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A Method of Estimating the *p*-adic Sizes of Common Zeros of Partial Derivative Polynomials Associated with an n^{th} Degree Form

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ABSTRACT

Let $\underline{x} = (x_1, x_2, ..., x_n)$ be a vector in a space Z^n where *Z* is the ring of integers and let *q* be a positive integer, *f* a polynomial in $\underline{x}$ with coefficients in *Z*. The exponential sum associated with f is defined as

$$S(f;q) = \sum \exp(2\pi i f(x)/q)$$

where the sum is taken over a complete set of residues modulo *q*.
The value of *S(f;q)* has been shown to depend on the estimate of the cardinality |*V*|, the number of elements contained in the set

$$V = \{\underline{x} \bmod q \mid \underline{f}_{\underline{x}} \equiv \underline{0} \bmod q\}$$

where $\underline{f}_{\underline{x}}$ is the partial derivatives of *f* with respect to $\underline{x}$. To determine the cardinality of *V*, the information on the *p*-adic sizes of common zeros of the partial derivatives polynomials need to be obtained.

This paper discusses a method of determining the *p*-adic sizes of the components of (ξ,η), a common root of partial derivatives polynomial of *f(x,y)* in of degree *n*, where *n* is odd based on the *p*-adic Newton polyhedron technique associated with the polynomial. The polynomial of degree *n* is of the form

$$f(x,y) = ax^n + bx^{n-1}y + cx^{n-2}y^2 + sx + ty + k$$

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