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Sigmoid Function in the Space of Univalent λ-Pseudo-(p, q)-Derivative Operators Related to Shell-Like Curves Connected with Fibonacci Numbers of Sakaguchi Type Functions

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ABSTRACT

In this work, sigmoid function in the space of univalent λ-pseudo-(p, q)-derivative operators related to shell-like curves connected with Fibonacci number of Sakaguchi type functions have been investigated. The initial coefficient bounds for $|a_2|$ and $|a_3|$ are obtained. The relevant connection to Fekete-Szegö inequalities for the class defined are further determined and some new corollaries are also given.

Keywords: Analytic function, Univalent function, $\lambda-(p, q)-$derivative operator, Subordination, Fibonacci numbers, Shell-like curves.

AMS Subject Classification: 30C45, 30C50, 33E99

1. Introduction

Let Γ denote the class of functions of the form

$$f(z) = z + a_2 z^2 + a_3 z^3 + a_4 z^4 + \cdots \tag{1}$$

which are analytic in the unit disc $\mathbb{E} = \{z \in \mathbb{C} : |z| < 1\}$ and normalized by $f(0) = f'(0) - 1 = 0$. Recall that, the subclasses of the univalent function $\mathcal{S}$ are starlike and convex functions denoted by $\mathcal{S}^*$ and $\mathcal{K}$ which satisfies the geometric conditions $\mathbb{R}\left\{\frac{zf'(z)}{f(z)}\right\} > 0$ and $\mathbb{R}\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > 0$, respectively.
An analytic function f is subordinate to an analytic function g in $\mathbb{E}$, written as $f \prec g$, if there exists an analytic function w defined on $\mathbb{E}$ with $w(0) = 0$ and $|w(z)| < 1$ satisfying $f(z) = g(w(z))$. It follows from Schwarz Lemma that

$$f(z) \prec g(z) \Longleftrightarrow f(0) = g(0) \text{ and } f(\mathbb{E}) \subset g(\mathbb{E}).$$

Duren (1983) gave the definition.
Sokół (1999) defined and investigated the class $\mathcal{SL}$ of shell-like function which is a subclass of $\mathcal{S}^*$ and the geometric condition of the class satisfy

$$\frac{zf'(z)}{f(z)} \prec \tilde{p}(z),$$

where $\tilde{p}(z) = \frac{1+\tau^2 z^2}{1-\tau z-\tau^2 z^2}$ and $\tau = \frac{1-\sqrt{5}}{2} \approx -0.618$. It is noted that $\tilde{p}(z)$ is not univalent in $\mathbb{E}$, but it is univalent in the disc $|z| < \frac{3-\sqrt{5}}{2} \approx 0.38$.
The class $\mathcal{SLM}_\alpha$ were introduced by Dziok et al. (2011) referred to as α-convex functions related to a shell-like curve connected with Fibonacci numbers which satisfy the geometric condition $\alpha\left(1 + \frac{zf''(z)}{f'(z)}\right) + (1-\alpha)\frac{zf'(z)}{f(z)} \prec \bar{p}(z)$ for $\alpha \in [0,1]$ and $\bar{p}(z)$ as define above, respectively. Also, Güney et al. (2018) studied subclasses of bi-univalent functions related to shell-like curves connected with Fibonacci numbers and their interesting results littered everywhere.

By taking $t = \tau z$, Raina and Sokół (2016) showed that

$$\tilde{p}(z) = \frac{1+\tau^2 z^2}{1-\tau z-\tau^2 z^2} = \left(\frac{t+1}{t}\right)\frac{t}{1-t+t^2} =$$

$$\frac{1}{\sqrt{5}}\left(\frac{t+1}{t}\right)\left(\frac{1}{1-(1-\tau)t} - \frac{1}{1-\tau t}\right).$$

Further simplification gives

$$\tilde{p}(z) = 1 + \sum_{n=1}^{\infty}(u_n + u_{n+1})\tau^n z^n$$

where $u_n = \frac{(1-\tau)^n - \tau^n}{\sqrt{5}}$ and $\tau \approx -0.618$ which implies that $u_0 = 0, u_1 = 1, u_{n+2} = u_n + u_{n+1}$ for $n = 0, 1, 2, ...$ Finally, we obtain

$$\tilde{p}(z) = 1+\sum_{n=1}^{\infty} \tilde{p}_n z^n = 1+\tau z+3\tau^2 z^2+4\tau^3 z^3+7\tau^4 z^4+11\tau^5 z^5+\cdots \quad (z \in \mathbb{E}). \tag{2}$$

Recently, Babalola (2016) gave a plausible contribution to the geometric function theory by defining a new subclass λ-pseudo starlike function of order β for $\beta \in (0, 1]$ satisfying the condition

$$\Re\left(\frac{z(f'(z))^\lambda}{f(z)}\right) > \beta, \quad (z \in \mathbb{E}), \tag{3}$$

where $\lambda \geq 1 \in \mathbb{R}$ and denoted by $\mathcal{L}_\lambda(\beta)$. Setting $\lambda = 2$ in (3) gives the product of two functions of bounded turning point and starlike function which satisfies the condition

$$\Re\left\{f'(z)\left(\frac{zf'(z)}{f(z)}\right)\right\} > \beta. \quad (z \in \mathbb{E}).$$

where $\beta \in (0, 1]$.

Frasin (2010) investigated the coefficient inequalities for certain class $\mathcal{S}(\alpha, s, t)$ of Sakaguchi type functions defined by condition

$$\Re\left\{\frac{(s-t)zf'(z)}{f(sz) - f(tz)}\right\} > \alpha, \quad (z \in \mathbb{E}) \tag{4}$$

for some complex numbers s, t with $s \neq t$ and $\alpha \in (0, 1]$.

Varying some parameters in (4), various subclasses of analytic functions studied by Owa et al. (2007), Sakaguchi (1959), Yasar and Yalcin (2012) were obtained. Altinkaya and Yalcin (2015) studied the comprehensive subclass of Sakaguchi type functions and the early few coefficient bounds for the class $\mathcal{S}_\Sigma^\lambda(\beta, s, t, h)$ were obtained.

Olatunji et al. (2017) used (3) and (4) to define a class $\mathcal{L}_\lambda^\beta(s, t, \Phi)$ whose geometric condition satisfies

$$\Re(f'(z))^\lambda\left(\frac{(s-t)z}{f(sz) - f(tz)}\right) > \beta, \quad (z \in \mathbb{E}),$$

where $s, t \in \mathbb{C}, s \neq t, \lambda \geq 1 \in \mathbb{R}, 0 \leq \beta < 1$, and $\Phi(z)$ is the modified sigmoid function $z \in \mathbb{E}$. The first few coefficient bounds were obtained and used to determine the Fekete-Szegö inequalities. Altinkaya and Özkan (2017) and Laxmi and Sharma (2017) also worked on λ-pseudo function and their results can not be ignored.

The theory of (p, q)-derivative operators have played an important role in differential equations, physics, mechanics and so on. Let $0 < q \leq p \leq 1$, then the (p, q)-bracket is defined by

$$[k]_{p,q} = \frac{p^k - q^k}{p - q} \; (q \neq p) \quad \text{and} \quad [k]_{p,p} = kp^{k-1}.$$

We note that $\lim_{q \to p} [k]_{p,q} = [k]_{p,p}$. For $0 < q \leq 1$, q-bracket $[k]_q$ for $k = 0, 1, 2, \dots$ is given by

$$[k]_q = [k]_{1,q} = \frac{1 - q^k}{1 - q} \; (q \neq 1), \quad [k]_1 = [k]_{1,1} = k,$$

which has been used by many researchers to define several subclasses of analytic functions in different perspectives and their interesting results are voluminous to discuss, see for details Ahuja et al. (2018), Altinkaya and Yalcin (2017), Chakrabarti and Jagannathan (1991), Jahangiri (2015), Magesh et al. (2018), Selvaraj et al. (2017) and Tang et al. (2018).
The (p, q)-derivative operator $D_{p,q}$ of a function $f \in \Gamma$ is given by

$$D_{p,q} f(z) = 1 + \sum_{k=2}^{\infty} [k]_{p,q} a_k z^{k-1} \tag{5}$$

which can be easily seen that

$$D_{p,q} f(z) = \frac{f(pz) - f(qz)}{(p - q)z}, \, p \neq q, z \neq 0, (D_{p,q} f)(0) = 1 \, \text{and} \, (D_{p,p} f)(z) = f'(z).$$

For more details, see Güney et al. (2018).

Special function has recently taken the attention of many researchers thesedays because of its wide application in real analysis, differential equations, functional analysis, algebra, topology and so on. It deals with an information process that is inspired by the way nervous system such as brain operates. It consist of large number of interconnected processing elements (neurons) working together to perform a specific task. The function can be categorized into three namely; sigmoid, threshold and ramp functions. Sigmoid function of the form $h(z) = \frac{1}{1+e^{-z}}$ is differentiable and is the most common function among all because of its gradient descendent algorithm. It can be evaluated in different ways, most especially by truncated series expansion.

It has the following characteristics:

(i) it output real numbers between 0 and 1,

(ii) it maps a very large input domain to a small range of output,

(iii) it never looses information

and

(iv) it increases monotonically.

The aforementioned characteristics make sigmoid function to be useful in geometric function theory. See the concepts in Fadipe-Joseph et al. (2013), Murugusundaramoorthy and Janani (2015), Olatunji (2016), Olatunji et al. (2017) and Olatunji et al. (2013).

Inspired by earlier work done by Olatunji et al. (2017), Güney et al. (2018), Raina and Sokół (2016) and Sokół (1999), in this paper, we studied the sigmoid function in the space of univalent λ-pseudo-(p,q)-derivative operators related to shell-like curves connected with Fibonacci number of Sakaguchi type functions. The initial coefficient bounds were determined and used to obtain the Fekete-Szegö inequalities.

For the purpose of our results, the following lemmas and definitions shall be necessary.

Lemma 1.1. *Fadipe-Joseph et al. (2013) Let h be a sigmoid function and*

$$\Phi(z) = 2h(z) = 1 + \sum_{m=1}^{\infty} \frac{(-1)^m}{2^m} \left(\sum_{n=1}^{\infty} \frac{(-1)^n}{n!} z^n \right), \tag{6}$$

then $\Phi(z) \in \mathcal{P}$, $|z| < 1$ where $|\Phi(z)|$ is a modified sigmoid function.

Lemma 1.2. *Fadipe-Joseph et al. (2013) Let h be a sigmoid function and*

$$\Phi_{m,n}(z) = 1 + \sum_{m=1}^{\infty} \frac{(-1)^m}{2^m} \left(\sum_{n=1}^{\infty} \frac{(-1)^n}{n!} z^n \right), \tag{7}$$

then $|\Phi_{m,n}(z)| < 2$.

Lemma 1.3. *If $\Phi z \in P$ and it is starlike, then f is a normalized univalent function of the form (1).*

Taking $m = 1$, Fadipe-Joseph et al. (2013) remarked the following:

Remark 1.1. *$\Phi(z) = 1 + \sum_{n=1}^{\infty} c_n z^n$ where $c_n = \frac{(-1)^n}{2n!}$ then $|c_n| \leq 2$, $n = 1, 2, 3, \cdots$ this result is sharp for each n.*
Let $\mathcal{P}$ be the class of Caratheodory function for which $p(0) = 1$ and $Re\,(p(z)) > 0$, $z \in \mathbb{E}$. It is known that if $p \in \mathcal{P}$ and $p(z) = 1 + p_1 z + p_2 z^2 + \cdots$ $(z \in \mathbb{E})$, then $|p_k| \leq 2\,(k \in \mathbb{N})$ Pommerenke (1975).

Definition 1.1. *Let $\Phi(z) \in \mathcal{P}$ be univalent, for $s \neq b, s, b \in \mathbb{C}$, $\lambda \geq 1$ and $\Phi(z)$ is the modified sigmoid function, a function $f \in \Gamma$ is said to be the class $\mathcal{G}_{p,q}^{\lambda}(\Phi(z), s, b, \bar{p}(z))$ if*

$$\left(D_{p,q} f\right)^{\lambda}(z) \left(\frac{(s-b)z}{f(sz) - f(bz)}\right) \prec \bar{p}(z) = \frac{1 + \tau^2 z^2}{1 - \tau z - \tau^2 z^2}. \tag{8}$$

2. Main Results

In the next session, we shall prove the following results:

Theorem 2.1. *Let $f \in \Gamma$ of the form (1) belong to the class $\mathcal{G}_{p,q}^{\lambda}(\Phi, s, b, \bar{p}(z))$, then*

$$|a_2| \leq \frac{|\tau|}{4(\lambda[2]_{p,q} - (s+b))} \tag{9}$$

and

$$|a_3| \leq \frac{|\tau| \left| \left((3\tau - 1) - \frac{\left(\frac{\lambda(\lambda-1)}{2}[2]_{p,q}^2 + (s+b)^2 - \lambda[2]_{p,q}(s+b)\right)\tau}{(\lambda[2]_{p,q} - (s+b))^2} \right) \right|}{16(\lambda[3]_{p,q} - (s^2 + sb + b^2))}. \tag{10}$$

Proof. Firstly, let $\Phi(z) = 1 + \frac{z}{2} - \frac{z^3}{24} + \frac{z^5}{240} - \frac{z^6}{64} + \frac{779 z^7}{20160} - \cdots$, and $\Phi(z) \prec \bar{p}$. Then, there exist an analytic function u such that $|u(z)| < 1$ in $\mathbb{E}$ and $\Phi(z) = \bar{p}(u(z))$. Therefore, the function

$$\Phi(z) = \frac{1 + u(z)}{1 - u(z)} = 1 + \frac{z}{2} - \frac{z^3}{24} + \frac{z^5}{240} - \frac{z^6}{64} + \frac{779 z^7}{20160} - \cdots. \tag{11}$$

is in the class $\mathcal{P}(0)$. It follows that

$$u(z) = \frac{\Phi(z) - 1}{\Phi(z) + 1} = \frac{z}{4} - \frac{z^2}{16} - \frac{z^3}{192} - \frac{5z^4}{768} - \frac{13 z^5}{15360} + \cdots. \tag{12}$$

In view of (8), (11) and (12), clearly

$$(D_{p,q}f)^{\lambda}(z)\left(\frac{(s-b)z}{f(sz)-f(bz)}\right)=\bar{p}\left(\frac{\Phi(z)-1}{\Phi(z)+1}\right). \tag{13}$$

Hence,

$$\frac{(s-b)z}{f(sz)-f(bz)}=1-(s+b)a_2z+((s+b)^2a_2^2-(s^2+sb+b^2)a_3)z^2+\cdots \tag{14}$$

Using the series expansion of $(D_{p,q}f)(z)$ in (5) and the expansion given by (1), to obtain

$$(\mathcal{D}_{p,q}f)^{\lambda}(z)\left(\frac{(s-b)z}{f(sz)-f(bz)}\right)=1+(\lambda[2]_{p,q}-(s+b))a_2z+$$

$$\left[\left(\frac{\lambda(\lambda-1)}{2}[2]_{p,q}^2+(s+b)^2-\lambda[2]_{p,q}(s+b)\right)a_2^2+(\lambda[3]_{p,q}-(s^2+sb+b^2))a_3\right]z^2+\cdots \tag{15}$$

and the right-hand side of (13) gives

$$\tilde{p}(u(z))=1+\tilde{p}_1\left(\frac{z}{4}-\frac{z^2}{16}-\frac{z^3}{192}-\frac{5z^4}{768}-\frac{13z^5}{15360}+\cdots\right)+$$

$$\tilde{p}_2\left(\frac{z}{4}-\frac{z^2}{16}-\frac{z^3}{192}-\frac{5z^4}{768}-\frac{13z^5}{15360}+\cdots\right)^2$$

$$+\tilde{p}_3\left(\frac{z}{4}-\frac{z^2}{16}-\frac{z^3}{192}-\frac{5z^4}{768}-\frac{13z^5}{15360}+\cdots\right)^3+\cdots \tag{16}$$

From (15) and (16), on equating the coefficients of z and z^2 in (13), to find that

$$(\lambda[2]_{p,q}-(s+b))\,a_2=\frac{\tau}{4} \tag{17}$$

and

$$\left(\frac{\lambda(\lambda-1)}{2}[2]_{p,q}^2+(s+b)^2-\lambda[2]_{p,q}(s+b)\right)a_2^2+$$

$$(\lambda[3]_{p,q}-(s^2+sb+b^2))a_3=\frac{\tau}{16}(3\tau-1). \tag{18}$$

Now, (17) gives

$$a_2=\frac{\tau}{4(\lambda[2]_{p,q}-(s+b))} \tag{19}$$

and

$$a_3=\frac{\tau}{16(\lambda)[3]_{p,q}-(s^2+sb+b^2)}\left((3\tau-1)-\frac{\left(\frac{\lambda(\lambda-1)}{2}[2]_{p,q}^2+(s+b)^2-\lambda[2]_{p,q}(s+b)\tau\right)}{(\lambda[2]_{p,q}-(s+b))^2}\right). \tag{20}$$

which complete the proof of the Theorem 2.1. □

Theorem 2.2. *Let $f \in \Gamma$ of the form (1) belong to the class $\mathcal{G}^{\lambda}_{p,q}(\Phi, s, b, \bar{p}(z))$, then*

$$\left|a_3 - \mu a_2^2\right| \leq$$

$$\frac{|\tau| \left|(3\tau-1)(\lambda[2]_{p,q}-(s+b))^2 - \left(\frac{\lambda(\lambda-1)}{2}[2]^2_{p,q} + (s+b)^2 - \lambda[2]_{p,q}(s+b)\right)\tau - \tau\mu(\lambda[3]_{p,q}-(s^2+sb+b^2))\right|}{16(\lambda[2]_{p,q}-(s+b))^2(\lambda[3]_{p,q}-(s^2+sb+b^2))}, \tag{21}$$

for $\mu \leq 1$.

Proof. $a_3 - \mu a_2^2 =$

$$\frac{\tau}{16(\lambda)[3]_{p,q}-(s^2+sb+b^2)}\left((3\tau-1) - \frac{\left(\frac{\lambda(\lambda-1)}{2}[2]^2_{p,q} + (s+b)^2 - \lambda[2]_{p,q}(s+b)\right)\tau}{(\lambda[2]_{p,q}-(s+b))^2}\right) - \frac{\tau^2\mu}{16(\lambda[2]_{p,q}-(s+b))^2}, \tag{22}$$

further simplification gives

$$= \frac{\tau\left((3\tau-1)(\lambda[2]_{p,q}-(s+b))^2 - \left(\frac{\lambda(\lambda-1)}{2}[2]^2_{p,q} + (s+b)^2 - \lambda[2]_{p,q}(s+b)\right)\tau - \tau\mu(\lambda[3]_{p,q}-(s^2+sb+b^2))\right)}{16(\lambda[2]_{p,q}-(s+b))^2(\lambda[3]_{p,q}-(s^2+sb+b^2))}, \tag{23}$$

finally we have

$$\left|a_3 - \mu a_2^2\right| =$$

$$\frac{|\tau| \left|(3\tau-1)(\lambda[2]_{p,q}-(s+b))^2 - \left(\frac{\lambda(\lambda-1)}{2}[2]^2_{p,q} + (s+b)^2 - \lambda[2]_{p,q}(s+b)\right)\tau - \tau\mu(\lambda[3]_{p,q}-(s^2+sb+b^2))\right|}{16(\lambda[2]_{p,q}-(s+b))^2(\lambda[3]_{p,q}-(s^2+sb+b^2))} \tag{24}$$

as required in (21). □

Taking $\lambda = 1$ in Theorem 2.1 to obtain

Corollary 2.1. *Let $f \in \Gamma$ of the form (1) belong to the class $\mathcal{G}_{p,q}(\Phi, s, b, \bar{p}(z))$, then*

$$|a_2| \leq \frac{|\tau|}{4([2]_{p,q}-(s+b))} \tag{25}$$

and

$$|a_3| \leq \frac{|\tau| \left|\left((3\tau-1) - \frac{\left(\frac{(-1)}{2}[2]^2_{p,q} + (s+b)^2 - [2]_{p,q}(s+b)\right)\tau}{([2]_{p,q}-(s+b))^2}\right)\right|}{16([3]_{p,q}-(s^2+sb+b^2))}. \tag{26}$$

Setting $s = 1$ in corollary 2.1, to obtain

Corollary 2.2. *Let $f \in \Gamma$ of the form (1) belong to the class $\mathcal{G}_{p,q}(\Phi, 1, b, \bar{p}(z))$, then*

$$|a_2| \leq \frac{|\tau|}{4([2]_{p,q} - (1+b))} \tag{27}$$

and

$$|a_3| \leq \frac{|\tau| \left| \left((3\tau - 1) - \frac{\left(\frac{(-1)}{2} [2]_{p,q}^2 + (1+b)^2 - [2]_{p,q}(1+b) \right) \tau}{([2]_{p,q} - (1+b))^2} \right) \right|}{16([3]_{p,q} - (1+b+b^2))}. \tag{28}$$

Putting $b = -1$ in corollary 2.2, to obtain

Corollary 2.3. *Let $f \in \Gamma$ of the form (1) belong to the class $\mathcal{G}_{p,q}(\Phi, 1, -1, \bar{p}(z))$, then*

$$|a_2| \leq \frac{|\tau|}{4([2]_{p,q})} \tag{29}$$

and

$$|a_3| \leq \frac{|\tau| \left| (3\tau - 1) + \frac{\tau}{2} \right|}{16([3]_{p,q} - 1)}. \tag{30}$$

Taking $\lambda = 1$ in Theorem 2.2, to obtain

Corollary 2.4. *Let $f \in \Gamma$ of the form (1) belong to the class $\mathcal{G}_{p,q}(\Phi, s, b, \bar{p}(z))$, then*

$$\left| a_3 - \mu a_2^2 \right| \leq$$

$$\frac{|\tau| \left| (3\tau - 1)([2]_{p,q} - (s+b))^2 - \left(\frac{(-1)}{2} [2]_{p,q}^2 + (s+b)^2 - [2]_{p,q}(s+b) \right) \tau - \tau\mu([3]_{p,q} - (s^2 + sb + b^2)) \right|}{16([2]_{p,q} - (s+b))^2([3]_{p,q} - (s^2 + sb + b^2))} \tag{31}$$

for $\mu \leq 1$.

Setting $s = 1$ in corollary 2.4 to get

Corollary 2.5. *Let $f \in \Gamma$ of the form (1) belong to the class $\mathcal{G}_{p,q}(\Phi, 1, b, \bar{p}(z))$, then*

$$\left| a_3 - \mu a_2^2 \right| \leq$$

$$\frac{|\tau| \left| (3\tau - 1)([2]_{p,q} - (1+b))^2 - \left(\frac{(-1)}{2} [2]_{p,q}^2 + (1+b)^2 - [2]_{p,q}(1+b) \right) \tau - \tau\mu([3]_{p,q} - (1+b+b^2)) \right|}{16([2]_{p,q} - (1+b))^2([3]_{p,q} - (1+b+b^2))}, \tag{32}$$

for $\mu \leq 1$.

Putting $b = -1$ in corollary 2.5, to get

Corollary 2.6. *Let $f \in \Gamma$ of the form (1) belong to the class $\mathcal{G}_{p,q}(\Phi, 1, -1, \bar{p}(z))$, then*

$$\left|a_3 - \mu a_2^2\right| \leq \frac{|\tau|\left|(3\tau-1)([2]_{p,q}\ +\frac{1}{2}[2]_{p,q}^2\tau - \tau\mu([3]_{p,q}-1)\right|}{16[2]_{p,q}^2([3]_{p,q}-1)}, \tag{33}$$

for $\mu \leq 1$.

3. Conclusion

Though, researchers have studied different subclasses of analytic functions associated with shell-like curves connected with Fibonacci numbers but it has not been investigated in terms of sigmoid function and λ-pseudo-(p, q)-derivative operator. The initial coefficient bounds, Fekete-Szegö inequalities and the few corollaries obtained in this paper have not been in existence.

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