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Vague Soft Expert Set and its Application in Decision Making

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ABSTRACT

In this paper, we recall the concept of vague soft expert set theory and its operations. We define the AND and OR operations of vague soft expert set and some properties. We prove the De Morgan's law on vague soft expert set. We then provide an illustrative example of vague soft expert set and its application in decision making.

Keywords: Fuzzy soft expert set theory, vague set, vague soft set, vague soft expert set.

1. Introduction

Fuzzy set was introduced by Zadeh (1965) as a mathematical tool to solve problems and vagueness in everyday life. It has been currently extended by Varnamkhasti and Hassan (2012, 2015) to neurogenetics and neuro-fuzzy inference (Varnamkhasti and Hassan (2013)). Molodtsov (1999) mentioned a soft set as a mathematical way to represent and solve these problems with uncertainties which traditional mathematical tools cannot handle. He has proved several applications of his theory in solving problems in economics, engineering, environment, social science, medical science and business management. Roy and Maji (2007) used this theory to solve some decision-making problems, thus extending decision-making beyond goal programming (Hassan and Ayop (2012), Hassan and Halim (2012), Hassan and Loon (2012), Hassan and Sahrin (2012), Hassan et al. (2012)). Furthermore, soft set has been developed rapidly to multiparametrized soft set (Alkhazaleh et al. (2011), Salleh et al. (2012)), soft intuitionistic fuzzy sets (Alhazaymeh et al. (2012)), probability theory (Zhu and Wen (2010)), trapezoidal fuzzy soft sets (Khalil and Hassan (2016, 2017)), Q-fuzzy soft sets by Adam and Hassan (2014b,e), followed by Q-fuzzy soft matrix by Adam and Hassan (2014c,d) and multi Q-fuzzy soft sets in Adam and Hassan (2014a, 2015, 2016a,b), while soft expert sets was proposed by Alkhazaleh and Salleh (2011). Vague set in Gau and Buehrer (1993) and vague soft set in Xu et al. (2010) was applied by Alhazaymeh and Hassan (2012a,b,c) to decision making, and further explored interval-valued vague soft set (Alhazaymeh and Hassan (2013a,c)), generalized vague soft expert set (Alhazaymeh and Hassan (2013b, 2014a,b,c)), and vague soft set relations and functions (Alhazaymeh and Hassan (2015)), and proceeded by Alzu'bi et al. (2015) and Tahat et al. (2015).

In this paper, we review and extend the concept of vague soft expert set theory (Hassan and Alhazaymeh (2013)) and soft expert sets (Bashir and Salleh (2012), Alkhazaleh and Salleh (2014), Sahin et al. (2015), Selvachandran and Salleh (2015), Al-Quran and Hassan (2016), Qayyum et al. (2016)) to further illustrate an application of this theory in decision making. The concept of vague soft expert set is explored as a more effective and useful mathematical tool, which is a combination between a vague set and a soft expert set, where the user can know the opinion of all experts in one model with a single operation. The organization of this paper is as follows: In Section 2 basic notions about vague soft expert set is reviewed. Section 3 discusses the AND and OR operations on vague soft expert set. We also study some De Morgan's law based on these operations. In Section 4 we discuss the propositions on vague soft expert set and some examples are elaborated. In Section 6 we present the application based on vague soft expert set. The last section summarizes all the contributions and points out future research work.

2. Preliminaries

In this section, we recall some basic notions related to vague soft expert set theory.

Let U be a universal, E be a set of parameters, X a set of expert (agents), and $O = \{1 = agree, 0 = disagree\}$ a set of opinions. Let $Z = E \times X \times O$ and $A \subset Z$.

Definition 2.1. *Alkhazaleh and Salleh (2011). A pair (F, A) is called a soft expert set over U, where F is a mapping given by*

$$F : A \rightarrow P(U)$$

where $P(U)$ denotes the power set of U.

Definition 2.2. *(Alkhazaleh and Salleh (2011)). An agree-soft expert set $(F, A)_1$ over U is a soft expert subset from (F, A) defined as follows:*

$$(F, A)_1 \quad = \quad \{F_1(\alpha) : \alpha \in E \times X \times \{1\}\}.$$

Definition 2.3. *(Alkhazaleh and Salleh (2011)). A disagree-soft expert set $(F, A)_0$ over U is a soft expert subset from (F, A) defined as follows:*

$$(F, A)_0 \quad = \quad \{F_0(\alpha) : \alpha \in E \times X \times \{0\}\}.$$

Definition 2.4. *(Hassan and Alhazaymeh (2013)). A pair (F, A) is called vague soft expert set over U, where F is a mapping given by*

$$F : A \longrightarrow V^U$$

where V^U denotes the set of all vague subsets of U.

Definition 2.5. *(Hassan and Alhazaymeh (2013)). For two vague soft expert sets (F, A) and (G, B) over U, (F, A) is called a vague soft expert subset of (G, B) if*
a. $A \subseteq B$,
b. $\forall \varepsilon \in A, F(\varepsilon)$ is a vague subset of $G(\varepsilon)$.

This relationship is denoted by $(F, A) \subseteq (G, B)$. In this case (G, B) is called a vague soft expert superset of (F, A).

Definition 2.6. *(Hassan and Alhazaymeh (2013)). Two vague soft expert sets (F, A) and (G, B) over U are said to be equal if (F, A) is a vague soft expert subset of (G, B) and (G, B) is a vague soft expert subset of (F, A).*

Definition 2.7. *(Hassan and Alhazaymeh (2013)). An agree-vague soft expert set $(F, A)_1$ over U is a vague soft expert subset of (F, A) defined as follows:*

$$(F, A)_1 \quad = \quad \{F_1(\alpha) : \alpha \in E \times X \times \{1\}\}.$$

Definition 2.8. *(Hassan and Alhazaymeh (2013)). A disagree-vague soft expert set $(F,A)_0$ over U is a vague soft expert subset of (F,A) defined as follows:*

$$(F,A)_0 \quad = \quad \{F_0(\alpha) : \alpha \in E \times X \times \{0\}\}.$$

Definition 2.9. *(Hassan and Alhazaymeh (2013)). The complement of a vague soft expert set (F,A) is denoted by $(F,A)^c$ and is defined by $(F,A)^c = (F^c, \rceil A)$ where $F^c : A \longrightarrow V^U$ is a mapping given by*

$$F^c(\alpha) \quad = \quad c(F(\alpha)), \forall \alpha \in A,$$

where c is a vague complement.

Definition 2.10. *(Hassan and Alhazaymeh (2013)). The union of two vague soft expert sets (F,A) and (G,B) over U, denoted by $(F,A)\cup(G,B)$, is the vague soft expert set (H,C) where $C = A \cup B$ and $\forall \varepsilon \in C$,*

$$H(\varepsilon) = \begin{cases} F(\varepsilon) & , if \varepsilon \in A - B \\ G(\varepsilon) & , if \varepsilon \in B - A \\ \max\left(F(\varepsilon), G(\varepsilon)\right) & , if \varepsilon \in A \cap B \end{cases}$$

Definition 2.11. *(Hassan and Alhazaymeh (2013)). The intersection of two vague soft expert sets (F,A) and (G,B) over U, denoted by $(F,A)\cap(G,B)$, is the vague soft expert set (H,C) where $C = A \cap B$ and $\forall \varepsilon \in C$,*

$$H(\varepsilon) = \begin{cases} F(\varepsilon) & , if \varepsilon \in A - B \\ G(\varepsilon) & , if \varepsilon \in B - A \\ \min\left(F(\varepsilon), G(\varepsilon)\right) & , if \varepsilon \in A \cap B \end{cases}$$

Example 2.1. *Let*
$A = \{(e_1,p,1),(e_2,p,0),(e_3,p,1),(e_1,q,1),(e_2,q,1),(e_1,r,0),(e_2,r,1),(e_3,r,1)\}$
and
$B = \{(e_1 p,1),(e_2,p,0),(e_1,q,1),(e_2,q,1),(e_1,r,0),(e_2,r,1)\}$.

Suppose (F,A) and (G,B) are two vague soft expert sets over U such that

$(F,A) = \{((e_1,p,1),\{\frac{u_1}{\langle 0.5,0.5\rangle},\frac{u_2}{\langle 0.6,0.8\rangle},\frac{u_3}{\langle 0,0\rangle}\}),((e_1,q,1),\{\frac{u_1}{\langle 0.3,0.6\rangle},\frac{u_2}{\langle 0.9,0.9\rangle},\frac{u_3}{\langle 1,1\rangle}\}),$
$((e_2,q,1),\{\frac{u_1}{\langle 0.3,0.4\rangle},\frac{u_2}{\langle 0.4,0.5\rangle},\frac{u_3}{\langle 0.6,0.8\rangle}\}),((e_2,r,1),\{\frac{u_1}{\langle 0.1,0.1\rangle},\frac{u_2}{\langle 0.8,0.9\rangle},\frac{u_3}{\langle 0.3,0.3\rangle}\}),$
$((e_3,p,1),\{\frac{u_1}{\langle 0.3,0.6\rangle},\frac{u_2}{\langle 0.6,0.6\rangle},\frac{u_3}{\langle 0.5,0.5\rangle}\}),((e_3,r,1),\{\frac{u_1}{\langle 0.1,0.2\rangle},\frac{u_2}{\langle 0.2,0.6\rangle},\frac{u_3}{\langle 0.4,0.6\rangle}\}),$
$((e_1,r,0),\{\frac{u_1}{\langle 0.3,0.7\rangle},\frac{u_2}{\langle 0.1,0.3\rangle},\frac{u_3}{\langle 0.3,0.6\rangle}\}),((e_2,p,0),\{\frac{u_1}{\langle 0.5,0.5\rangle},\frac{u_2}{\langle 0.6,0.8\rangle},\frac{u_3}{\langle 0,0\rangle}\})\}.$

$(G,B) = \{((e_1,p,1),\{\frac{u_1}{\langle 0.3,0.5\rangle},\frac{u_2}{\langle 0.6,0.8\rangle},\frac{u_3}{\langle 0,0\rangle}\}),((e_1,q,1),\{\frac{u_1}{\langle 0.4,0.5\rangle},\frac{u_2}{\langle 0.9,0.9\rangle},\frac{u_3}{\langle 1,1\rangle}\}),$
$((e_2,q,1),\{\frac{u_1}{\langle 0.4,0.4\rangle},\frac{u_2}{\langle 0.5,0.5\rangle},\frac{u_3}{\langle 0.7,0.7\rangle}\}),((e_2,r,1),\{\frac{u_1}{\langle 0.1,0.1\rangle},\frac{u_2}{\langle 0.9,0.9\rangle},\frac{u_3}{\langle 0.4,0.4\rangle}\}),$
$((e_1,r,0),\{\frac{u_1}{\langle 0.4,0.5\rangle},\frac{u_2}{\langle 0.2,0.2\rangle},\frac{u_3}{\langle 0.4,0.4\rangle}\}),((e_2,p,0),\{\frac{u_1}{\langle 0.5,0.5\rangle},\frac{u_2}{\langle 0.7,0.7\rangle},\frac{u_3}{\langle 0,0\rangle}\})\}.$

By using basic vague union (maximum) we have $(F,A) \cup (G,B) = (H,C)$ where

$(H,C) = \{((e_1,p,1),\{\frac{u_1}{\langle 0.5,0.5\rangle},\frac{u_2}{\langle 0.6,0.8\rangle},\frac{u_3}{\langle 0,0\rangle}\}),((e_1,q,1),\{\frac{u_1}{\langle 0.4,0.5\rangle},\frac{u_2}{\langle 0.9,0.9\rangle},\frac{u_3}{\langle 1,1\rangle}\}),$
$((e_2,q,1),\{\frac{u_1}{\langle 0.4,0.4\rangle},\frac{u_2}{\langle 0.5,0.5\rangle},\frac{u_3}{\langle 0.7,0.7\rangle}\}),((e_2,r,1),\{\frac{u_1}{\langle 0.1,0.1\rangle},\frac{u_2}{\langle 0.9,0.9\rangle},\frac{u_3}{\langle 0.4,0.4\rangle}\}),$
$((e_3,p,1),\{\frac{u_1}{\langle 0.3,0.6\rangle},\frac{u_2}{\langle 0.6,0.4\rangle},\frac{u_3}{\langle 0.5,0.5\rangle}\}),((e_3,r,1),\{\frac{u_1}{\langle 0.1,0.2\rangle},\frac{u_2}{\langle 0.2,0.6\rangle},\frac{u_3}{\langle 0.4,0.6\rangle}\}),$
$((e_1,r,0),\{\frac{u_1}{\langle 0.4,0.5\rangle},\frac{u_2}{\langle 0.2,0.2\rangle},\frac{u_3}{\langle 0.4,0.4\rangle}\}),((e_2,p,0),\{\frac{u_1}{\langle 0.5,0.5\rangle},\frac{u_2}{\langle 0.7,0.7\rangle},\frac{u_3}{\langle 0,0\rangle}\})\}.$

Example 2.2. *Consider Example 2.1 By using basic vague intersection (minimum) we have $(F,A) \cap (G,B) = (H,C)$ where*

$(H,C) = \{((e_1,p,1),\{\frac{u_1}{\langle 0.3,0.5\rangle},\frac{u_2}{\langle 0.6,0.8\rangle},\frac{u_3}{\langle 0,0\rangle}\}),((e_1,q,1),\{\frac{u_1}{\langle 0.3,0.6\rangle},\frac{u_2}{\langle 0.9,0.9\rangle},\frac{u_3}{\langle 1,1\rangle}\}),$
$((e_2,q,1),\{\frac{u_1}{\langle 0.3,0.4\rangle},\frac{u_2}{\langle 0.4,0.5\rangle},\frac{u_3}{\langle 0.6,0.8\rangle}\}),((e_2,r,1),\{\frac{u_1}{\langle 0.1,0.1\rangle},\frac{u_2}{\langle 0.5,0.9\rangle},\frac{u_3}{\langle 0.3,0.4\rangle}\}),$
$((e_3,p,1),\{\frac{u_1}{\langle 0.3,0.6\rangle},\frac{u_2}{\langle 0.6,0.6\rangle},\frac{u_3}{\langle 0.5,0.5\rangle}\}),((e_3,r,1),\{\frac{u_1}{\langle 0.1,0.2\rangle},\frac{u_2}{\langle 0.2,0.6\rangle},\frac{u_3}{\langle 0.4,0.5\rangle}\}),$
$((e_1,r,0),\{\frac{u_1}{\langle 0.3,0.7\rangle},\frac{u_2}{\langle 0.1,0.3\rangle},\frac{u_3}{\langle 0.3,0.6\rangle}\}),((e_2,p,0),\{\frac{u_1}{\langle 0.3,0.6\rangle},\frac{u_2}{\langle 0.6,0.7\rangle},\frac{u_3}{\langle 0,0.5\rangle}\})\}.$

3. AND and OR operations

In this section we define the AND and OR operations of vague soft expert sets and study their properties using a few examples.

Let U be a universal, E be a set of parameters, X a set of expert (agents), and $O = \{1 = agree, 0 = disagree\}$ a set of opinions. Let $Z = E \times X \times O$ and $A \subset Z$.

Definition 3.1. *If (F,A) and (G,B) are two vague soft expert sets over U then "(F,A) AND (G,B)" denoted by $(F,A) \wedge (G,B)$, is defined by*

$$(F,A) \wedge (G,B) = (H, A \times B)$$

where $H(\alpha,\beta) = F(\alpha) \cap G(\beta)\ \forall \alpha,\beta \in A \times B$ and $\cap$ the vague soft expert intersection.

Example 3.1. *Suppose that a company produces new types of products and wants to take the opinion of some experts about these products. Let $U = \{u_1,u_2,u_3\}$ be a set of products, $E = \{e_1,e_2,e_3\}$ a set of decision parameters where $e_i = \{1,2,3\}$ denotes the parameters "easy to use","quality" and "cheap". Let $\{p,q,r\}$ be a set of experts. Suppose that*

$A = \{(e_1,p,1),(e_3,p,1),(e_3,r,1),(e_2,p,0)\}$ *and*
$B = \{(e_1,p,1),(e_3,p,1),(e_2,p,0)\}$.

Suppose (F,A) and (G,B) are two vague soft expert sets over U such that
$(F,A) =$
$$\left\{\left((e_1,p,1),\left\{\frac{u_1}{\langle 0.5,0.5\rangle},\frac{u_2}{\langle 0.6,0.8\rangle},\frac{u_3}{\langle 0,0\rangle}\right\}\right),\left((e_3,p,1),\left\{\frac{u_1}{\langle 0.3,0.6\rangle},\frac{u_2}{\langle 0.6,0.6\rangle},\frac{u_3}{\langle 0.5,0.5\rangle}\right\}\right),\right.$$
$$\left.\left((e_3,r,1),\left\{\frac{u_1}{\langle 0.3,0.7\rangle},\frac{u_2}{\langle 0.1,0.3\rangle},\frac{u_3}{\langle 0.3,0.6\rangle}\right\}\right),\left((e_2,p,0),\left\{\frac{u_1}{\langle 0.5,0.5\rangle},\frac{u_2}{\langle 0.6,0.8\rangle},\frac{u_3}{\langle 0,0\rangle}\right\}\right)\right\},$$

and

$$(G,B)=\Bigg\{\Big((e_1,p,1),\Big\{\tfrac{u_1}{\langle 0.4,0.6\rangle},\tfrac{u_2}{\langle 0.7,0.7\rangle},\tfrac{u_3}{\langle 1,1\rangle}\Big\}\Big),$$
$$\Big((e_3,p,1),\Big\{\tfrac{u_1}{\langle 0.4,0.4\rangle},\tfrac{u_2}{\langle 0.3,0.7\rangle},\tfrac{u_3}{\langle 0.4,0.7\rangle}\Big\}\Big),$$
$$\Big((e_2,p,0),\Big\{\tfrac{u_1}{\langle 0.4,0.7\rangle},\tfrac{u_2}{\langle 0.6,0.7\rangle},\tfrac{u_3}{\langle 0,0\rangle}\Big\}\Big)\Bigg\}.$$

Then

$$(F,A)\wedge(G,B)=(H,A\times B)=$$
$$\Bigg\{\Big(((e_1,p,1),(e_1,p,1)),\Big\{\tfrac{u_1}{\langle 0.4,0.6\rangle},\tfrac{u_2}{\langle 0.6,0.8\rangle},\tfrac{u_3}{\langle 0,1\rangle}\Big\}\Big),$$
$$\Big(((e_1,p,1),(e_3,p,1)),\Big\{\tfrac{u_1}{\langle 0.4,0.5\rangle},\tfrac{u_2}{\langle 0.3,0.8\rangle},\tfrac{u_3}{\langle 0,0.7\rangle}\Big\}\Big),$$
$$\Big(((e_1,p,1),(e_2,p,0)),\Big\{\tfrac{u_1}{\langle 0.4,0.7\rangle},\tfrac{u_2}{\langle 0.6,0.8\rangle},\tfrac{u_3}{\langle 0,0\rangle}\Big\}\Big),$$
$$\Big(((e_3,p,1),(e_1,p,1)),\Big\{\tfrac{u_1}{\langle 0.3,0.6\rangle},\tfrac{u_2}{\langle 0.6,0.6\rangle},\tfrac{u_3}{\langle 0.5,1\rangle}\Big\}\Big),$$
$$\Big(((e_3,p,1),(e_3,p,1)),\Big\{\tfrac{u_1}{\langle 0.3,0.6\rangle},\tfrac{u_2}{\langle 0.3,0.7\rangle},\tfrac{u_3}{\langle 0.4,0.7\rangle}\Big\}\Big),$$
$$\Big(((e_3,p,1),(e_2,p,0)),\Big\{\tfrac{u_1}{\langle 0.3,0.7\rangle},\tfrac{u_2}{\langle 0.6,0.7\rangle},\tfrac{u_3}{\langle 0,0.5\rangle}\Big\}\Big),$$
$$\Big(((e_3,r,1),(e_1,p,1)),\Big\{\tfrac{u_1}{\langle 0.3,0.7\rangle},\tfrac{u_2}{\langle 0.1,0.7\rangle},\tfrac{u_3}{\langle 0.3,1\rangle}\Big\}\Big),$$
$$\Big(((e_3,r,1),(e_3,p,1)),\Big\{\tfrac{u_1}{\langle 0.3,0.7\rangle},\tfrac{u_2}{\langle 0.3,0.7\rangle},\tfrac{u_3}{\langle 0.3,0.7\rangle}\Big\}\Big),$$
$$\Big(((e_3,r,1),(e_2,p,0)),\Big\{\tfrac{u_1}{\langle 0.3,0.7\rangle},\tfrac{u_2}{\langle 0.1,0.7\rangle},\tfrac{u_3}{\langle 0,0.6\rangle}\Big\}\Big),$$
$$\Big(((e_2,p,0),(e_1,p,1)),\Big\{\tfrac{u_1}{\langle 0.4,0.6\rangle},\tfrac{u_2}{\langle 0.6,0.8\rangle},\tfrac{u_3}{\langle 0,1\rangle}\Big\}\Big),$$
$$\Big(((e_2,p,0),(e_3,p,1)),\Big\{\tfrac{u_1}{\langle 0.4,0.5\rangle},\tfrac{u_2}{\langle 0.3,0.8\rangle},\tfrac{u_3}{\langle 0,0.7\rangle}\Big\}\Big),$$
$$\Big(((e_2,p,0),(e_2,p,0)),\Big\{\tfrac{u_1}{\langle 0.4,0.7\rangle},\tfrac{u_2}{\langle 0.6,0.8\rangle},\tfrac{u_3}{\langle 0,0\rangle}\Big\}\Big)\Bigg\}.$$

Definition 3.2. *If (F,A) and (G,B) are two vague soft expert sets over U then "(F,A) OR (G,B)" denoted by $(F,A)\vee(G,B)$ is defined by*

$$(F,A)\vee(G,B)=(H,A\times B)$$

where $H(\alpha,\beta)=F(\alpha)\cup G(\beta)$ $\forall\alpha,\beta\in A\times B$ and $\cup$ the vague soft expert union.

Example 3.2. *Consider Example 3.1. Let*

$$A = \{(e_1,p,1),(e_3,p,1),(e_3,r,1),(e_2,p,0)\} \text{ and } B = \{(e_1,p,1),(e_3,p,1),(e_2,p,0)\}.$$

Suppose (F,A) *and* (G,B) *are two vague soft expert sets over* U *such that*

$$(F,A) = \left\{ \left((e_1,p,1), \left\{\frac{u_1}{\langle 0.5,0.5\rangle}, \frac{u_2}{\langle 0.6,0.8\rangle}, \frac{u_3}{\langle 0,0\rangle}\right\}\right), \left((e_3,p,1), \left\{\frac{u_1}{\langle 0.3,0.6\rangle}, \frac{u_2}{\langle 0.6,0.6\rangle}, \frac{u_3}{\langle 0.5,0.5\rangle}\right\}\right),\right.$$
$$\left.\left((e_3,r,1), \left\{\frac{u_1}{\langle 0.3,0.7\rangle}, \frac{u_2}{\langle 0.1,0.3\rangle}, \frac{u_3}{\langle 0.3,0.6\rangle}\right\}\right), \left((e_2,p,0), \left\{\frac{u_1}{\langle 0.5,0.5\rangle}, \frac{u_2}{\langle 0.6,0.8\rangle}, \frac{u_3}{\langle 0,0\rangle}\right\}\right)\right\},$$

and

$$(G,B) = \left\{ \left((e_1,p,1), \left\{\frac{u_1}{\langle 0.4,0.6\rangle}, \frac{u_2}{\langle 0.7,0.7\rangle}, \frac{u_3}{\langle 1,1\rangle}\right\}\right), \left((e_3,p,1), \left\{\frac{u_1}{\langle 0.4,0.4\rangle}, \frac{u_2}{\langle 0.3,0.7\rangle}, \frac{u_3}{\langle 0.4,0.7\rangle}\right\}\right),\right.$$
$$\left.\left((e_2,p,0), \left\{\frac{u_1}{\langle 0.4,0.7\rangle}, \frac{u_2}{\langle 0.6,0.7\rangle}, \frac{u_3}{\langle 0,0\rangle}\right\}\right)\right\}.$$

Then $(F,A)\bigvee(G,B) = (H, A\times B) =$

$$\left\{ \left(((e_1,p,1),(e_1,p,1)), \left\{\frac{u_1}{\langle 0.5,0.5\rangle}, \frac{u_2}{\langle 0.7,0.8\rangle}, \frac{u_3}{\langle 1,0\rangle}\right\}\right),\right.$$
$$\left(((e_1,p,1),(e_3,p,1)), \left\{\frac{u_1}{\langle 0.5,0.4\rangle}, \frac{u_2}{\langle 0.6,0.7\rangle}, \frac{u_3}{\langle 0.4,0\rangle}\right\}\right),$$
$$\left(((e_1,p,1),(e_2,p,0)), \left\{\frac{u_1}{\langle 0.4,0.7\rangle}, \frac{u_2}{\langle 0.6,0.7\rangle}, \frac{u_3}{\langle 0,0\rangle}\right\}\right),$$
$$\left(((e_3,p,1),(e_1,p,1)), \left\{\frac{u_1}{\langle 0.3,0.6\rangle}, \frac{u_2}{\langle 0.6,0.6\rangle}, \frac{u_3}{\langle 0.5,0.5\rangle}\right\}\right),$$
$$\left(((e_3,p,1),(e_3,p,1)), \left\{\frac{u_1}{\langle 0.4,0.4\rangle}, \frac{u_2}{\langle 0.6,0.6\rangle}, \frac{u_3}{\langle 1,0.5\rangle}\right\}\right),$$
$$\left(((e_3,p,1),(e_2,p,0)), \left\{\frac{u_1}{\langle 0.4,0.4\rangle}, \frac{u_2}{\langle 0.6,0.6\rangle}, \frac{u_3}{\langle 0.5,0\rangle}\right\}\right),$$
$$\left(((e_3,r,1),(e_1,p,1)), \left\{\frac{u_1}{\langle 0.4,0.6\rangle}, \frac{u_2}{\langle 0.7,0.3\rangle}, \frac{u_3}{\langle 1,0.6\rangle}\right\}\right),$$
$$\left(((e_3,r,1),(e_3,p,1)), \left\{\frac{u_1}{\langle 0.4,0.4\rangle}, \frac{u_2}{\langle 0.3,0.3\rangle}, \frac{u_3}{\langle 0.4,0.6\rangle}\right\}\right),$$
$$\left(((e_3,r,1),(e_2,p,0)), \left\{\frac{u_1}{\langle 0.4,0.7\rangle}, \frac{u_2}{\langle 0.6,0.3\rangle}, \frac{u_3}{\langle 0.3,0\rangle}\right\}\right),$$
$$\left(((e_2,p,0),(e_1,p,1)), \left\{\frac{u_1}{\langle 0.4,0.4\rangle}, \frac{u_2}{\langle 0.7,0.3\rangle}, \frac{u_3}{\langle 1,0.7\rangle}\right\}\right),$$
$$\left(((e_2,p,0),(e_3,p,1)), \left\{\frac{u_1}{\langle 0.4,0.4\rangle}, \frac{u_2}{\langle 0.6,0.7\rangle}, \frac{u_3}{\langle 0.4,0.7\rangle}\right\}\right),$$
$$\left.\left(((e_2,p,0),(e_3,p,1)), \left\{\frac{u_1}{\langle 0.4,0.4\rangle}, \frac{u_2}{\langle 0.7,0.3\rangle}, \frac{u_3}{\langle 0.4,0\rangle}\right\}\right)\right\}.$$

4. De Morgan's Law on Vague Soft Expert Sets

In this section we give some De Morgan's law on union and intersection of vague soft expert sets and we prove the De Morgan's law on vague soft expert sets. De Morgan's law and its variants have been widely used in soft sets such as by Adam and Hassan (2014b,e) on Q-fuzzy soft sets, multi Q-fuzzy soft sets (Adam and Hassan (2015)), and neutrosophic vague soft expert set (Al-Quran and Hassan (2016)).

Proposition 4.1. *If* (F, A) *and* (G, B) *are two vague soft expert sets over* U, *then*

1. $((F, A) \wedge (G, B))^c = (F, A)^c \vee (G, B)^c$
2. $((F, A) \vee (G, B))^c = (F, A)^c \wedge (G, B)^c$

Proof. 1. From the right hand side, we have

$$\begin{aligned} (F, A)^c \vee (G, B)^c &= ((F^c, \rceil A) \vee (G^c, \rceil B)) \\ &= (J, (\rceil A \times \rceil B)), where J(\rceil\alpha, \rceil\beta) = F^c(\rceil\alpha) \cup G^c(\rceil\beta). \\ &= (J, \rceil(A \times B)). \end{aligned}$$

Here $\rceil(A \times B)$ means not in A and not in B at the same time.
Suppose that $(F, A) \wedge (G, B) = (H, A \times B)$.
Therefore, $((F, A) \wedge (G, B))^c = (H, A \times B)^c = (H^c, \rceil(A \times B))$
Suppose we take $\forall (\alpha, \beta) \in (A \times B)$. We therefore have
$H^c(\rceil\alpha, \rceil\beta) = U - H(\alpha, \beta),$
$= U - [F(\alpha) \bigcap G(\beta)]$
$= [U - F(\alpha)] \bigcup [U - G(\beta)]$
$= [F^c(\rceil\alpha)] \bigcup [G^c(\rceil\beta)]$
$= J(\rceil\alpha, \rceil\beta)$
Hence $((F, A) \wedge (G, B))^c = (J, \rceil(A \times B))$.

From the discussion above, we thus have
$((F, A) \wedge (G, B))^c = (F, A)^c \vee (G, B)^c$.

2. From the right hand side, we have

$$\begin{aligned} (F, A)^c \wedge (G, B)^c &= ((F^c, \rceil A) \wedge (G^c, \rceil B)) \\ &= (K, (\rceil A \times \rceil B)), where K(\rceil\alpha, \rceil\beta) = F^c(\rceil\alpha) \bigcap G^c(\rceil\beta), \\ &= (K, \rceil(A \times B)). \end{aligned}$$

Suppose that $(F, A) \vee (G, B) = (H, A \times B)$.
Therefore, $((F, A) \vee (G, B))^c = (H, A \times B)^c = (H^c, \rceil(A \times B))$.

Suppose we take $\forall(\alpha,\beta)\in(A\times B)$. We therefore have

$$\begin{aligned}H^c(\rceil\alpha,\rceil\beta) &= U - H(\alpha,\beta),\\ &= U-[F(\alpha)\bigcup G(\beta)]\\ &= [U-F(\alpha)]\bigcap[U-G(\beta)]\\ &= [F^c(\rceil\alpha)]\bigcap[G^c(\rceil\beta)]\\ &= K(\rceil\alpha,\rceil\beta).\end{aligned}$$

Hence $((F,A)\vee(G,B))^c = (K,\rceil(A\times B))$.

From the discussion above, we thus have
$((F,A)\vee(G,B))^c = (F,A)^c\wedge(G,B)^c$.

□

5. Some propositions on vague soft sets

Proposition 5.1. *If (F,A) , (G,B) and (H,C) are three vague soft expert sets over U, then*

1. $(F,A)\cap((G,B)\cap(H,C)) = ((F,A)\cap(G,B))\cap(H,C)$,
2. $(F,A)\cap(F,A) = (F,A)$.

Proof. 1. We need to prove that
$(F,A)\cap((G,B)\cap(H,C)) = ((F,A)\cap(G,B))\cap(H,C)$.

By using the Definition 2.11, we have

$$((G,B)\cap(H,C))(\varepsilon) = \begin{cases} G(\varepsilon), & \text{if } \varepsilon\in B-C\\ H(\varepsilon), & \text{if } \varepsilon\in C-B\\ \min(G(\varepsilon)\cap H(\varepsilon)), & \text{if } \varepsilon\in B\cap C.\end{cases}$$

We consider the case when $\varepsilon\in B\cap C$ as the other cases are trivial. Thus

$$(G,B)\cap(H,C)(\varepsilon) = (G(\varepsilon)\cap H(\varepsilon), B\cup C).$$

We also consider here the case when $\varepsilon\in A$ as the other cases are trivial. Thus

$$\begin{aligned}(F,A)\cap((G,B)\cap(H,C))(\varepsilon) &= (F(\varepsilon)\cap(G(\varepsilon)\cap H(\varepsilon)), A\cup(B\cup C))\\ &= ((F(\varepsilon)\cap G(\varepsilon))\cap H(\varepsilon),(A\cup B)\cup C)\\ &= ((F,A)\cap(G,B))\cap(H,C)(\varepsilon).\end{aligned}$$

2. The proof is straightforward.

□

Proposition 5.2. *If* (F, A) , (G, B) *and* (H, C) *are three vague soft expert sets over* U*, then*

1. $(F, A) \cup ((G, B) \cap (H, C)) = ((F, A) \cup (G, B)) \cap ((F, A) \cup (H, C))$,
2. $(F, A) \cap ((G, B) \cup (H, C)) = ((F, A) \cap (G, B)) \cup ((F, A) \cap (H, C))$.

Proof. 1. We need to prove that
$(F, A) \cup ((G, B) \cap (H, C)) = ((F, A) \cup (G, B)) \cap ((F, A) \cup (H, C))$,

By using the Definitions 2.10 and 2.11, we have

$$((G, B) \cap (H, C))(\varepsilon) = \begin{cases} G(\varepsilon), & \text{if } \varepsilon \in B - C \\ H(\varepsilon), & \text{if } \varepsilon \in C - B \\ \max(G(\varepsilon) \cap H(\varepsilon)), & \text{if } \varepsilon \in B \cap C. \end{cases}$$

We consider the case when $\varepsilon \in B \cap C$ as the other cases are trivial, then we have

$$(G, B) \cap (H, C)(\varepsilon) = (G(\varepsilon) \cap H(\varepsilon), B \cup C).$$

We also consider here the case when $\varepsilon \in A$ as the other cases are trivial, then we have

$$(F, A) \cup ((G, B) \cap (H, C))(\varepsilon) = (F(\varepsilon) \cup (G(\varepsilon) \cap H(\varepsilon)), A \cup (B \cup C))$$

$$= ((F(\varepsilon) \cup G(\varepsilon)), A \cup B) \cap (F(\varepsilon) \cup H(\varepsilon), A \cup C)$$

$$= ((F, A) \cup (G, B)) \cap ((F, A) \cup (H, C))(\varepsilon).$$

2. We want to prove that
$(F, A) \cap ((G, B) \cup (H, C)) = ((F, A) \cap (G, B)) \cup ((F, A) \cap (H, C))$,

By using the Definitions 2.10 and 2.11, we have

$$((G, B) \cup (H, C))(\varepsilon) = \begin{cases} G(\varepsilon), & \text{if } \varepsilon \in B - C \\ H(\varepsilon), & \text{if } \varepsilon \in C - B \\ \min(G(\varepsilon) \cup H(\varepsilon)), & \text{if } \varepsilon \in B \cap C. \end{cases}$$

We consider the case when $\varepsilon \in B \cap C$ as the other cases are trivial, then we have

$$(G, B) \cup (H, C)(\varepsilon) = (G(\varepsilon) \cup H(\varepsilon), B \cup C).$$

We also consider here the case when $\varepsilon \in A$ as the other cases are trivial, then we have

$$(F, A) \cap ((G, B) \cup (H, C))(\varepsilon) = (F(\varepsilon) \cap (G(\varepsilon) \cup H(\varepsilon)), A \cup (B \cup C))$$

$$= ((F(\varepsilon) \cap G(\varepsilon)), A \cup B) \cup (F(\varepsilon) \cap H(\varepsilon), A \cup C)$$
$$= ((F, A) \cap (G, B)) \cup ((F, A) \cap (H, C))(\varepsilon).$$

□

Proposition 5.3. *Let (F, A), (G, B) and (H, C) be vague soft expert sets. Then*

1. $(F, A) \vee ((G, B) \wedge (H, C)) = ((F, A) \vee (G, B)) \wedge ((F, A) \vee (H, C))$.
2. $(F, A) \wedge ((G, B) \vee (H, C)) = ((F, A) \wedge (G, B)) \vee ((F, A) \wedge (H, C))$.

Proof. 1. For any $\alpha \in A$, $\beta \in B$ and $\gamma \in C$ and by using Proposition 5.2 (a) and Definitions 2.10 and 2.11, we have

$$\begin{aligned}(F, A) \vee ((G, B) \wedge (H, C)) &= (F, A) \cup ((G, B) \cap (H, C)) \\ &= ((F, A) \cup (G, B)) \cap ((F, A) \cup (H, C)) \\ &= ((F, A) \vee (G, B)) \wedge ((F, A) \vee (H, C)).\end{aligned}$$

Hence $(F, A) \vee ((G, B) \wedge (H, C)) = ((F, A) \vee (G, B)) \wedge ((F, A) \vee (H, C))$.

2. For any $\alpha \in A$, $\beta \in B$ and $\gamma \in C$ and by using Proposition 5.2 (b) and Definitions 2.10 and 2.11, we have

$$\begin{aligned}(F, A) \wedge ((G, B) \vee (H, C)) &= (F, A) \cap ((G, B) \cup (H, C)) \\ &= ((F, A) \cap (G, B)) \cup ((F, A) \cap (H, C)) \\ &= ((F, A) \wedge (G, B)) \vee ((F, A) \wedge (H, C)).\end{aligned}$$

Hence $(F, A) \wedge ((G, B) \vee (H, C)) = ((F, A) \wedge (G, B)) \vee ((F, A) \wedge (H, C))$.

□

6. An application of vague soft expert set in decision making

Maji and Roy (2002) applied the theory of soft sets to solve a decision making problem using rough mathematics. In this section, we present an application of vague soft expert set theory in a decision making problem based on the Definitions 2.7 and 2.8. The problem we consider is as below.

Assume that a company wants to fill a position. There are three candidates who form the universe $U = \{u_1, u_2, u_3\}$. The hiring committee considers a set of parameters, $E = \{e_1, e_2, e_3\}$, where the parameters $e_i (i = 1, 2, 3)$ stand for "experience", "computer knowledge" and "speak fluently" respectively. Let

$X = \{p, q, r\}$ be a set of experts (committee members). After a serious discussion the committee constructs the following vague soft expert set.

$$(F,Z) = \{((e_1,p,1),\{\frac{u_1}{\langle 0.5,0.5\rangle},\frac{u_2}{\langle 0.6,0.8\rangle},\frac{u_3}{\langle 0,0\rangle}\}),((e_1,q,1),\{\frac{u_1}{\langle 0.3,0.6\rangle},\frac{u_2}{\langle 0.9,0.9\rangle},\frac{u_3}{\langle 1,1\rangle}\}),$$
$$((e_1,r,1),\{\frac{u_1}{\langle 0.4,0.8\rangle},\frac{u_2}{\langle 0.2,0.2\rangle},\frac{u_3}{\langle 0.9,0.9\rangle}\}),((e_2,p,1),\{\frac{u_1}{\langle 0.2,0.8\rangle},\frac{u_2}{\langle 0.4,0.7\rangle},\frac{u_3}{\langle 0.5,0.6\rangle}\}),$$
$$((e_2,q,1),\{\frac{u_1}{\langle 0.3,0.4\rangle},\frac{u_2}{\langle 0.4,0.5\rangle},\frac{u_3}{\langle 0.6,0.8\rangle}\}),((e_2,r,1),\{\frac{u_1}{\langle 0.1,0.1\rangle},\frac{u_2}{\langle 0.8,0.9\rangle},\frac{u_3}{\langle 0.3,0.4\rangle}\})\},$$
$$((e_3,p,1),\{\frac{u_1}{\langle 0.3,0.6\rangle},\frac{u_2}{\langle 0.6,0.6\rangle},\frac{u_3}{\langle 0.5,0.5\rangle}\}),((e_3,q,1),\{\frac{u_1}{\langle 0.1,0.2\rangle},\frac{u_2}{\langle 0.2,0.6\rangle},\frac{u_3}{\langle 0.4,0.6\rangle}\}),$$
$$((e_3,r,1),\{\frac{u_1}{\langle 0.7,0.3\rangle},\frac{u_2}{\langle 0.1,0.3\rangle},\frac{u_3}{\langle 0.3,0.6\rangle}\}),((e_1,p,0),\{\frac{u_1}{\langle 0.1,0.1\rangle},\frac{u_2}{\langle 0.8,0.9\rangle},\frac{u_3}{\langle 0.3,0.3\rangle}\}),$$
$$((e_1,q,0),\{\frac{u_1}{\langle 0.3,0.4\rangle},\frac{u_2}{\langle 0.4,0.5\rangle},\frac{u_3}{\langle 0.6,0.8\rangle}\}),((e_1,r,0),\{\frac{u_1}{\langle 0.3,0.7\rangle},\frac{u_2}{\langle 0.1,0.3\rangle},\frac{u_3}{\langle 0.3,0.6\rangle}\}),$$
$$((e_2,p,0),\{\frac{u_1}{\langle 0.5,0.5\rangle},\frac{u_2}{\langle 0.6,0.8\rangle},\frac{u_3}{\langle 0,0\rangle}\}),((e_2,q,0),\{\frac{u_1}{\langle 0.3,0.6\rangle},\frac{u_2}{\langle 0.9,0.9\rangle},\frac{u_3}{\langle 1,1\rangle}\}),$$
$$((e_2,r,0),\{\frac{u_1}{\langle 0.4,0.8\rangle},\frac{u_2}{\langle 0.2,0.2\rangle},\frac{u_3}{\langle 0.9,0.9\rangle}\}),((e_3,p,0),\{\frac{u_1}{\langle 0.3,0.6\rangle},\frac{u_2}{\langle 0.6,0.6\rangle},\frac{u_3}{\langle 0.5,0.5\rangle}\}),$$
$$((e_3,q,0),\{\frac{u_1}{\langle 0.1,0.2\rangle},\frac{u_2}{\langle 0.2,0.6\rangle},\frac{u_3}{\langle 0.4,0.6\rangle}\}),((e_3,r,0),\{\frac{u_1}{\langle 0.3,0.7\rangle},\frac{u_2}{\langle 0.1,0.3\rangle},\frac{u_3}{\langle 0.3,0.6\rangle}\}).$$

In Table 1 and Table 2 we present the agree-vague soft expert set and disagree-vague soft expert set respectively.

Table 1: Agree-vague soft expert set

U	u_1	u_2	u_3
(e_1,p)	$\langle 0.5, 0.5\rangle$	$\langle 0.6, 0.8\rangle$	$\langle 0, 0\rangle$
(e_1,q)	$\langle 0.3, 0.6\rangle$	$\langle 0.9, 0.9\rangle$	$\langle 1, 1\rangle$
(e_1,r)	$\langle 0.4, 0.8\rangle$	$\langle 0.2, 0.2\rangle$	$\langle 0.9, 0.9\rangle$
(e_2,p)	$\langle 0.2, 0.8\rangle$	$\langle 0.4, 0.7\rangle$	$\langle 0.5, 0.6\rangle$
(e_2,q)	$\langle 0.3, 0.4\rangle$	$\langle 0.4, 0.5\rangle$	$\langle 0.6, 0.8\rangle$
(e_2,r)	$\langle 0.1, 0.1\rangle$	$\langle 0.8, 0.9\rangle$	$\langle 0.3, 0.4\rangle$
(e_3,p)	$\langle 0.3, 0.6\rangle$	$\langle 0.6, 0.6\rangle$	$\langle 0.5, 0.5\rangle$
(e_3,q)	$\langle 0.1, 0.2\rangle$	$\langle 0.2, 0.6\rangle$	$\langle 0.4, 0.6\rangle$
(e_3,r)	$\langle 0.7, 0.3\rangle$	$\langle 0.1, 0.3\rangle$	$\langle 0.3, 0.6\rangle$
$c_j = \sum_{i=1}^{n}(\bar{u}_{ij} - \underline{u}_{ij})$	$c_1 = -1.4$	$c_2 = -1.3$	$c_3 = -0.9$

Table 2: Disagree-vague soft expert set

U	u_1	u_2	u_3
(e_1, p)	$\langle 0.1, 0.1\rangle$	$\langle 0.8, 0.9\rangle$	$\langle 0.3, 0.3\rangle$
(e_1, q)	$\langle 0.3, 0.4\rangle$	$\langle 0.4, 0.5\rangle$	$\langle 0.6, 0.8\rangle$
(e_1, r)	$\langle 0.3, 0.7\rangle$	$\langle 0.1, 0.3\rangle$	$\langle 0.3, 0.6\rangle$
(e_2, p)	$\langle 0.5, 0.5\rangle$	$\langle 0.6, 0.8\rangle$	$\langle 0, 0\rangle$
(e_2, q)	$\langle 0.3, 0.6\rangle$	$\langle 0.9, 0.9\rangle$	$\langle 1, 1\rangle$
(e_2, r)	$\langle 0.4, 0.8\rangle$	$\langle 0.2, 0.2\rangle$	$\langle 0.9, 0.9\rangle$
(e_3, p)	$\langle 0.3, 0.6\rangle$	$\langle 0.6, 0.6\rangle$	$\langle 0.5, 0.5\rangle$
(e_3, q)	$\langle 0.1, 0.2\rangle$	$\langle 0.2, 0.6\rangle$	$\langle 0.4, 0.6\rangle$
(e_3, r)	$\langle 0.3, 0.7\rangle$	$\langle 0.1, 0.3\rangle$	$\langle 0.3, 0.6\rangle$
$k_j = \sum_{i=1}^{n}(\bar{u}_{ij} - \underline{u}_{ij})$	$k_1 = -2$	$k_2 = -1.2$	$k_3 = -1$

The following algorithm, modified from Alkhazaleh and Salleh (2011) may be followed by the company to fill the position.

1. Input the vague soft expert set (F, Z).
2. Find an agree-vague soft expert set and a disagree-vague soft expert set.
3. Find $c_j = \sum_{i=1}^{n}(\bar{u}_{ij} - \underline{u}_{ij})$ for agree-vague soft expert set.
4. Find $k_j = \sum_{i=1}^{n}(\bar{u}_{ij} - \underline{u}_{ij})$ for disagree-vague soft expert set.
5. Find $s_j = c_j - k_j$.
6. Find m, for which $s_m = max s_j$. Then s_m is the optimal choice object. If m has more than one value, then any one of them could be chosen by the company using its option.

Now we use this algorithm to find the best choice for the company to fill the position. From Table 1 and Table 2 we have the following Table 3:

Table 3: $s_j = c_j - k_j$

i	c_j	k_j	s_j
1	-1.4	-2	0.6
2	-1.3	-1.2	-0.1
3	-0.9	-1	0.1

Then max $s_j = s_1$, so the committee will choose candidate 1 for the job.

In the case of soft expert set, the data sets would have to be binary. The soft expert set would then be defined as follows.

$(F, Z) = \{((e_1, p, 1), \{u_1, u_3\}), ((e_1, q, 1), \{u_2, u_3\}), ((e_1, r, 1), \{u_2, u_3\}),$
$((e_2, p, 1), \{u_3\}), ((e_2, q, 1), \{u_1, u_2, u_3\}), ((e_2, r, 1), \{u_1, u_2, u_3\})\},$
$((e_3, p, 1), \{u_2, u_3\}), ((e_3, q, 1), \{u_1, u_3\}), ((e_3, r, 1), \{u_1, u_2, \}),$
$((e_1, p, 0), \{u_1, u_2, u_3\}), ((e_1, q, 0), \{u_1, u_2, u_3\}), ((e_1, r, 0), \{u_2, u_3\}),$
$((e_2, p, 0), \{u_1, u_2, u_3\}), ((e_2, q, 0), \{u_1, u_2, u_3\}), ((e_2, r, 0), \{u_2, u_3\}),$
$((e_3, p, 0), \{u_1, u_2, u_3\}), ((e_3, q, 0), \{u_1, u_3\}), ((e_3, r, 0), \{u_2, u_3\}).$

The values of s_j would be as in Table 4. The maximum value would be that of s_1 and s_3 . Thus there is a certain hesitancy to choose a candidate to fill up the vacant position. We have thus illustrated that a decision can be clearly

Table 4: $s_j = c_j - k_j$

i	c_j	k_j	s_j
1	5	6	-1
2	6	8	-2
3	8	9	-1

reached using vague soft expert set compared to that of soft expert set without hesitancy.

7. Conclusion

In this paper, the basic concepts of a vague soft expert set are reviewed, and some basic operations on vague soft expert set are given with examples on union and intersection of vague soft expert sets. Operations of AND and OR on vague soft expert sets are illustrated and the concept of De Morgan's law of vague soft expert set is also proven. In addition, this new extension not only provides a significant addition to existing theories for handling uncertainties, but also leads to potential areas of further research and pertinent applications.

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