

## An Introduction to Neutrix Composition of Distributions and Delta Function

<sup>1,2</sup>Adem Kilicman and <sup>3</sup>Brian Fisher

<sup>1</sup>Faculty of Science, Universiti Putra Malaysia,  
43400 UPM Serdang, Selangor, Malaysia

<sup>2</sup>Institute for Mathematical Research, University Putra Malaysia,  
43400 UPM Serdang, Selangor, Malaysia

<sup>3</sup>University of Leicester, Leicester, LE1 7RH, England

E-mail: [akilicman@putra.upm.edu.my](mailto:akilicman@putra.upm.edu.my) and [fbr@le.ac.uk](mailto:fbr@le.ac.uk)

### ABSTRACT

The composition of the distribution  $\delta^{(s)}(x)$  and an infinitely differentiable function  $f(x)$  having a simple zero at the point  $x = x_0$  is defined by Gel'fand Shilov by the

equation  $\delta^{(s)}(f(x)) = \frac{1}{|f'(x_0)|} \left[ \frac{1}{f'(x)} \frac{1}{dx} \right]^s \delta(x - x_0)$ . It is shown how this definition

can be extended to functions  $f(x)$  which are not necessarily infinitely differentiable or not having simple zeros at the point  $x = x_0$ , by defining  $\delta^{(s)}(f(x))$  as the limit or neutrix limit of the sequence  $\{\delta_n^{(s)}(f(x))\}$ , where  $\{\delta_n(x)\}$  is a certain sequence of infinitely differentiable functions converging to the Dirac delta-function  $\delta(x)$ . A number of examples are given.

Keywords: Distribution, delta-function, composition of distributions, neutrix, neutrix limit.

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