Malaysian Journal of Mathematical Sciences 10(S) February: 111–121 (2016)
Special Issue: The 3rd International Conference on Mathematical Applications in Engineering 2014 (ICMAE'14)

MALAYSIAN JOURNAL OF MATHEMATICAL SCIENCES

Journal homepage: http://einspem.upm.edu.my/journal

Evolution of the Two-Mode Entangled States with An Atomic Coupler

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ABSTRACT

We develop a model to investigate the behavior of atomic coupler consists of two waveguides, each of which includes a localized or a trapped atom with a two-mode pair coherent state source. The waveguides are placed close enough to each other to allow energy exchange between them. The two atoms, in the different waveguides, are located adjacent to each other. In each waveguide, one mode propagates along and interacts with the atom inside in a standard way as the Jaynes- Cummings model (JCM). The atom-mode system, in each waveguide, interacts with the other one via the evanescent wave. Pair coherent states (PCS) are one of the most important states in quantum information theory since it includes sufficient entanglement causing the violation of the Bell inequality. Moreover, the PCS possesses prominent nonclassical properties such as sub-Poissonian statistics, correlation in the number fluctuations, squeezing and violations of the Cauchy Schwartz inequalities

Keywords: Atomic Quantum Coupler, Entanglement,Pair Coherent State.

1. Introduction

The quantum directional coupler is a device composed of two (or more) waveguides placed close enough to allow exchange of energy between them via evanescent waves(Jensen, 1982). The rate of flow of the exchanged energy can be controlled by the device design and the intensity of the input flux. The outgoing fields from the coupler can be measured in standard ways to quantify the nonclassical effects(Faisal , 2010). Quite recently, this device has attracted much attention in optics communication and quantum computing network, which require data transmission and ultra-high-speed data processing. Furthermore, the directional coupler has been experimentally implemented, e.g., in planar structures, dual optical fibers and certain organic polymers. For more details related to the quantum properties of the fields in the directional couplers the reader can consult and the references therein. The interaction between the radiation field and the matter (i.e. atom), namely the Jaynes Cummings model (JCM)(Jaynes , 1963),(Perina, 2000) is an important topic in quantum optics and quantum information theories. The simplest form of the JCM is a two-level atom interacting with a single-mode radiation field. The JCM is a rich source of the nonclassical effects, e.g., the revival collapse phenomenon (RCP), sub-Poissonian statistics and quadrature squeezing. Furthermore, the JCM has been experimentally implemented by various means such as the one-atom mazer, NMR refocusing, the Rydberg atom in a superconducting cavity, the trapped ion and the micromaser. Various extensions to the JCM have been reported including two two-level atoms interacting with radiation field(s).Trapped atoms or molecules are promising systems for quantum information processing and communications. They can serve as convenient and robust quantum memories for photons, providing thereby an interface between static and flying qubits. Coupling cold atoms to a radiation field sustained by an optical waveguide has already been addressed in various contexts. For example, hollow optical glass fibers have been used to guide atoms over long distances, specifically when employing a red detuned light field filling out the hollow core. A substrate-based atom waveguide can also be realized by using guided two-color evanescent light fields. Moreover, the coupling of atomic dipoles to the evanescent field of tapered optical fibers has been demonstrated in (Faisal, 2008),(Bennet, 1993),(Eberly, 2007),(Faisal, 2003),(Meunier, 2005).

2. Model Formalism

Now we are in a position to develop the model of AQC with PCS, which is the main object of the project. The atomic coupler consists of two waveguides, each of which includes a localized and/or a trapped atom. The waveguides are placed close enough to each other to allow interchanging energy between them. The two atoms (in the different waveguides) are located very adjacent to each other. In each waveguide one mode propagates along and interacts with the atom inside in a standard way as the JCM. The atom-mode in each waveguide interacts with the other one via the evanescent wave. The fields exited from the coupler can be examined as single or compound modes by means of homodyne detection to observe the squeezing of vacuum fluctuations, or by means of a set of photodetectors to measure photon antibunching and sub-Poissonian photon statistics in the standard ways. The model will be based on the framework of the rotating wave approximation (RWA) the Hamiltonian describing the AQC with the pair coherent states can be expressed as:

$$\frac{\hat{H}}{\hbar} = \hat{H}_0 + \hat{H}_I, \quad \hat{H}_0 = \sum_{j=0}^{2} \omega_j \hat{a}_j^\dagger \hat{a}_j + \frac{\omega_a}{2}(\hat{\sigma}_z^{(1)} + \hat{\sigma}_z^{(2)}), \tag{1}$$

$$\hat{H}_I = \sum_{j=1}^{2} \lambda_j (\hat{a}_j \hat{\sigma}_+^{(j)} + \hat{a}_j^\dagger \hat{\sigma}_-^{(j)}) + \lambda_3 (\hat{a}_1 \hat{a}_2^\dagger \hat{\sigma}_+^{(1)} \hat{\sigma}_-^{(2)} + \hat{a}_1^\dagger \hat{a}_2 \hat{\sigma}_-^{(1)} \hat{\sigma}_+^{(2)}), \tag{2}$$

where $\hat{H}_0$ and $\hat{H}_I$ are the free and the interaction parts of the Hamiltonian, $\hat{\sigma}_\pm^{(j)}$ and $\hat{\sigma}_z^{(j)}$ are the Pauli spin operators of the jth atom ($j = 1, 2$); $\hat{a}_j$ ($\hat{a}_j^\dagger$) is the annihilation (creation) operator of the jth-mode with the frequency ω_j and ω_a is the atomic transition frequency (we consider that the frequencies of the two atoms are equal) and λ_1 (λ_2) is the atom-field coupling constant in the first (second) waveguide in the framework of the JCM. The interaction between the modes in the two waveguides occurs through the evanescent wave with the coupling constant λ_3. This term is the only one, which is conservative and can execute switching between the two waveguides. Thus it plays an essential role in the behavior of the AQC.

We should stress that the switching mechanism occurs through the two JCMs (in the two waveguides) and can be obtained by applying the RWA in each individual waveguide. In other words, the quantity$\lambda_3(\hat{a}_1 \hat{a}_2^\dagger \hat{\sigma}_+^{(1)} \hat{\sigma}_-^{(2)} + \hat{a}_1^\dagger \hat{a}_2 \hat{\sigma}_-^{(1)} \hat{\sigma}_+^{(2)}$ is non -conservative and hence it is cancelled out. Finally, the treatment of the switching mechanism in (1) is related to the notion of coupler; however, the existence of atoms in the waveguides has been taken into account. In (1) the treatment is considered only at the moment when the two fields interacting with atoms in the waveguides. Also when we treat the atoms (fields) classically the Hamiltonian (1) tends to that of the linear directional coupler (two-atom interaction). The interaction of two two-level atoms with the two modes has been considered in the optical cavity earlier (Faisal, 2008),(Eberly, 2007),(Casagrande, 2007), however, in the sense different from that presented above. For instance, as a sum of two separate Jaynes-Cummings

Hamiltonians to investigate the entanglement as well as the entanglement transfer from a bipartite continuous-variable (CV) system to a pair of localized qubits (Casagrande, 2007). Also, the quantum properties of the system of two two-level atoms interacting with the two nondegenerate cavity modes when the atoms and the field are initially in the atomic superposition states and the pair-coherent state has been investigated in.

Next, we evaluate the wave function for the Hamiltonian (1). We consider two types of the initial field states, namely, two-mode squeezed vacuum state and pair coherent state. The pair-coherent state(Satyanarayana, 1989) is one of the most important states in quantum information theory since it includes sufficient entanglement causing the violation of the Bell inequality. Moreover, the PCS possesses prominent nonclassical properties such as sub-Poissonian statistics, correlation in the number fluctuations, squeezing and violations of the Cauchy Schwartz inequalities [18]. Additionally, the PCS is a non-Gaussian state and its Wigner function exhibits significant negative values(Agarwal, 1986). The PCS has been generated by various means. For instance, it can be generated via the competition of four-wave mixing and two-photon absorption in a nonlinear medium(Zheng , 2001). Also the PCS has been realized in the trapped ion system (Zheng , 2001). In this case the trapped ion is excited bichromatically by three laser beams along different directions in the xy plane of the ion trap. If the initial vibrational state of motion of the ion is prepared in a Fock state, then the steady state of the system is a pure state given by a product of the atomic ground state with the PCS of the vibrational motion . A similar scheme has been given in(Zheng , 2001). Mathematically, the PCS is an eigenstates for both pairs of annihilation operator and the number difference operator. This can be expressed as(G.S., 1986)

$$|\xi, q\rangle = N_q \sum_{n=0}^{\infty} \frac{\xi^n}{\sqrt{n!(n+q)!}} |n, n+q\rangle, \tag{3}$$

$$N_q = \sum_{n=0}^{\infty} [\frac{|\xi^n|^{2n}}{n!(n+q)!}]^{-1/2}, where \xi = |\xi| \exp(i\phi_1) \tag{4}$$

The two-mode squeezed vacuum state (TMS) has the form:

$$|r\rangle = \frac{1}{\cosh(\frac{r}{2})} \sum_{n=0}^{\infty} (\tanh r/2)^n \exp(i\phi_2 n)|n, n\rangle, \tag{5}$$

whereϕ_1 is the phase. Throughout the calculation we use the generic state:

$$|\psi_f(0)\rangle = \sum_{n=0}^{\infty} C_n |n, n+q\rangle, \tag{6}$$

where the probability amplitude C_n can take either one of the those of (2) and (3). Also for the sake of simplicity we take $\phi_j = 0; j = 1; 2$.

3. Derivation of the Wavefunction

We consider two types of the initial atomic states as

$$|\psi_1(0)\rangle_a = \sin\theta_1 |e_1, e_2\rangle + \cos\theta_1 |g_1, g_2\rangle \tag{7}$$

$$|\psi_2(0)\rangle_a = \sin\theta_2 |e_1, e_2\rangle + \cos\theta_2 |g_1, g_2\rangle, \tag{8}$$

where the subscript a denotes the atomic system;$|e_j\rangle$ and $|g_j\rangle$ for the excited and the ground atomic states of the jth atom, respectively. The variables θ_1 andθ_2 are phases, which can be specified to provide different forms of the initial atomic states. we restrict the study to the resonance case$2\omega_a = \omega_1 + \omega_2$, also, one can easily prove that$[\hat{H}_0, \hat{H}_I]$. We use the technique given in (Faisal , 2010) for solving the dynamical equation of the system. This technique is sensitive for the initial atomic states. Therefore we have to solve the Schrdingers equation for times. We write down the analytical solution for these cases as follows.

i) For the atomic states$|e_1, e_2\rangle$ the wavefunction takes the form

$$\begin{aligned} |\,\Psi_1(t)\rangle &= \sum_{n=0}^{\infty} C_n \Big[X_1^{(1)}(t, n, n+q) \,|\, e_1, e_2, n, n+q\rangle + X_2^{(1)}(t, n, n+q) \,|\, e_1, g_2, n, n+q+1\rangle \\ &+ X_3^{(1)}(t, n, n+q) \,|\, g_1, e_2, n+1, n+q\rangle + X_4^{(1)}(t, n, n+q) \,|\, g_1, g_2, n+1, n+q+1\rangle \Big] \end{aligned} \tag{9}$$

From the initial conditions, the exact forms of the coefficients X_j can be expressed as:

$$\begin{aligned}
X_1^{(1)}(t,n,n+q) &= \frac{1}{2}\exp(i\frac{t}{2}c_2^{(1)})\left[\cos(t\Omega_-^{(1)}) - i\frac{c_2^{(1)}}{2\Omega_-^{(1)}}\sin(t\Omega_-^{(1)})\right] \\
&+\frac{1}{2}\exp(-i\frac{t}{2}c_2^{(1)})\left[\cos(t\Omega_+^{(1)}) + i\frac{c_2^{(1)}}{2\Omega_+^{(1)}}\sin(t\Omega_+^{(1)})\right], \quad (10) \\
X_2^{(1)}(t,n,n+q) &= -\frac{i\sin(t\Omega_-^{(1)})}{2\Omega_-^{(1)}}[\lambda_2-\lambda_1]\sqrt{n+1}\exp(i\frac{t}{2}c_2^{(1)}) \\
&-\frac{i\sin(t\Omega_+^{(1)})}{2\Omega_+^{(1)}}[\lambda_2+\lambda_1]\sqrt{n+1}\exp(-i\frac{t}{2}c_2^{(1)}), \quad (11) \\
X_3^{(1)}(t,n,n+q) &= \frac{i\sin(t\Omega_-^{(1)})}{2\Omega_-^{(1)}}[\lambda_2-\lambda_1]\sqrt{n+1}\exp(i\frac{t}{2}c_2^{(1)}) \\
&-\frac{i\sin(t\Omega_+^{(1)})}{2\Omega_+^{(1)}}[\lambda_2+\lambda_1]\sqrt{n+1}\exp(-i\frac{t}{2}c_2^{(1)}), \quad (12) \\
X_4^{(1)}(t,n,n+q) &= \frac{1}{2}\exp(i\frac{t}{2}c_2^{(1)})\left[-\cos(t\Omega_-^{(1)}) + i\frac{c_2^{(1)}}{2\Omega_-^{(1)}}\sin(t\Omega_-^{(1)})\right] \\
&+\frac{1}{2}\exp(-i\frac{t}{2}c_2^{(1)})\left[\cos(t\Omega_+^{(1)}) + i\frac{c_2^{(1)}}{2\Omega_+^{(1)}}\sin(t\Omega_+^{(1)})\right], \quad (13)
\end{aligned}$$

where

$$c_1^{(1)} = 2\lambda_1\lambda_2\sqrt{(n+1)(n+q+1)}, c_2^{(1)} = \lambda_3\sqrt{(n+1)(n+q+1)}, \quad (14)$$

$$\Omega_\pm^{(1)} = \sqrt{\frac{c_2^{(1)2}}{4} + (\lambda_1\sqrt{n+1} \pm \lambda_2\sqrt{n+q+1})^2}. \quad (15)$$

ii)For the atomic states $|g_1, g_2\rangle$ the wavefunction takes the form

$$\begin{aligned}
\mid \Psi_2(t)\rangle &= \sum_{n=0}^{\infty} C_n \Big[X_1^{(2)}(t,n,n+q) \mid e_1, e_2, n-1, n+q-1\rangle + X_2^{(2)}(t,n,n+q) \mid e_1, g_2, n-1, n+q\rangle \\
&+ X_3^{(2)}(t,n,n+q) \mid g_1, e_2, n, n+q-1\rangle + X_4^{(2)}(t,n,n+q) \mid g_1, g_2, n, n+q\rangle\Big], \quad (16)
\end{aligned}$$

From the initial conditions, the exact forms of the coefficients X_j can be expressed as:

$$\begin{aligned}
X_1^{(2)}(t,n,n+q) &= \frac{1}{2}\exp(i\frac{t}{2}c_2^{(2)})\left[-\cos(t\Omega_-^{(2)}) + i\frac{c_2^{(2)}}{2\Omega_-^{(2)}}\sin(t\Omega_-^{(2)})\right] \\
&+\frac{1}{2}\exp(-i\frac{t}{2}c_2^{(2)})\left[-\cos(t\Omega_+^{(2)}) + i\frac{c_2^{(2)}}{2\Omega_+^{(2)}}\sin(t\Omega_+^{(2)})\right] \\
, X_2^{(2)}(t,n,n+q) &= \frac{i\sin(t\Omega_-^{(2)})}{2\Omega_-^{(2)}}\left[(\lambda_2\sqrt{n+q} - \lambda_1\sqrt{n})\exp(i\frac{t}{2}c_2^{(2)})\right] \\
&-\frac{i\sin(t\Omega_+^{(2)})}{2\Omega_+^{(2)}}\left[(\lambda_2\sqrt{n+q} + \lambda_1\sqrt{n})\exp(-i\frac{t}{2}c_2^{(2)})\right] \\
, X_3^{(2)}(t,n,n+q) &= \frac{-i\sin(t\Omega_-^{(2)})}{2\Omega_-^{(2)}}\left[(\lambda_2\sqrt{n+q} - \lambda_1\sqrt{n})\exp(i\frac{t}{2}c_2^{(2)})\right] \\
&-\frac{i\sin(t\Omega_+^{(2)})}{2\Omega_+^{(2)}}\left[(\lambda_2\sqrt{n+q} + \lambda_1\sqrt{n})\exp(-i\frac{t}{2}c_2^{(2)})\right] \\
, X_4^{(2)}(t,n,n+q) &= \frac{1}{2}\exp(i\frac{t}{2}c_2^{(2)})\left[\cos(t\Omega_-^{(2)}) - i\frac{c_2^{(2)}}{2\Omega_-^{(2)}}\sin(t\Omega_-^{(2)})\right] \\
&+\frac{1}{2}\exp(-i\frac{t}{2}c_2^{(2)})\left[\cos(t\Omega_+^{(2)}) + i\frac{c_2^{(2)}}{2\Omega_+^{(2)}}\sin(t\Omega_+^{(2)})\right] \quad (17)
\end{aligned}$$

where

$$c_1^{(2)} = 2\lambda_1\lambda_2\sqrt{(n)(n+q+1)}, c_2^{(2)} = \lambda_3\sqrt{(n)(n+q+1)}, \quad (18)$$

$$\Omega_\pm^{(2)} = \sqrt{\frac{c_2^{(2)2}}{4} + (\lambda_1\sqrt{n} \pm \lambda_2\sqrt{n+q})^2}. \quad (19)$$

iii) For the atomic states$|g_1, e_2\rangle$ the wavefunction takes the form

$$\begin{aligned}
\mid \Psi_3(t)\rangle &= \sum_{n=0}^{\infty} C_n\left[X_1^{(3)}(t,n,n+q) \mid e_1, e_2, n-1, n+q-1\rangle + X_2^{(3)}(t,n,n+q) \mid e_1, g_2, n-1, n+q+1\rangle\right. \\
&\left. + X_3^{(3)}(t,n,n+q) \mid g_1, e_2, n, n+q\rangle + X_4^{(3)}(t,n,n+q) \mid g_1, g_{2,} n, n+q+1\rangle\right], \quad (20)
\end{aligned}$$

From the initial conditions, the exact forms of the coefficients X_j can be expressed as:

$$\begin{aligned}
X_1^{(3)}(t,n,n+q) &= \frac{i\sin(t\Omega_-^{(3)})}{2\Omega_-^{(3)}}[\lambda_2\sqrt{n+q+1}-\lambda_1\sqrt{n}]\exp(-i\frac{t}{2}c_2^{(3)}) \\
&\quad -\frac{i\sin(t\Omega_+^{(3)})}{2\Omega_+^{(3)}}[\lambda_2\sqrt{n+q+1}-\lambda_1\sqrt{n}]\exp(-i\frac{t}{2}c_2^{(3)}), \\
X_2^{(3)}(t,n,n+q) &= -\frac{1}{2}\exp(i\frac{t}{2}c_2^{(3)})\left[\cos(t\Omega_-^{(3)})+i\frac{c_2^{(3)}}{2\Omega_-^{(3)}}\sin(t\Omega_-^{(3)})\right] \\
&\quad +\frac{1}{2}\exp(-i\frac{t}{2}c_2^{(3)})\left[\cos(t\Omega_+^{(3)})+i\frac{c_2^{(3)}}{2\Omega_+^{(3)}}\sin(t\Omega_+^{(3)})\right], \\
X_3^{(3)}(t,n,n+q) &= \frac{1}{2}\exp(i\frac{t}{2}c_2^{(3)})\left[\cos(t\Omega_-^{(3)})+i\frac{c_2^{(3)}}{2\Omega_-^{(3)}}\sin(t\Omega_-^{(3)})\right] \\
&\quad +\frac{1}{2}\exp(-i\frac{t}{2}c_2^{(3)})\left[\cos(t\Omega_+^{(3)})-i\frac{c_2^{(3)}}{2\Omega_+^{(3)}}\sin(t\Omega_+^{(3)})\right], \\
X_4^{(3)}(t,n,n+q) &= -\frac{i\sin(t\Omega_-^{(3)})}{2\Omega_-^{(3)}}[\lambda_2-\sqrt{n+q+1}+\lambda_1\sqrt{n}]\exp(-i\frac{t}{2}c_2^{(3)}) \\
&\quad -\frac{i\sin(t\Omega_+^{(3)})}{2\Omega_+^{(3)}}[\lambda_2\sqrt{n+q+1}-\lambda_1\sqrt{n}]\exp(-i\frac{t}{2}c_2^{(3)}),
\end{aligned} \tag{21}$$

where

$$c_1^{(3)} = 2\lambda_1\lambda_2\sqrt{n(n+q+1)}, c_2^{(3)} = \lambda_3\sqrt{n(n+q+1)}, \tag{22}$$

$$\Omega_\pm^{(3)} = \sqrt{\frac{c_2^{(3)2}}{4}+(\lambda_1\sqrt{n}\pm\lambda_2\sqrt{n+q+1})^2}. \tag{23}$$

iv) For the atomic states $|e_1, g_2\rangle$ the wavefunction takes the form

$$\begin{aligned} \mid \Psi_4(t)\rangle &= \sum_{n=0}^{\infty} C_n \Big[X_1^{(4)}(t,n,n+q) \mid e_1, e_2, n, n+q\rangle + X_2^{(4)}(t,n,n+q-1) \mid e_1, g_2, n, n+q\rangle \\ &+ X_3^{(4)}(t,n,n+q) \mid g_1, e_2, n+1, n+q-1\rangle + X_4^{(4)}(t,n,n+q) \mid g_1, g_2, n+1, n+q\rangle \Big], \end{aligned} \tag{24}$$

From the initial conditions, the exact forms of the coefficientsX_J can be expressed as:

$$\begin{aligned} X_1^{(4)}(t,n,n+q) &= \frac{i\sin(t\Omega_-^{(4)})}{2\Omega_-^{(4)}}[\lambda_1\sqrt{n-1} - \lambda_2\sqrt{n+q}]\exp(-i\frac{t}{2}c_2^{(4)}) \\ &\quad -\frac{i\sin(t\Omega_+^{(4)})}{2\Omega_+^{(4)}}[\lambda_1\sqrt{n+1} - \lambda_2\sqrt{n+q}]\exp(-i\frac{t}{2}c_2^{(4)}), \\ X_2^{(4)}(t,n,n+q) &= -\frac{1}{2}\exp(i\frac{t}{2}c_2^{(4)})\left[\cos(t\Omega_-^{(4)}) + i\frac{c_2^{(4)}}{2\Omega_-^{(4)}}\sin(t\Omega_-^{(4)})\right] \\ &\quad +\frac{1}{2}\exp(-i\frac{t}{2}c_2^{(4)})\left[\cos(t\Omega_+^{(4)}) - i\frac{c_2^{(4)}}{2\Omega_+^{(4)}}\sin(t\Omega_+^{(4)})\right], \\ X_3^{(4)}(t,n,n+q) &= \frac{1}{2}\exp(i\frac{t}{2}c_2^{(4)})\left[\cos(t\Omega_-^{(4)}) + i\frac{c_2^{(4)}}{2\Omega_-^{(4)}}\sin(t\Omega_-^{(4)})\right] \\ &\quad +\frac{1}{2}\exp(-i\frac{t}{2}c_2^{(4)})\left[\cos(t\Omega_+^{(4)}) - i\frac{c_2^{(4)}}{2\Omega_+^{(4)}}\sin(t\Omega_+^{(4)})\right], \\ X_4^{(4)}(t,n,n+q) &= -\frac{i\sin(t\Omega_-^{(4)})}{2\Omega_-^{(4)}}[\lambda_1\sqrt{n+q+1} - \lambda_2\sqrt{n}]\exp(-i\frac{t}{2}c_2^{(4)}) \\ &\quad -\frac{i\sin(t\Omega_+^{(4)})}{2\Omega_+^{(4)}}[\lambda_1\sqrt{n+1} + \lambda_2\sqrt{n+q}]\exp(-i\frac{t}{2}c_2^{(4)}), \end{aligned} \tag{25}$$

where

$$c_1^{(4)} = 2\lambda_1\lambda_2\sqrt{(n+1)(n+q)}, c_2^{(4)} = \lambda_3\sqrt{(n+1)(n+q)}, \tag{26}$$

$$\Omega_\pm^{(4)} = \sqrt{\frac{c_2^{(4)2}}{4} + (\lambda_1\sqrt{n+1} \pm \lambda_2\sqrt{n+q})^2}. \tag{27}$$

4. Conclusion

We have developed, model of the AQC and wavefunction with pair coherent states, the exact solution for the equations of motion and the switching mechanism in AQC is sensitive to the type of the initial atomic states. Based on the information given in literature review, one can note that the AQC is within the reach of current technology. Also it may be of interest in the framework of quantum information such as data transmission (Faisal , 2010), quantum gates(Zheng , 2001) as well as entanglement generation

Acknowledgments

This work is supported by the research management centre, grantsEDW B11-199-0677 and EDW B11-186-0664.

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