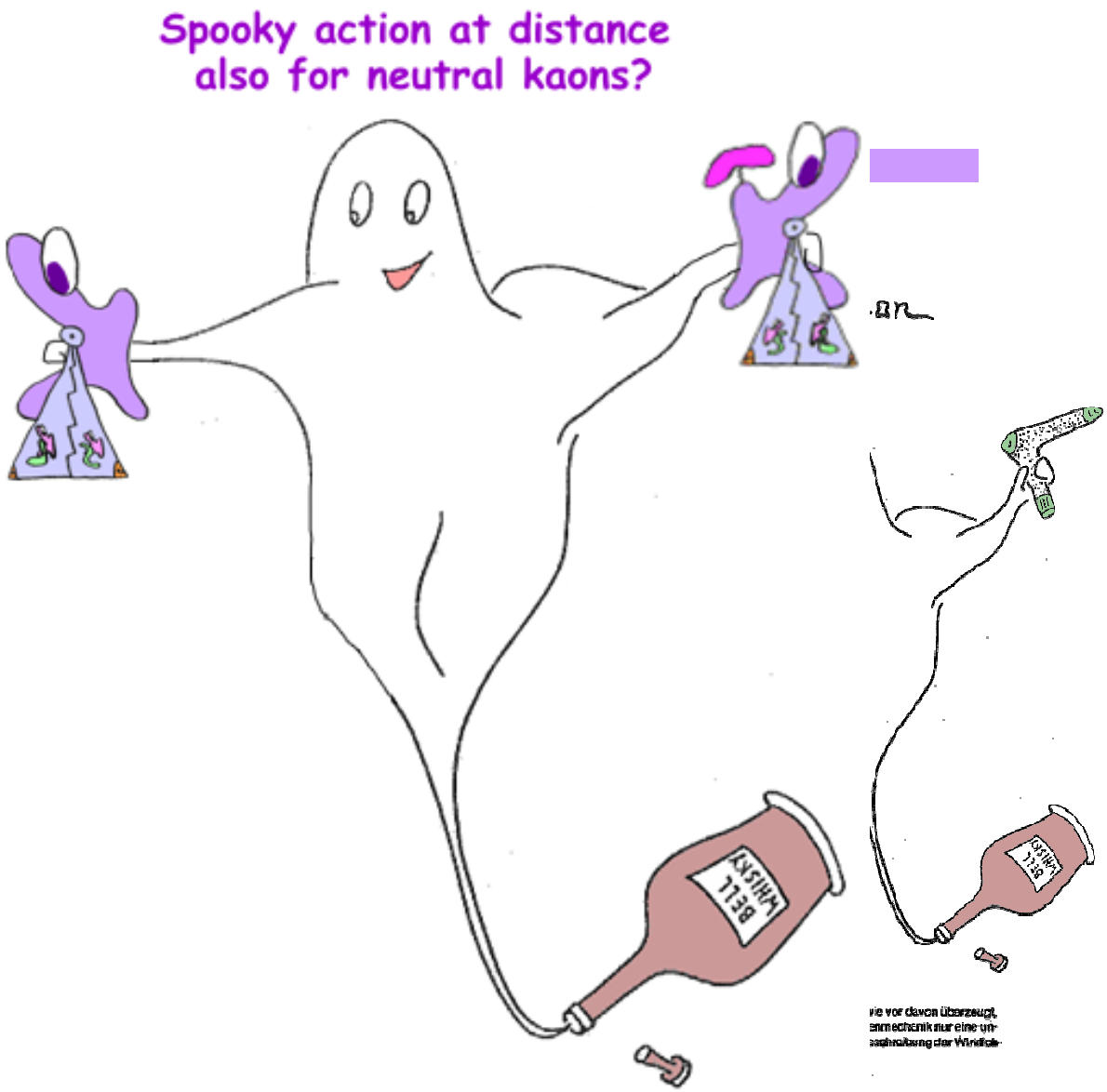
- **Bertlmann, Grimus, Hiesmayr,** ***Bell inequality and CP violation in the neutral kaon system,*** **Phys. Lett. A 289, 21 (2001)**
- **B.C. Hiesmayr.** ***A generalized Bell inequality and decoherence for the K0 anti-K0 system.*** **Found. of Phys. Lett. 14, 231 (2001).**
- **Bertlmann, Hiesmayr,** ***Bell inequalities for entangled kaons and their unitary time evolution,*** **Phys. Rev. A 63, 062112 (2001)**
- **B.C. Hiesmayr,** ***Nonlocality and Entanglement in a strange system,*** **European Journal C 50, 73-79 (2007)**
- **Bertlmann, Bramon, Garbarino, Hiesmayr,** ***Violation of a BI in particle physics experimentally verified?,*** **Phys. Lett. A 332, 355 (2004)**

A little

Les chaussettes
de M. Bertlmar
et la nature
de la réalité

Fondation Hugot
juin 17 1980

from: J.S. Bell, "Bertlmann's
Journal de Physique, To
No. 3, mars 1981.
reprinted in: J.S. Bell, "Speak
mechanics", Caml



Drawn by R.A. Bertlmann to the 60th birthday of John Bell