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Classification of a Subclass of 10-Dimensional Complex Filiform Leibniz Algebras

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ABSTRACT

This article deals with the isomorphism classes and invariants of filiform Leibniz algebras arising from naturally graded non Lie filiform Leibniz algebras. It has been divided into two subclasses denoted by FLb_n and SLb_n in fixed dimension n. In the paper we will focus on studying of FLb_n. To study the structure of FLb_n one needs to describe its isomorphisms classes. One of the approaches to solve the isomorphism problem has been made earlier by using algebraic invariants. The paper implements the approach to FLb_{10}. The isomorphism criteria by using invariant functions are given. The class FLb_{10} is split into its disjoint subsets. Some of these subset consists of a union of parametric family of orbits while the others are just single orbits. For parametric family of orbit cases, the invariants separating the orbits are given.

Keywords: Isomorphism class, isomorphism criterion, invariant function.

1. Introduction

The theory of Lie algebras is one of the most developed part of modern algebra. Due to a consideration of non antisymmetric version of Lie algebras, Loday (1993) introduced a more general object called Leibniz algebra. If the antisymmetricity is assumed, the identity which the Leibniz algebra satisfies is equivalent to the Jacobi identity. Hence, any Lie algebra is a Leibniz algebra. In order to study a certain class of algebras, it is important to describe the class at least in low-dimensions up to an isomorphism precision. However, to classify all Leibniz algebras in a fixed dimension also is not an easy task. Some difficulties arise even one considers nilpotent Leibniz algebras of dimension 5. Therefore in this work, a special subclass of nilpotent Leibniz algebras, called filiform Leibniz algebras is considered. First the notion of filiform Leibniz algebra has been introduced in Ayupov and Omirov (2001).

The class of all filiform Leibniz algebra structures on n-dimensional vector space is denoted by Lb_n. The set Lb_n is represented as a disjoint union of three subclasses which are invariant with respect to the action of linear group ("base change") $Lb_n = FLb_n \cup SLb_n \cup TLb_n$. It is sufficient to classify each of them separately. The classes of FLb_n and SLb_n have been classified up to dimension $n \leq 9$ (see Obaiys et al. (2010),Rakhimov and Said Husain (2011a),Rakhimov and Said Husain (2011b),Gomez and Omirov (2015) and Kasim et al. (2014)). While the class of filiform Leibniz algebras appearing from the naturally graded filiform Lie algebras denoted by TLb_n has been classified up to dimension $n \leq 8$ (see Rakhimov and Hassan (2011), Abdulkareem et al. (2013) and Abdulkareem et al. (2015)).

As it has been mentioned above in this paper we will deal with non-Lie Leibniz algebras case. This class is split into two subclasses: FLb_n and SLb_n in fixed dimension n. An approach to the classification problem in terms of algebraic invariants in this case has been proposed in Rakhimov and Bekbaev (2010). In this article, the approach of Rakhimov and Bekbaev (2010) is implemented to FLb_{10}. The organization of the article is as follows. The first section contains some facts on filiform Leibniz algebras. In the second section, the isomorphism criterion for FLb_{10} is presented. The isomorphism classes and invariants for FLb_{10} are given in Section 3.

2. Preliminaries

This section contains some definitions and facts which we make use in the paper.

Let X be a set and G be a group, i.e., there is a function $\sigma : G \times X \longrightarrow X$ such that

- $\sigma(gh, x) = \sigma(g, \sigma(g, x))$, for all $g, h \in G$ and $x \in X$.
- $\sigma(e, x) = x$, for all $x \in X$ and e is the identity element of G.

We write $g * x$ for the action of G on X or simply gx if the context is clear.

Definition 1. *A function $f : X \to K$ is said to be an invariant (or orbit) function with respect to an action of a group G on a set X if $f(gx) = f(x)$ for $g \in G$.*

Let V be an n-dimensional vector space over an algebraically closed field K (char$K$=0). Bilinear maps $V \times V \to V$ form an n^3-dimensional vector space $Hom(V \otimes V, V)$ which can be considered together with its natural structure of an affine algebraic variety over K and denoted by $Alg_n(K) \cong K^{n^3}$. An n-dimensional algebra structure L over K on V can be considered as an element $\lambda(L)$ of $Alg_n(K)$ via the bilinear mapping λ: $V \otimes V \to V$ defining a binary algebraic operation on L. A pair $L = (V, \lambda)$ is an algebra structure on $n-$dimensional vector space V over a field K with λ as an algebra law on V. Let $\{e_1, e_2, ..., e_n\}$ be a basis of the algebra L, then the table of multiplication of L is represented by a point (γ_{ij}^k) of the affine space K^{n^3} as follows:

$$\lambda(e_i, e_j) = \textstyle\sum_{k=1}^{n} \gamma_{ij}^k e_k \text{ where } i, j = 1, 2, ..., n.$$

The elements γ_{ij}^k of K are called structure constants of L. We consider the action $(g*\lambda)(x, y) = g(\lambda(g^{-1}(x), g^{-1}(y)))$ of the linear reductive group $GL_n(K)$ on $Alg_n(K)$.

Definition 2. *Two algebra structures λ_1 and λ_2 on V are said to be isomorphic, if there is $g \in GL_n(K)$ such that*

$$\lambda_2(x, y) = (g * \lambda_1)(x, y) = g^{-1}(\lambda_1(g(x), g(y)))$$

for all $x, y \in V$.

Elements of the orbits under this action are isomorphic to each algebras from $Alg_n(K)$. The set of all laws isomorphic to λ is denoted by $O(\lambda)$ (the orbit of λ).

Definition 3. *An algebra L over a field K is called a Leibniz algebra, if its bilinear operation $\lambda(\cdot,\cdot)$ satisfies the following Leibniz identity:*

$$\lambda(x, \lambda(y, z)) = \lambda(\lambda(x, y), z) - \lambda(\lambda(x, z), y),$$

for all $x, y, z \in L$.

We denote the set of all Leibniz algebra laws on V by LB_n. The set LB_n is subvariety of $Alg_n(K)$ specified by Leibniz identity above.

Later on all algebras are suppose to be over the field of complex numbers $\mathbb{C}$ and the bracket notation is used to denote the law: $[\cdot,\cdot] = \lambda(\cdot,\cdot)$. Let L be a Leibniz algebra. We define the lower central series as follows:

$$L^1 = L,\ L^{k+1} = [L^k, L],\ k \in \mathbb{N}.$$

Definition 4. *A Leibniz algebra L is said to be nilpotent if there exists an integer $s \in \mathbb{N}$ such that*

$$L^1 \supset L^2 \supset \ldots \supset L^s = 0.$$

Definition 5. *An n-dimensional Leibniz algebra L is said to be filiform if $dim\ L^i = n - i$, where $2 \leq i \leq n$.*

The class of $(n+1)$-dimensional filiform Leibniz algebras, FLb_{n+1} arising from naturally graded non Lie filiform Leibniz algebras admits a so-called adapted basis $\{e_0, e_1, .., e_n\}$ such that the table of multiplication with respect to this basis is given as follows (see Gomez and Omirov (2015), Rakhimov and Bekbaev (2010)):

$$FLb_{n+1} = \begin{cases} [e_0, e_0] = e_2, & \\ [e_i, e_0] = e_{i+1}, & 1 \leq i \leq n-1, \\ [e_0, e_1] = \alpha_3 e_3 + \alpha_4 e_4 + \ldots + \alpha_{n-1} e_{n-1} + \theta e_n, & \\ [e_j, e_1] = \alpha_3 e_{j+2} + \alpha_4 e_{j+3} + \ldots + \alpha_{n+1-j} e_n, & 1 \leq j \leq n-2, \\ \quad \alpha_3, \alpha_4, \ldots, \alpha_n, \theta \in \mathbb{C}. & \end{cases}$$

Elements of FLb_{n+1} we denote by $L(\alpha_3, \alpha_4, \ldots, \alpha_n, \theta)$.

Let L be a filiform Leibniz algebra and $\{e_0, e_1, e_2, \ldots, e_n\}$ be its adapted basis.

Definition 6. *The basis transformation $f \in GL(V)$ is said to be adapted for the structure of L, if the basis $f(e_0), f(e_1), ..., f(e_n)$ is adapted.*

A subgroup of $GL_n(K)$ consisting of all linear transformations sending one adapted basis to another is said to be adapted for the structure of L. The subgroup is denoted by G_{ad}. In G_{ad} we study the following transformations of FLb_{n+1} called elementary:

$$\text{first type - } \tau(a,b,k) = \begin{cases} f(e_0) = e_0 + ae_k, \\ f(e_1) = e_1 + be_k, \\ f(e_{i+1}) = [f(e_i), f(e_0)], \quad 1 \leq i \leq n-1, \ 2 \leq k \leq n, \\ f(e_2) = [f(e_0), f(e_0)], \end{cases}$$

$$\text{second type - } \nu(a,b) = \begin{cases} f(e_0) = ae_0 + be_1, \\ f(e_1) = (a+b)e_1 + b(\theta - \alpha_n)e_{n-1}, \quad a(a+b) \neq 0 \\ f(e_{i+1}) = [f(e_i), f(e_0)], \quad 1 \leq i \leq n-1, \\ f(e_2) = [f(e_0), f(e_0)], \end{cases}$$

where $a, b, d, \alpha_n \in \mathbb{C}$.

Proposition 2.1.

1. *Let f be an adapted transformation of FLb_{n+1}, then*

$$f = \tau(a_n, b_n, n) \circ \tau(a_{n-1}, a_{n-1}, n-1) \circ ... \circ \tau(a_2, a_2, 2) \circ \nu(a_0, a_1).$$

2. *The transformation*

$$\tau(a_n, b_n, n) \circ \tau(a_{n-1}, a_{n-1}, n-1) \circ ... \circ \tau(a_2, a_2, 2)$$

preserves the structure constants of algebras from FLb_{n+1}.

The above proposition means that the transformation

$$\tau(a_n, b_n, n) \circ \tau(a_{n-1}, a_{n-1}, n-1) \circ ... \circ \tau(a_2, a_2, 2)$$

fixes the structure constants of FLb_{n+1}. Thus the action of G_{ad} on algebras from FLb_{n+1} can be reduced to the action of elementary transformations of the second type. The proof of the proposition is straightforward.

Theorem 1. *Two algebras $L(\alpha)$ and $L(\alpha')$ from FLb_{n+1} where $\alpha = (\alpha_3, \alpha_4, ..., \alpha_n, \theta)$ and $\alpha = (\alpha'_3, \alpha'_4, ..., \alpha'_n, \theta')$ are isomorphic, if and only if there exist complex numbers A and B, such that $A(A+B) \neq 0$ and the following conditions hold:*

$$\alpha'_t = \frac{1}{A^{t-2}}\varphi_t\left(\frac{B}{A};\alpha\right), \quad 3 \leq t \leq n, \tag{1}$$

$$\theta = \frac{1}{A^{n-2}}\varphi_{n+1}\left(\frac{B}{A};\alpha\right), \tag{2}$$

where

$$\begin{aligned}\varphi_t(y;z) &= \varphi_t(y;z_3,z_4,...,z_n,z_{n+1}),\\ &= \Bigg((1+y)z_t - \sum_{k=3}^{t-1}(C_{k-1}^{k-2}yz_{t+2-k} + C_{k-1}^{k-3}y^2\sum_{i_1=k+2}^{t} z_{t+3-i_1}\cdot z_{i_1+1-k}\\ &\quad + C_{k-1}^{k-4}y^3\sum_{i_2=k+3}^{t}\sum_{i_1=k+3}^{i_2} z_{t+3-i_2}\cdot z_{i_2+3-i_1}\cdot z_{i_1-k} + ... + C_{k-1}^{1}y^{k-2}\\ &\quad \sum_{i_{k-3}=2k-2}^{t}\sum_{i_{k-4}=2k-2}^{i_{k-3}}\cdots\sum_{i_1=2k-2}^{i_2} z_{t+3-i_{k-3}}\cdot z_{i_{k-3}+3-i_{k-4}}\cdot ...\cdot z_{i_2+3-i_1}\\ &\quad \cdot z_{i_1+5-2k} + y^{k-1}\sum_{i_{k-2}=2k-1}^{t}\sum_{i_{k-3}=2k-1}^{i_{k-2}}\cdots\sum_{i_1=2k-1}^{i_2} z_{t+3-i_{k-2}}\cdot z_{i_{k-2}+3-i_{k-3}}\\ &\quad \cdot ...\cdot z_{i_2+3-i_1}\cdot z_{i_1+4-2k})\cdot\varphi_k(y;z)\Bigg),\end{aligned}$$

for $3 \leq t \leq n$.

To simplify notations for transition from $L(\alpha)$ to $L(\alpha')$, we write $\alpha' = \rho\left(\frac{1}{A},\frac{B}{A};\alpha\right)$,

where

$$\rho\left(\frac{1}{A},\frac{B}{A};\alpha\right) = \left(\rho_1\left(\frac{1}{A},\frac{B}{A};\alpha\right), \rho_2\left(\frac{1}{A},\frac{B}{A};\alpha\right), ..., \rho_{n-1}\left(\frac{1}{A},\frac{B}{A};\alpha\right)\right),$$

and

$$\begin{aligned}\rho_t(x,y;z) &= x^t\varphi_{t+2}(y;z), \quad 1 \leq t \leq n-2,\\ \rho_{n-1}(x,y;z) &= x^{n-2}\varphi_{n+1}(y;z).\end{aligned}$$

The complete details are given by Said Husain (2011). We make use the properties of the operator ρ given in Rakhimov and Bekbaev (2010):

1. $\rho(1,0;\cdot)$ is the identity operator.
2. $\rho\left(\frac{1}{A_2},\frac{B_2}{A_2};\rho\left(\frac{1}{A_1},\frac{B_1}{A_1};\alpha\right)\right)=\rho\left(\frac{1}{A_1A_2},\frac{A_1B_2+A_2B_1+B_1B_2}{A_1A_2};\alpha\right)$.
3. If $\alpha'=\rho\left(\frac{1}{A},\frac{B}{A};\alpha\right)$ then $\alpha=\rho\left(A,-\frac{B}{A+B};\alpha'\right)$.

Let us introduce the following notations:

$$
\begin{aligned}
&\Delta_3=\alpha_3, && \Delta'_3=\alpha'_3,\\
&\Delta_4=\alpha_4+2\alpha_3^2, && \Delta'_4=\alpha'_4+2\alpha_3'^2,\\
&\Delta_5=\alpha_5-5\alpha_3^3, && \Delta'_5=\alpha'_5-5\alpha_3'^3,\\
&\Delta_6=\alpha_6+14\alpha_3^4, && \Delta'_6=\alpha'_6+14\alpha_3'^4,\\
&\Delta_7=\alpha_7-42\alpha_3^5, && \Delta'_7=\alpha'_7-42\alpha_3'^5,\\
&\Delta_8=\alpha_8+132\alpha_3^6, && \Delta'_8=\alpha'_8+132\alpha_3'^6,\\
&\Delta_9=\alpha_9-429\alpha_3^7, && \Delta'_9=\alpha'_9-429\alpha_3'^7,\\
&\Theta_9=\theta-\alpha_9, && \Theta'_9=\theta'-\alpha'_9.
\end{aligned}
$$

Note that there is an adapted base change sending the structure constant $\alpha_3,\alpha_4,...,\alpha_n,\theta$ to $\Delta_3,\Delta_4,...,\Delta_n,\Theta_n$. Elements of FLb_{n+1} with respect to this basis we denote by $L(\mathbf{\Delta}):=L(\Delta_3,\Delta_4,...,\Delta_n,\Theta_n)$, where $\mathbf{\Delta}=(\Delta_3,\Delta_4,...,\Delta_n,\Theta_n)$.

3. Isomorphism Criterion

In this section we give criteria for elements of FLb_{10} to be isomorphic in terms of structure constants $\Delta_3,\Delta_4,...,\Delta_9,\Theta_9$. However, all algebraic expressions involved in the isomorphism criterion are very huge. To avoid the huge expression in the system of equations, we introduce the functions defined below:

$$
\begin{aligned}
f_5(\mathbf{X}) &= x_5 + 5x_3x_4;\\
f_6(\mathbf{X}) &= x_6 + 6x_3x_5 + 9\,x_3^2x_4 + 3\,x_4^2;\\
f_7(\mathbf{X}) &= x_7 + 7\,x_3x_6 + 7\,x_4x_5 + 7\,x_3^3x_4 + 14\,x_3^2x_5 + 28\,x_3x_4^2;\\
f_8(\mathbf{X}) &= x_8 + 8\,x_3x_7 + 8\,x_4x_6 + 20\,x_3^2x_6 + 4\,x_5^2 + 2\,x_3^4x_4 + 16\,x_3^3x_5\\
&\quad + 72\,x_3x_4x_5 + 108\,x_3^2x_4^2 + 12\,x_4^3;\\
f_9(\mathbf{X}) &= x_9 + 90\,x_3x_4x_6 + 9\,x_3x_8 + 165\,x_3x_4^3 + 45\,x_3x_5^2 + 315\,x_4x_3^2x_5\\
&\quad + 27\,x_7x_3^2 + 30\,x_3^3x_6 + 225\,x_4^2x_3^3 + 9\,x_3^4x_5 + 9\,x_4x_7 + 45\,x_4^2x_5 + 9\,x_5x_6,
\end{aligned}
$$

where $\mathbf{X} = (x_3, x_4, x_5, x_6, x_7, x_8, x_9)$.

By using the functions along with Proposition 2.1 the adapted base change for elements $L(\mathbf{\Delta}')$ and $L(\mathbf{\Delta})$ of FLb_{10} is rewritten as follows.

Proposition 3.1. *Two algebras $L(\mathbf{\Delta}')$ and $L(\mathbf{\Delta})$ from FLb_{10}, are isomorphic if and only if there exist complex numbers A and B such that $A(A+B) \neq 0$ and the following conditions hold:*

$$
\begin{aligned}
\Delta_3' &= \frac{1}{A}\left(1+\frac{B}{A}\right)\Delta_3, & \Delta_4' &= \frac{1}{A^2}\left(1+\frac{B}{A}\right)\Delta_4,\\
f_5(\mathbf{\Delta}') &= \frac{1}{A^3}\left(1+\frac{B}{A}\right)f_5(\mathbf{\Delta}), & f_6(\mathbf{\Delta}') &= \frac{1}{A^4}\left(1+\frac{B}{A}\right)f_6(\mathbf{\Delta}),\\
f_7(\mathbf{\Delta}') &= \frac{1}{A^5}\left(1+\frac{B}{A}\right)f_7(\mathbf{\Delta}), & f_8(\mathbf{\Delta}') &= \frac{1}{A^6}\left(1+\frac{B}{A}\right)f_8(\mathbf{\Delta}),\\
f_9(\mathbf{\Delta}') &= \frac{1}{A^7}\left(1+\frac{B}{A}\right)f_9(\mathbf{\Delta}), & \Theta_9' &= \frac{1}{A^7}\Theta_9.
\end{aligned}
$$

4. Isomorphism Classes

To classify FLb_{10}, we split it into its disjoint subsets subject to conditions on structure constants $\Delta_3, \Delta_4, ..., \Delta_9, \Theta_9$ as follows:

$$
\begin{aligned}
U_1 &= \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 \neq 0, \Delta_4 \neq 0\}, \\
U_2 &= \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 \neq 0, \Delta_4 = 0, f_5(\boldsymbol{\Delta}) \neq 0, f_6(\Delta) \neq 0\}, \\
U_3 &= \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 \neq 0, \Delta_4 = 0, f_5(\boldsymbol{\Delta}) \neq 0, f_6(\boldsymbol{\Delta}) = 0, f_8(\boldsymbol{\Delta}) \neq 0\}, \\
U_4 &= \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 \neq 0, \Delta_4 = 0, f_5(\boldsymbol{\Delta}) \neq 0, f_6(\boldsymbol{\Delta}) = f_8(\boldsymbol{\Delta}) = 0, \Theta_9 \neq 0\}, \\
U_5 &= \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 \neq 0, \Delta_4 = 0, f_5(\boldsymbol{\Delta}) \neq 0, f_6(\boldsymbol{\Delta}) = f_8(\boldsymbol{\Delta}) = \Theta_9 = 0\}, \\
U_6 &= \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 \neq 0, \Delta_4 = f_5(\boldsymbol{\Delta}) = 0, f_6(\boldsymbol{\Delta}) \neq 0, f_7(\boldsymbol{\Delta}) \neq 0\}, \\
U_7 &= \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 \neq 0, \Delta_4 = f_5(\boldsymbol{\Delta}) = 0, f_6(\boldsymbol{\Delta}) \neq 0, f_7(\boldsymbol{\Delta}) = 0, f_8(\boldsymbol{\Delta}) \neq 0\}, \\
U_8 &= \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 \neq 0, \Delta_4 = f_5(\boldsymbol{\Delta}) = 0, f_6(\boldsymbol{\Delta}) \neq 0, f_7(\boldsymbol{\Delta}) = f_8(\boldsymbol{\Delta}) = 0, \\
&\qquad \Theta_9 \neq 0\}, \\
U_9 &= \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 \neq 0, \Delta_4 = f_5(\boldsymbol{\Delta}) = 0, f_6(\boldsymbol{\Delta}) \neq 0, f_7(\boldsymbol{\Delta}) = f_8(\boldsymbol{\Delta}) \\
&\qquad = \Theta_9 = 0\}, \\
U_{10} &= \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 \neq 0, \Delta_4 = f_5(\boldsymbol{\Delta}) = f_6(\boldsymbol{\Delta}) = 0, f_7(\boldsymbol{\Delta}) \neq 0, f_8(\boldsymbol{\Delta}) \neq 0\}, \\
U_{11} &= \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 \neq 0, \Delta_4 = f_5(\boldsymbol{\Delta}) = f_6(\boldsymbol{\Delta}) = 0, f_7(\boldsymbol{\Delta}) \neq 0, f_8(\boldsymbol{\Delta}) = 0, \\
&\qquad f_9(\boldsymbol{\Delta}) \neq 0, \Theta_9 \neq 0\}, \\
U_{12} &= \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 \neq 0, \Delta_4 = f_5(\boldsymbol{\Delta}) = f_6(\boldsymbol{\Delta}) = 0, f_7(\boldsymbol{\Delta}) \neq 0, f_8(\boldsymbol{\Delta}) = 0, \\
&\qquad f_9(\boldsymbol{\Delta}) \neq 0, \Theta_9 = 0\},
\end{aligned}
$$

$$U_{13} = \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 \neq 0, \Delta_4 = f_5(\boldsymbol{\Delta}) = f_6(\boldsymbol{\Delta}) = 0, f_7(\boldsymbol{\Delta}) \neq 0, f_8(\boldsymbol{\Delta}) = f_9(\boldsymbol{\Delta}) = 0, \Theta_9 \neq 0\},$$
$$U_{14} = \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 \neq 0, \Delta_4 = f_5(\boldsymbol{\Delta}) = f_6(\boldsymbol{\Delta}) = f_7(\boldsymbol{\Delta}) = 0, f_8(\boldsymbol{\Delta}) \neq 0, f_9(\boldsymbol{\Delta}) \neq 0\},$$
$$U_{15} = \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 \neq 0, \Delta_4 = f_5(\boldsymbol{\Delta}) = f_6(\boldsymbol{\Delta}) = f_7(\boldsymbol{\Delta}) = 0, f_8(\boldsymbol{\Delta}) \neq 0, f_9(\boldsymbol{\Delta}) = 0, \Theta_9 \neq 0\},$$
$$U_{16} = \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 \neq 0, \Delta_4 = f_5(\boldsymbol{\Delta}) = f_6(\boldsymbol{\Delta}) = f_7(\boldsymbol{\Delta}) = f_8(\boldsymbol{\Delta}) = 0, f_9(\boldsymbol{\Delta}) \neq 0, \Theta_9 \neq 0\},$$
$$U_{17} = \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 = 0, \Delta_4 \neq 0, f_5(\boldsymbol{\Delta}) \neq 0\},$$
$$U_{18} = \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 = 0, \Delta_4 \neq 0, f_5(\boldsymbol{\Delta}) = 0, f_6(\boldsymbol{\Delta}) \neq 0, f_7(\boldsymbol{\Delta}) \neq 0\},$$
$$U_{19} = \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 = 0, \Delta_4 \neq 0, f_5(\boldsymbol{\Delta}) = 0, f_6(\boldsymbol{\Delta}) \neq 0, f_7(\boldsymbol{\Delta}) = 0, f_9(\boldsymbol{\Delta}) \neq 0\},$$
$$U_{20} = \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 = 0, \Delta_4 \neq 0, f_5(\boldsymbol{\Delta}) = 0, f_6(\boldsymbol{\Delta}) \neq 0, f_7(\boldsymbol{\Delta}) = f_9(\boldsymbol{\Delta}) = 0, \Theta_9 \neq 0\},$$
$$U_{21} = \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 = 0, \Delta_4 \neq 0, f_5(\boldsymbol{\Delta}) = 0, f_6(\boldsymbol{\Delta}) \neq 0, f_7(\boldsymbol{\Delta}) = f_9(\boldsymbol{\Delta}) = \Theta_9 = 0\},$$
$$U_{22} = \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 = 0, \Delta_4 \neq 0, f_5(\boldsymbol{\Delta}) = f_6(\boldsymbol{\Delta}) = 0, f_7(\boldsymbol{\Delta}) \neq 0, f_8(\boldsymbol{\Delta}) \neq 0\},$$
$$U_{23} = \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 = 0, \Delta_4 \neq 0, f_5(\boldsymbol{\Delta}) = f_6(\boldsymbol{\Delta}) = 0, f_7(\boldsymbol{\Delta}) \neq 0, f_8(\boldsymbol{\Delta}) = 0, f_9(\boldsymbol{\Delta}) \neq 0\},$$
$$U_{24} = \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 = 0, \Delta_4 \neq 0, f_5(\boldsymbol{\Delta}) = f_6(\boldsymbol{\Delta}) = 0, f_7(\boldsymbol{\Delta}) \neq 0, f_8(\boldsymbol{\Delta}) = f_9(\boldsymbol{\Delta}) = 0, \Theta_9 \neq 0\},$$
$$U_{25} = \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 = 0, \Delta_4 \neq 0, f_5(\boldsymbol{\Delta}) = f_6(\boldsymbol{\Delta}) = f_7(\boldsymbol{\Delta}) = 0, f_8(\boldsymbol{\Delta}) \neq 0, f_9(\boldsymbol{\Delta}) \neq 0\},$$
$$U_{26} = \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 = 0, \Delta_4 \neq 0, f_5(\boldsymbol{\Delta}) = f_6(\boldsymbol{\Delta}) = f_7(\boldsymbol{\Delta}) = 0, f_8(\boldsymbol{\Delta}) \neq 0, f_9(\boldsymbol{\Delta}) = 0, \Theta_9 \neq 0\},$$
$$U_{27} = \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 = 0, \Delta_4 \neq 0, f_5(\boldsymbol{\Delta}) = f_6(\boldsymbol{\Delta}) = f_7(\boldsymbol{\Delta}) = f_8(\boldsymbol{\Delta}) = 0, f_9(\boldsymbol{\Delta}) \neq 0, \Theta_9 \neq 0\},$$
$$U_{28} = \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 = \Delta_4 = 0, f_5(\boldsymbol{\Delta}) \neq 0, f_6(\boldsymbol{\Delta}) \neq 0\},$$
$$U_{29} = \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 = \Delta_4 = 0, f_5(\boldsymbol{\Delta}) \neq 0, f_6(\boldsymbol{\Delta}) = 0, f_7(\boldsymbol{\Delta}) \neq 0, f_8(\boldsymbol{\Delta}) \neq 0\},$$
$$U_{30} = \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 = \Delta_4 = 0, f_5(\boldsymbol{\Delta}) \neq 0, f_6(\boldsymbol{\Delta}) = 0, f_7(\boldsymbol{\Delta}) \neq 0, f_8(\boldsymbol{\Delta}) = 0, f_9(\boldsymbol{\Delta}) \neq 0, \Theta_9 \neq 0\},$$
$$U_{31} = \{L(\boldsymbol{\Delta}) \in FLb_{10} \colon \Delta_3 = \Delta_4 = 0, f_5(\boldsymbol{\Delta}) \neq 0, f_6(\boldsymbol{\Delta}) = 0, f_7(\boldsymbol{\Delta}) \neq 0, f_8(\boldsymbol{\Delta}) = 0, f_9(\Delta) \neq 0, \Theta_9 = 0\},$$

$$U_{32} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon \Delta_3 = \Delta_4 = 0, f_5(\mathbf{\Delta}) \neq 0, f_6(\mathbf{\Delta}) = 0, f_7(\mathbf{\Delta}) \neq 0, f_8(\mathbf{\Delta}) = f_9(\mathbf{\Delta}) = 0, \Theta_9 \neq 0\},$$

$$U_{33} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon \Delta_3 = \Delta_4 = 0, f_5(\mathbf{\Delta}) \neq 0, f_6(\mathbf{\Delta}) = f_7(\mathbf{\Delta}) = 0, f_8(\mathbf{\Delta}) \neq 0, f_9(\mathbf{\Delta}) \neq 0\},$$

$$U_{34} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon \Delta_3 = \Delta_4 = 0, f_5(\mathbf{\Delta}) \neq 0, f_6(\mathbf{\Delta}) = f_7(\mathbf{\Delta}) = 0, f_8(\mathbf{\Delta}) \neq 0, f_9(\mathbf{\Delta}) = 0, \Theta_9 \neq 0\},$$

$$U_{35} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon \Delta_3 = \Delta_4 = 0, f_5(\mathbf{\Delta}) \neq 0, f_6(\mathbf{\Delta}) = f_7(\mathbf{\Delta}) = 0, f_8(\mathbf{\Delta}) = 0, f_9(\mathbf{\Delta}) \neq 0\},$$

$$U_{36} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon \Delta_3 = \Delta_4 = f_5(\mathbf{\Delta}) = 0, f_6(\mathbf{\Delta}) \neq 0, f_7(\mathbf{\Delta}) \neq 0\},$$

$$U_{37} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon \Delta_3 = \Delta_4 = f_5(\mathbf{\Delta}) = 0, f_6(\mathbf{\Delta}) \neq 0, f_7(\mathbf{\Delta}) = 0, f_8(\mathbf{\Delta}) \neq 0, f_9(\mathbf{\Delta}) \neq 0\},$$

$$U_{38} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon \Delta_3 = \Delta_4 = f_5(\mathbf{\Delta}) = 0, f_6(\mathbf{\Delta}) \neq 0, f_7(\mathbf{\Delta}) = 0, f_8(\mathbf{\Delta}) \neq 0, f_9(\mathbf{\Delta}) = 0, \Theta_9 \neq 0\},$$

$$U_{39} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon \Delta_3 = \Delta_4 = f_5(\mathbf{\Delta}) = 0, f_6(\mathbf{\Delta}) \neq 0, f_7(\mathbf{\Delta}) = f_8(\mathbf{\Delta}) = 0, f_9(\mathbf{\Delta}) \neq 0, \Theta_9 \neq 0\},$$

$$U_{40} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon \Delta_3 = \Delta_4 = f_5(\mathbf{\Delta}) = f_6(\mathbf{\Delta}) = 0, f_7(\mathbf{\Delta}) \neq 0, f_8(\mathbf{\Delta}) \neq 0\},$$

$$U_{41} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon \Delta_3 = \Delta_4 = f_5(\mathbf{\Delta}) = 0, f_6(\mathbf{\Delta}) = 0, f_7(\mathbf{\Delta}) \neq 0, f_8(\mathbf{\Delta}) = 0, f_9(\mathbf{\Delta}) \neq 0, \Theta_9 \neq 0\},$$

$$U_{42} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon \Delta_3 = \Delta_4 = f_5(\mathbf{\Delta}) = 0, f_6(\mathbf{\Delta}) = 0, f_7(\mathbf{\Delta}) = 0, f_8(\mathbf{\Delta}) \neq 0, f_9(\mathbf{\Delta}) \neq 0\},$$

$$U_{43} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon \Delta_3 \neq 0, \Delta_4 = f_5(\mathbf{\Delta}) = f_6(\mathbf{\Delta}) = 0, f_7(\mathbf{\Delta}) \neq 0, f_8(\mathbf{\Delta}) = f_9(\mathbf{\Delta}) = \Theta_9 = 0\},$$

$$U_{44} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon \Delta_3 \neq 0, \Delta_4 = f_5(\mathbf{\Delta}) = f_6(\mathbf{\Delta}) = f_7(\mathbf{\Delta}) = 0, f_8(\mathbf{\Delta}) \neq 0, f_9(\mathbf{\Delta}) = \Theta_9 = 0\},$$

$$U_{45} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon \Delta_3 \neq 0, \Delta_4 = f_5(\mathbf{\Delta}) = f_6(\mathbf{\Delta}) = f_7(\mathbf{\Delta}) = f_8(\mathbf{\Delta}) = 0, f_9(\mathbf{\Delta}) \neq 0, \Theta_9 = 0\},$$

$$U_{46} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon \Delta_3 \neq 0, \Delta_4 = f_5(\mathbf{\Delta}) = f_6(\mathbf{\Delta}) = f_7(\mathbf{\Delta}) = f_8(\mathbf{\Delta}) = f_9(\mathbf{\Delta}) = 0, \Theta_9 \neq 0\},$$

$$U_{47} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon \Delta_3 \neq 0, \Delta_4 = f_5(\mathbf{\Delta}) = f_6(\mathbf{\Delta}) = f_7(\mathbf{\Delta}) = f_8(\mathbf{\Delta}) = f_9(\mathbf{\Delta}) = \Theta_9 = 0\},$$

$$U_{48} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon \Delta_3 = \Delta_4 \neq 0, f_5(\mathbf{\Delta}) = f_6(\mathbf{\Delta}) = 0, f_7(\mathbf{\Delta}) \neq 0, f_8(\mathbf{\Delta}) = f_9(\mathbf{\Delta}) = \Theta_9 = 0\},$$

$$U_{49} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon \Delta_3 = 0, \Delta_4 \neq 0, f_5(\mathbf{\Delta}) = f_6(\mathbf{\Delta}) = f_7(\mathbf{\Delta}) = 0, f_8(\mathbf{\Delta}) \neq 0, f_9(\mathbf{\Delta}) = \Theta_9 = 0\},$$

$$
\begin{aligned}
U_{50} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon & \Delta_3 = 0, \Delta_4 \neq 0, f_5(\mathbf{\Delta}) = f_6(\mathbf{\Delta}) = f_7(\mathbf{\Delta}) = f_8(\mathbf{\Delta}) = 0, \\
& f_9(\mathbf{\Delta}) \neq 0, \Theta_9 = 0\}, \\
U_{51} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon & \Delta_3 = 0, \Delta_4 \neq 0, f_5(\mathbf{\Delta}) = f_6(\mathbf{\Delta}) = f_7(\mathbf{\Delta}) = f_8(\mathbf{\Delta}) \\
& = f_9(\mathbf{\Delta}) = 0, \Theta_9 \neq 0\}, \\
U_{52} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon & \Delta_3 = 0, \Delta_4 \neq 0, f_5(\mathbf{\Delta}) = f_6(\mathbf{\Delta}) = f_7(\mathbf{\Delta}) = f_8(\mathbf{\Delta}) \\
& = f_9(\mathbf{\Delta}) = \Theta_9 = 0\}, \\
U_{53} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon & \Delta_3 = \Delta_4 = 0, f_5(\mathbf{\Delta}) \neq 0, f_6(\mathbf{\Delta}) = f_7(\mathbf{\Delta}) \neq 0, f_8(\mathbf{\Delta}) \\
& = f_9(\mathbf{\Delta}) = \Theta_9 = 0\}, \\
U_{54} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon & \Delta_3 = \Delta_4 = 0, f_5(\mathbf{\Delta}) \neq 0, f_6(\mathbf{\Delta}) = f_7(\mathbf{\Delta}) = 0, f_8(\mathbf{\Delta}) \neq 0, \\
& f_9(\mathbf{\Delta}) = 0, \Theta_9 = 0\}, \\
U_{55} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon & \Delta_3 = \Delta_4 = 0, f_5(\mathbf{\Delta}) \neq 0, f_6(\mathbf{\Delta}) = f_7(\mathbf{\Delta}) = f_8(\mathbf{\Delta}) \\
& = f_9(\mathbf{\Delta}) = 0, \Theta_9 \neq 0\}, \\
U_{56} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon & \Delta_3 = \Delta_4 = 0, f_5(\mathbf{\Delta}) \neq 0, f_6(\mathbf{\Delta}) = f_7(\mathbf{\Delta}) = f_8(\mathbf{\Delta}) \\
& = f_9(\mathbf{\Delta}) = \Theta_9 = 0\}, \\
U_{57} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon & \Delta_3 = \Delta_4 = f_5(\mathbf{\Delta}) = 0, f_6(\mathbf{\Delta}) \neq 0, f_7(\mathbf{\Delta}) = 0, f_8(\mathbf{\Delta}) \neq 0, \\
& f_9(\mathbf{\Delta}) = \Theta_9 = 0\}, \\
U_{58} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon & \Delta_3 = \Delta_4 = f_5(\mathbf{\Delta}) = 0, f_6(\mathbf{\Delta}) \neq 0, f_7(\mathbf{\Delta}) = f_8(\mathbf{\Delta}) = 0, \\
& f_9(\mathbf{\Delta}) \neq 0, \Theta_9 = 0\}, \\
U_{59} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon & \Delta_3 = \Delta_4 = f_5(\mathbf{\Delta}) = 0, f_6(\mathbf{\Delta}) \neq 0, f_7(\mathbf{\Delta}) = f_8(\mathbf{\Delta}) \\
& = f_9(\mathbf{\Delta}) = 0, \Theta_9 \neq 0\}, \\
U_{60} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon & \Delta_3 = \Delta_4 = f_5(\mathbf{\Delta}) = 0, f_6(\mathbf{\Delta}) \neq 0, f_7(\mathbf{\Delta}) = f_8(\mathbf{\Delta}) \\
& = f_9(\mathbf{\Delta}) = \Theta_9 = 0\}, \\
U_{61} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon & \Delta_3 = \Delta_4 = f_5(\mathbf{\Delta}) = f_6(\mathbf{\Delta}) = 0, f_7(\mathbf{\Delta}) \neq 0, f_8(\mathbf{\Delta}) = 0, \\
& f_9(\mathbf{\Delta}) \neq 0, \Theta_9 = 0\}, \\
U_{62} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon & \Delta_3 = \Delta_4 = f_5(\mathbf{\Delta}) = f_6(\mathbf{\Delta}) = 0, f_7(\mathbf{\Delta}) \neq 0, f_8(\mathbf{\Delta}) \\
& = f_9(\mathbf{\Delta}) = 0, \Theta_9 \neq 0\}, \\
U_{63} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon & \Delta_3 = \Delta_4 = f_5(\mathbf{\Delta}) = f_6(\mathbf{\Delta}) = 0, f_7(\mathbf{\Delta}) \neq 0, f_8(\mathbf{\Delta}) \\
& = f_9(\mathbf{\Delta}) = \Theta_9 = 0\}, \\
U_{64} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon & \Delta_3 = \Delta_4 = f_5(\mathbf{\Delta}) = f_6(\mathbf{\Delta}) = f_7(\mathbf{\Delta}) = 0, f_8(\mathbf{\Delta}) \neq 0, \\
& f_9(\mathbf{\Delta}) = 0, \Theta_9 \neq 0\}, \\
U_{65} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon & \Delta_3 = \Delta_4 = f_5(\mathbf{\Delta}) = f_6(\mathbf{\Delta}) = f_7(\mathbf{\Delta}) = 0, f_8(\mathbf{\Delta}) \neq 0, \\
& f_9(\mathbf{\Delta}) = \Theta_9 = 0\}, \\
U_{66} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon & \Delta_3 = \Delta_4 = f_5(\mathbf{\Delta}) = f_6(\mathbf{\Delta}) = f_7(\mathbf{\Delta}) = f_8(\mathbf{\Delta}) = 0, \\
& f_9(\mathbf{\Delta}) \neq 0, \Theta_9 \neq 0\},
\end{aligned}
$$

$$U_{67} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon \Delta_3 = \Delta_4 = f_5(\mathbf{\Delta}) = f_6(\mathbf{\Delta}) = f_7(\mathbf{\Delta}) = f_8(\mathbf{\Delta}) = 0, \\ f_9(\mathbf{\Delta}) \neq 0, \Theta_9 = 0\},$$
$$U_{68} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon \Delta_3 = \Delta_4 = f_5(\mathbf{\Delta}) = f_6(\mathbf{\Delta}) = f_7(\mathbf{\Delta}) = f_8(\mathbf{\Delta}) \\ = f_9(\mathbf{\Delta}) = 0, \Theta_9 \neq 0\},$$
$$U_{69} = \{L(\mathbf{\Delta}) \in FLb_{10} \colon \Delta_3 = \Delta_4 = f_5(\mathbf{\Delta}) = f_6(\mathbf{\Delta}) = f_7(\mathbf{\Delta}) = f_8(\mathbf{\Delta}) \\ = f_9(\mathbf{\Delta}) = \Theta_9 = 0\}.$$

The action of G_{ad} on FLb_{10} restricted to each of the subsets U_i, $i = 1, 2, ..., 69$ leaves each of the subsets stable. Therefore, one can give an isomorphism criterion for each of these subsets separately. It seemed some of these subsets are given as one orbit, they are $U_{43} - U_{69}$, and the others are represented as a union of the orbits under the action G_{ad}, they are $U_1 - U_{42}$. The proofs of these criteria for the subsets are similar, therefore we give the proof only for two cases: one of them is the case where the subset is represented as one orbit and another case where the subset is given as a union of infinitely many orbits.

Proposition 4.1.

(i) Two algebras $L(\mathbf{\Delta})$ and $L(\mathbf{\Delta}')$ from U_{12} are isomorphic if and only if

$$\frac{f_7^3(\mathbf{\Delta})}{\Delta_3 f_9^2(\mathbf{\Delta})} = \frac{f_7^3(\mathbf{\Delta}')}{\Delta_3' f_9^2(\mathbf{\Delta}')}. \tag{3}$$

(ii) The representatives of the orbits in U_{12} given by algebras $L(1, 0, 0, 0, \lambda, -8\lambda, 1 + 45\lambda, 0)$, where $\lambda \in \mathbb{C}^*$.

Proof.

(i) **Necessity**. Let $L(\mathbf{\Delta})$ and $L(\mathbf{\Delta}')$ be isomorphic. There are complex numbers A and B such that $A(A+B) \neq 0$ and $\mathbf{\Delta}' = \rho\left(\frac{1}{A}, \frac{B}{A}; \mathbf{\Delta}\right)$, i.e., the system of equations of Proposition 3.1 hold. The system for U_{12} is given as follows:

$$\begin{aligned}
\Delta_3' &= \frac{1}{A}\left(1+\frac{B}{A}\right)\Delta_3,\\
\Delta_4' &= f_5\left(\boldsymbol{\Delta}'\right) = f_6\left(\boldsymbol{\Delta}'\right) = f_8\left(\boldsymbol{\Delta}'\right) = \Theta_9' = 0,\\
f_7\left(\boldsymbol{\Delta}'\right) &= \frac{1}{A^5}\left(1+\frac{B}{A}\right) f_7\left(\boldsymbol{\Delta}\right),\\
f_9\left(\boldsymbol{\Delta}'\right) &= \frac{1}{A^7}\left(1+\frac{B}{A}\right) f_9\left(\boldsymbol{\Delta}\right).
\end{aligned} \tag{4}$$

Then by substituting the values of $\Delta_3', f_7\left(\boldsymbol{\Delta}'\right)$ and $f_9\left(\boldsymbol{\Delta}'\right)$ into the expressions $\dfrac{f_7^3\left(\boldsymbol{\Delta}'\right)}{\Delta_3' f_9^2\left(\boldsymbol{\Delta}'\right)}$, we obtain the required equation as follows:

$$\begin{aligned}
\frac{f_7^3\left(\boldsymbol{\Delta}'\right)}{\Delta_3' f_9^2\left(\boldsymbol{\Delta}'\right)} &= \frac{\left[\frac{1}{A^5}\left(1+\frac{B}{A}\right) f_7\left(\boldsymbol{\Delta}\right)\right]^3}{\frac{1}{A}\left(1+\frac{B}{A}\right)\Delta_3} \cdot \frac{1}{\left[\frac{1}{A^7}\left(1+\frac{B}{A}\right) f_9\left(\boldsymbol{\Delta}\right)\right]^2}\\
&= \frac{f_7^3\left(\boldsymbol{\Delta}\right)}{\Delta_3 f_9^2\left(\boldsymbol{\Delta}\right)}.
\end{aligned}$$

Sufficiency. Let the equality (3) hold.

First note that there are $A_0, B_0 \in \mathbb{C}$ with $A_0(A_0+B_0) \neq 0$ and $A_0', B_0' \in \mathbb{C}$ with $A_0'(A_0'+B_0') \neq 0$ such that

$$\rho\left(\frac{1}{A_0}, \frac{B_0}{A_0}; \boldsymbol{\Delta}\right) = \boldsymbol{\Delta}_0$$

and

$$\rho\left(\frac{1}{A_0'}, \frac{B_0'}{A_0'}; \boldsymbol{\Delta}'\right) = \boldsymbol{\Delta}_0',$$

where

$$L(\boldsymbol{\Delta}_0) = L\left(1,0,0,0,\lambda,-8\lambda,1+45\lambda,0\right) \text{ with } \lambda = \frac{f_7^3\left(\boldsymbol{\Delta}\right)}{\Delta_3 f_9^2\left(\boldsymbol{\Delta}\right)}$$

and

$$L(\boldsymbol{\Delta}_0') = L\left(1,0,0,0,\lambda',-8\lambda',1+45\lambda',0\right) \text{ with } \lambda' = \frac{f_7^3\left(\boldsymbol{\Delta}'\right)}{\Delta_3' f_9^2\left(\boldsymbol{\Delta}'\right)}.$$

Namely,

$$A_0 = \sqrt{\frac{f_9(\boldsymbol{\Delta})}{f_7(\boldsymbol{\Delta})}} \quad B_0 = \sqrt{\frac{f_9(\boldsymbol{\Delta})}{f_7(\boldsymbol{\Delta})}}\left(\sqrt{\frac{f_9(\boldsymbol{\Delta})}{\Delta_3^2 f_7(\boldsymbol{\Delta})}} - 1\right)$$

and

$$A_0' = \sqrt{\frac{f_9(\boldsymbol{\Delta'})}{f_7(\boldsymbol{\Delta'})}} \quad B_0' = \sqrt{\frac{f_9(\boldsymbol{\Delta'})}{f_7(\boldsymbol{\Delta'})}}\left(\sqrt{\frac{f_9(\boldsymbol{\Delta'})}{\Delta_3'^2 f_7(\boldsymbol{\Delta'})}} - 1\right).$$

Due to (3) one has $L(\boldsymbol{\Delta}_0) = L(\boldsymbol{\Delta}_0')$. To prove the sufficiency of the equation (3) we give $A, B \in \mathbb{C}$ with $A(A+B) \neq 0$ such that

$$\boldsymbol{\Delta'} = \rho\left(\frac{1}{A}, \frac{B}{A}; \boldsymbol{\Delta}\right).$$

This procedure can be shown schematically as follows:

$$\begin{array}{ccc} L(\boldsymbol{\Delta}) & \xrightarrow{(A,B)} & L(\boldsymbol{\Delta'}) \\ {\scriptstyle (A_0,B_0)}\downarrow & & \downarrow{\scriptstyle (A_0',B_0')} \\ L(\boldsymbol{\Delta}_0) & = & L(\boldsymbol{\Delta}_0'), \end{array}$$

Hence, by using the properties of the operator $\rho(\cdot,\cdot;\cdot)$ given above we have

$$\begin{aligned}
\boldsymbol{\Delta}' &= \rho\left(A_0', -\frac{B_0'}{A_0'+B_0'}; \boldsymbol{\Delta}_0'\right) \\
&= \rho\left(A_0', -\frac{B_0'}{A_0'+B_0'}; \boldsymbol{\Delta}_0\right) \\
&= \rho\left(A_0', -\frac{B_0'}{A_0'+B_0'}; \rho\left(\frac{1}{A_0}, \frac{B_0}{A_0}; \boldsymbol{\Delta}\right)\right) \\
&= \rho\left(\frac{1}{\frac{1}{A_0'}}, \frac{\frac{-B_0'}{A_0'(A_0'+B_0')}}{\frac{1}{A_0'}}; \rho\left(\frac{1}{A_0}, \frac{B_0}{A_0}; \boldsymbol{\Delta}\right)\right) \\
&= \rho\left(\frac{A_0'}{A_0}, \frac{B_0A_0' - B_0'A_0}{A_0(A_0'+B_0')}; \boldsymbol{\Delta}\right).
\end{aligned}$$

Therefore,

$$\begin{aligned} A &= \tfrac{A_0}{A_0'} = \tfrac{A_0}{A_0'} = \sqrt{\frac{f_7(\mathbf{\Delta}')\, f_9(\mathbf{\Delta})}{f_7(\mathbf{\Delta})\, f_9(\mathbf{\Delta}')}}, \\ B &= \tfrac{B_0 A_0' - A_0 B_0'}{A_0'(A_0'+B_0')} = \sqrt{\frac{f_7(\mathbf{\Delta}')\, f_9(\mathbf{\Delta})}{f_7(\mathbf{\Delta})\, f_9(\mathbf{\Delta}')}} \left(\sqrt{\frac{\Delta_3'^2 f_7(\mathbf{\Delta}')\, f_9(\mathbf{\Delta})}{\Delta_3^2 f_7(\mathbf{\Delta})\, f_9(\mathbf{\Delta}')}} - 1 \right). \end{aligned}$$

(ii) It is obvious.

Proposition 4.2. *All algebras of the subset U_{43} are isomorphic to* $L(1, 0, 0, 0, 1, -8, 45, 0)$.

Proof.

In single orbit U_{43}, where $\Delta_4 = \Delta_5 = \Delta_6 = \Theta_9 = 0$, let $A^4 = \dfrac{f_7(\mathbf{\Delta})}{\Delta_3}$ and $B = A\left(\dfrac{A}{\Delta_3} - 1\right)$. By substituting the values of $A, B, \Delta_4, \Delta_5, \Delta_6$ and Θ_9 into adapted change of basis $\{e_0', e_1', e_2', ..., e_9'\}$ in FLb_{10}, we obtain new basis for U_{43} as follows:

$$\begin{aligned} e_0' &= Ae_0 + Be_1, \\ e_1' &= (A+B)\, e_1, \\ e_2' &= (A+B)(Ae_2 + B(e_3 - 2\,e_4 + 5\,e_5 - 14\,e_6 + 43\,e_7 - 140\,e_8 + 474e_9), \\ e_3' &= (A+B)\,(A^2 e_3 + 2ABe_4 + B(-4A+B)e_5 + 2B(5A-2B)e_6 + 14B \\ &\quad (-2A+B)e_7 + 2B(43A - 24B)e_8 + B(-280A + 167B)e_9), \\ e_4' &= (A+B)\,(A^3 e_4 + 3\,A^2 Be_5 + 3\,AB(-2\,A+B)e_6 + 3AB(5A-4B)e_7 + B^3 e_7 \\ &\quad - 42AB(A+B)e_8 - 6B^3 e_8 + 3B(43A^2 - 48AB + 9B^2)e_9), \\ e_5' &= (A+B)\,A(A^2(Ae_5 + 4Be_6) - 2AB(4A-3B)e_7 + 4B(5A^2 - 6AB + B^2)e_8 \\ &\quad - 7AB(8A - 12B)e_9) - B^3(24A+B)e_9), \\ e_6' &= (A+B)\,A^2(A^3 e_6 + 5B(A^2 e_7 - 2A(A-B)e_8 + (5A^2 - 8AB + 2B^2)e_9)), \\ e_7' &= (A+B)\,A^4(A^2 e_7 + 6ABe_8 - 3B(4Ae_9 - 5B)e_9), \\ e_8' &= (A+B)\,A^6(Ae_8 + 7Be_9), \\ e_9' &= (A+B)\,A^8 e_9. \end{aligned}$$

Then by substituting the values of $A, B, \Delta_4, \Delta_5, \Delta_6$ and Θ_9 into the table of multiplication, we have the following:

$$
\begin{aligned}
&[e_0', e_0'] = e_2,\\
&[e_i', e_0'] = e_{i+1} \quad \text{for} \quad 1 \leqslant i \leqslant 8,\\
&[e_0', e_1'] = e_3 - 2e_4 + 5e_5 - 14e_6 + 43e_7 - 140e_8 + 438e_9,\\
&[e_1', e_1'] = e_3 - 2e_4 + 5e_5 - 14e_6 + 43e_7 - 140e_8 + 438e_9,\\
&[e_2', e_1'] = e_4 - 2e_5 + 5e_6 - 14e_7 + 43e_8 - 140e_9,\\
&[e_3', e_1'] = e_5 - 2e_6 + 5e_7 - 14e_8 + 43e_9,\\
&[e_4', e_1'] = e_6 - 2e_7 + 5e_8 - 14e_9,\\
&[e_5', e_1'] = e_7 - 2e_8 + 5e_9,\\
&[e_6', e_1'] = e_8 - 2e_9,\\
&[e_7', e_1'] = e_9.
\end{aligned}
$$

This means that for $L(\Delta_3, \Delta_4, ..., \Delta_9, \Theta_9) \in U_{43}$, we have

$$\rho\left(\frac{1}{A}, \frac{B}{A}; \mathbf{\Delta}\right) = L\left(1, 0, 0, 0, 1, -8, 45, 0\right).$$

As it has been stated earlier that the disjoint subsets of FLb_{10} above are invariant with respect to the action of G_{ad}, therefore the isomorphism problem can be studied for each of them separately. As a result, we obtain 27 isolated classes and 42 parametric families of algebras. The final result we give as a table as follows.

Table 1: Parametric Family of Orbits in FLb_{10}.

U_i	Representative	Invariant Functions
U_1	$L(1, 1, \lambda_1 - 5, \lambda_2 - 6\lambda_1 + 18, \lambda_3 - 7\lambda_2$ $+21\lambda_1 - 66, \lambda_4 - 8\lambda_3 + 28\lambda_2 - 48\lambda_1$ $-4\lambda_1^2 + 242, \lambda_5 - 9\lambda_4 + 36\lambda_3 - 84\lambda_2$ $+954\lambda_1 - 9\lambda_1^2 - 9\lambda_1\lambda_2 - 4332, \lambda_6)$	$F_1(\mathbf{X}) = \frac{x_3}{x_4^2} f_5(\mathbf{X}), F_2(\mathbf{X}) = \frac{x_3^2}{x_4^3} f_6(\mathbf{X}),$ $F_3(\mathbf{X}) = \frac{x_3^3}{x_4^4} f_7(\mathbf{X}), F_4(\mathbf{X}) = \frac{x_3^4}{x_4^5} f_8(\mathbf{X}),$ $F_5(\mathbf{X}) = \frac{x_3^5}{x_4^6} f_9(\mathbf{X}), F_6(\mathbf{X}) = \frac{x_3^6}{x_4^7} \Theta_9$
U_2	$L(1, 0, \lambda_1, -5\lambda_1, \lambda_1\lambda_2 + 21\lambda_1, \lambda_1\lambda_3$ $-8\lambda_1\lambda_2 - 84\lambda_1 - 4\lambda_1^2, \lambda_1\lambda_4 + 45\lambda_1\lambda_2$ $-9\lambda_1\lambda_3 + 1842\lambda_1 - 36\lambda_1^2, \lambda_5)$	$F_1(\mathbf{X}) = \frac{x_5^3(\mathbf{X})}{x_3 f_6^2(\mathbf{X})}, F_2(\mathbf{X}) = \frac{f_5(\mathbf{X}) f_7(\mathbf{X})}{f_6^2(\mathbf{X})},$ $F_3(\mathbf{X}) = \frac{f_5^2(\mathbf{X}) f_8(\mathbf{X})}{f_6^3(\mathbf{X})}, F_4(\mathbf{X}) = \frac{f_5^2(\mathbf{X}) f_9(\mathbf{X})}{f_6^4(\mathbf{X})},$ $F_5(\mathbf{X}) = \frac{f_5^7(\mathbf{X})\Theta_9}{f_6^7(\mathbf{X})}.$
U_3	$L(1, 0, \lambda_1, -6\lambda_1, 28\lambda_1 + \lambda_1^2\lambda_2, -120\lambda_1$ $-8\lambda_1^2\lambda_2 - 39\lambda_1^2, 495\lambda_1 + 45\lambda_1^2\lambda_2$ $+36\lambda_1^2 + \lambda_3, \lambda_4)$	$F_1(\mathbf{X}) = \frac{f_5^5(\mathbf{X})}{x_3^3 f_8^2(\mathbf{X})}, F_2(\mathbf{X}) = \frac{x_3 f_7(\mathbf{X})}{f_5^2(\mathbf{X})},$ $F_3(\mathbf{X}) = \frac{x_3^2 f_9(\mathbf{X})}{f_5^3(\mathbf{X})}, F_4(\mathbf{X}) = \frac{f_5^{14}(\mathbf{X})\Theta_9}{x_3^7 f_8^7(\mathbf{X})}.$

U_i	Representative	Invariant Functions
U_4	$L(1,0,\lambda_1,-6\lambda_1,28\lambda_1+\lambda_1\lambda_2,-120\lambda_1$ $-8\lambda_1\lambda_2-40\lambda_1^2,495\lambda_1+45\lambda_1\lambda_2+45\lambda_1^2$ $+\lambda_1^3\lambda_3,\lambda_1^3)$	$F_1(\mathbf{X})=\frac{f_5^7(\mathbf{X})}{x_3^7\Theta_9^2}, F_2(\mathbf{X})=\frac{x_3f_7(\mathbf{X})}{f_5^2(\mathbf{X})}, F_3(\mathbf{X})=\frac{x_3^2f_9(\mathbf{X})}{f_5^3(\mathbf{X})}.$
U_5	$L(1,0,1,-6,\lambda_1-56,-8\lambda_1+512,72\lambda_1$ $+\lambda_2-4455,0)$	$F_1(\mathbf{X})=\frac{x_3f_7(\mathbf{X})}{f_5^2(\mathbf{X})}, F_2(\mathbf{X})=\frac{x_3^2f_9(\mathbf{X})}{f_5^3(\mathbf{X})}.$
U_6	$L(1,0,0,\lambda_1,-6\lambda_1,\lambda_1\lambda_2+20\lambda_1,\lambda_1^2\lambda_3$ $-9\lambda_1\lambda_2-48\lambda_1,\lambda_4)$	$F_1(\mathbf{X})=\frac{f_6^4(\mathbf{X})}{x_3f_7^3(\mathbf{X})}, F_2(\mathbf{X})=\frac{f_6(\mathbf{X})f_8(\mathbf{X})}{f_7^2(\mathbf{X})}, F_3(\mathbf{X})=\frac{x_3f_9(\mathbf{X})}{f_6^2(\mathbf{X})}, F_4(\mathbf{X})=\frac{f_6^7(\mathbf{X})\Theta_9}{f_7^7(\mathbf{X})}.$
U_7	$L(1,0,0,\lambda_1,-7\lambda_1,\lambda_1^2+36\lambda_1,\lambda_1^2\lambda_2$ $-9\lambda_1^2-165\lambda_1,\lambda_3)$	$F_1(\mathbf{X})=\frac{x_3^2f_8^3(\mathbf{X})}{f_6^5(\mathbf{X})}, F_2(\mathbf{X})=\frac{x_3f_9(\mathbf{X})}{f_6^2(\mathbf{X})}, F_3(\mathbf{X})=\frac{x_3^7\Theta_9f_8^7(\mathbf{X})}{f_6^{14}(\mathbf{X})}.$
U_8	$L(1,0,0,\lambda_1,-7\lambda_1,36\lambda_1,\lambda_1^2\lambda_2-165\lambda_1,\lambda_1^2)$	$F_1(\mathbf{X})=\frac{f_6^7(\mathbf{X})}{\Theta_9^3x_3^7}, F_2(\mathbf{X})=\frac{x_3f_9(\mathbf{X})}{f_6^2(\mathbf{X})}.$
U_9	$L(1,0,0,1,-7,36,\lambda-165,0)$	$F(\mathbf{X})=\frac{x_3f_9(\mathbf{X})}{f_6^2(\mathbf{X})}.$
U_{10}	$L(1,0,0,0,\lambda_1,-7\lambda_1,\lambda_1\lambda_2+36\lambda_1,\lambda_3)$	$F_1(\mathbf{X})=\frac{f_7^5(\mathbf{X})}{x_3f_8^4(\mathbf{X})}, F_2(\mathbf{X})=\frac{f_7(\mathbf{X})f_9(\mathbf{X})}{f_8^2(\mathbf{X})}, F_3(\mathbf{X})=\frac{f_7^7(\mathbf{X})\Theta_9}{f_8^7(\mathbf{X})}.$
U_{11}	$L(1,0,0,0,\lambda_1,-8\lambda_1,\lambda_1\lambda_2+45\lambda_1,\lambda_2^3)$	$F_1(\mathbf{X})=\frac{f_9^{12}(\mathbf{X})}{x_3f_7^{11}(\mathbf{X})\Theta_9^4}, F_2(\mathbf{X})=\frac{f_9^7(\mathbf{X})}{f_7^7(\mathbf{X})\Theta_9^2}.$
U_{12}	$L\,(1,0,0,0,\lambda,-8\lambda,1+45\lambda,0)$	$F(\mathbf{X})=\frac{f_7^3\,(\mathbf{X})}{x_3f_9^2\,(\mathbf{X})}.$
U_{13}	$L\,(1,0,0,0,\lambda,-8\lambda,45\lambda,\lambda)$	$F(\mathbf{X})=\frac{f_7^{7/3}(\mathbf{X})}{x_3^{7/3}\Theta_9^{4/3}}.$
U_{14}	$L\,(1,0,0,0,0,\lambda_1,-8\lambda_1,\lambda_2)$	$F_1(\mathbf{X})=\frac{f_8^6(\mathbf{X})}{x_3f_9^5(\mathbf{X})}, F_2(\mathbf{X})=\frac{f_8^7(\mathbf{X})\Theta_9}{f_9^7(\mathbf{X})}.$
U_{15}	$L\,(1,0,0,0,0,\lambda,-9\lambda,\lambda)$	$F(\mathbf{X})=\frac{f_8^{7/2}(\mathbf{X})}{x_3^{7/2}\Theta_9^{5/2}}.$
U_{16}	$L\,(1,0,0,0,0,0,\lambda,\lambda)$	$F(\mathbf{X})=\frac{f_9^7(\mathbf{X})}{x_3^7\Theta_9^6}.$
U_{17}	$L(0,1,1,\lambda_1-3,\lambda_2-7,\lambda_3-8\lambda_1+8,$ $\lambda_4-9\lambda_2-9\lambda_1+45,\lambda_5)$	$F_1(\mathbf{X})=\frac{x_4f_6(\mathbf{X})}{f_5^2(\mathbf{X})}, F_2(\mathbf{X})=\frac{x_4^2f_7(\mathbf{X})}{f_5^3(\mathbf{X})}, F_3(\mathbf{X})=\frac{x_4^3f_8(\mathbf{X})}{f_5^4(\mathbf{X})}, F_4(\mathbf{X})=\frac{x_4^4f_9(\mathbf{X})}{f_5^5(\mathbf{X})}, F_5(\mathbf{X})=\frac{\Theta_9x_4^7}{f_5^7(\mathbf{X})}.$
U_{18}	$L(0,1,0,\lambda_1-3,\lambda_1,\lambda_1^2\lambda_2-8\lambda_1$ $+12,\lambda_1\lambda_3-9\lambda_1,\lambda_4)$	$F_1(\mathbf{X})=\frac{f_6^3(\mathbf{X})}{f_7^2(\mathbf{X})x_4}, F_2(\mathbf{X})=\frac{x_4f_8(\mathbf{X})}{f_6^2(\mathbf{X})}, F_3(\mathbf{X})=\frac{f_6^2(\mathbf{X})f_9(\mathbf{X})}{f_7^3(\mathbf{X})}, \lambda_4=\frac{\Theta_9f_6^7(\mathbf{X})}{f_7^7(\mathbf{X})}.$
U_{19}	$L(0,1,0,\lambda_1-3,0,\lambda_1^2-8\lambda_1$ $+12,\lambda_1^3,\lambda_3)$	$F_1(\mathbf{X})=\frac{f_6^5(\mathbf{X})}{x_4^3f_9^2(\mathbf{X})}, F_2(\mathbf{X})=\frac{x_4f_8(\mathbf{X})}{f_6^2(\mathbf{X})}, F_3(\mathbf{X})=\frac{\Theta_9f_6^{14}(\mathbf{X})}{x_4^7f_9^7(\mathbf{X})}.$
U_{20}	$L(0,1,0,\lambda_1-3,0,\lambda_1^2\lambda_2-8\lambda_1+12,0,\lambda_1^3)$	$F_1(\mathbf{X})=\frac{f_6^7(\mathbf{X})}{\Theta_9^2x_4^7}, F_2(\mathbf{X})=\frac{x_4f_8(\mathbf{X})}{f_6^2(\mathbf{X})}.$
U_{21}	$L\,(0,1,0,-2,0,\lambda+4,0,0)$	$F(\mathbf{X})=\frac{x_4f_8(\mathbf{X})}{f_6^2(\mathbf{X})}.$

U_i	Representative	Invariant Functions
U_{22}	$L(0,1,0,-3,\lambda_1,\lambda_1+12,\lambda_1\lambda_2-9\lambda_1,\lambda_2)$	$F_1(\mathbf{X})=\frac{f_7^4(\mathbf{X})}{x_4f_8^3(\mathbf{X})}, F_2(\mathbf{X})=\frac{f_7(\mathbf{X})f_9(\mathbf{X})}{f_8^2(\mathbf{X})}, F_3(\mathbf{X})=\frac{f_7^7(\mathbf{X})\Theta_9}{f_8^7(\mathbf{X})}.$
U_{23}	$L(0,1,0,-3,\lambda_1,12,\lambda_1^2-9\lambda_1,\lambda_2)$	$F_1(\mathbf{X})=\frac{x_4^2f_9^3(\mathbf{X})}{f_7^5(\mathbf{X})}, F_2(\mathbf{X})=\frac{x_4^7\Theta_9f_9^7(\mathbf{X})}{f_7^{14}(\mathbf{X})}.$
U_{24}	$L\left(0,1,0,-3,\lambda,12,-9\lambda,\lambda^2\right)$	$F(\mathbf{X})=\frac{f_7^7(\mathbf{X})}{x_4^7\Theta_9^3}.$
U_{25}	$L\left(0,1,0,-3,0,\lambda_1+12,\lambda_1,\lambda_2\right)$	$F_1(\mathbf{X})=\frac{f_8^5(\mathbf{X})}{x_4f_9^4(\mathbf{X})}, F_2(\mathbf{X})=\frac{\Theta_9f_8^7(\mathbf{X})}{f_9^7(\mathbf{X})}.$
U_{26}	$L(0,1,0,-3,0,\lambda+12,0,\lambda^2)$	$F(\mathbf{X})=\frac{x_4^{14}\Theta_9^8}{f_8^{14}(\mathbf{X})}.$
U_{27}	$L\left(0,1,0,-3,0,12,\lambda^2,\lambda^3\right)$	$F(\mathbf{X})=\frac{\Theta_9^5x_4^7}{f_9^7(\mathbf{X})}.$
U_{28}	$L(0,0,1,1,\lambda_1,\lambda_2-4,\lambda_3-9,\lambda_4)$	$F_1(\mathbf{X})=\frac{f_5(\mathbf{X})f_7(\mathbf{X})}{f_6^2(\mathbf{X})}, F_2(\mathbf{X})=\frac{f_5^2(\mathbf{X})f_8(\mathbf{X})}{f_6^3(\mathbf{X})}, F_3(\mathbf{X})=\frac{f_5^3(\mathbf{X})f_9(\mathbf{X})}{f_6^4(\mathbf{X})}, F_4(\mathbf{X})=\frac{\Theta_9f_5^7(\mathbf{X})}{f_6^7(\mathbf{X})}.$
U_{29}	$L\left(0,0,1,0,\lambda_1,\lambda_1-4,\lambda_1\lambda_2,\lambda_3\right)$	$F_1(\mathbf{X})=\frac{f_7^3(\mathbf{X})}{f_5(\mathbf{X})f_8^2(\mathbf{X})}, F_2(\mathbf{X})=\frac{f_7(\mathbf{X})f_9(\mathbf{X})}{f_8^2(\mathbf{X})}, F_3(\mathbf{X})=\frac{f_7^7(\mathbf{X})\Theta_9}{f_8^7(\mathbf{X})}.$
U_{30}	$L(0,0,1,0,\lambda_1,-4,\lambda_1^2\lambda_2,\lambda_1^3)$	$F_1(\mathbf{X})=\frac{f_7^7(\mathbf{X})}{\Theta_9^2f_5^7(\mathbf{X})}, F_2(\mathbf{X})=\frac{f_5(\mathbf{X})f_9(\mathbf{X})}{f_7^2(\mathbf{X})}.$
U_{31}	$L\left(0,0,1,0,1,-4,\lambda,0\right)$	$F(\mathbf{X})=\frac{f_5(\mathbf{X})f_9(\mathbf{X})}{f_7^2(\mathbf{X})}.$
U_{32}	$L\left(0,0,1,0,\lambda,-4,0,\lambda^3\right)$	$F(\mathbf{X})=\frac{f_7^7(\mathbf{X})}{\Theta_9^2f_5^7(\mathbf{X})}.$
U_{33}	$L\left(0,0,1,0,0,\lambda_1-4,\lambda_1,\lambda_2\right)$	$F_1(\mathbf{X})=\frac{f_8^4(\mathbf{X})}{f_5(\mathbf{X})f_9^3(\mathbf{X})}, F_2(\mathbf{X})=\frac{\Theta_9f_8^7(\mathbf{X})}{f_9^7(\mathbf{X})}.$
U_{34}	$L\left(0,0,1,0,0,\lambda-4,0,\lambda^2\right)$	$F(\mathbf{X})=\frac{f_8^7(\mathbf{X})}{f_5^7(\mathbf{X})\Theta_9^3}.$
U_{35}	$L(0,0,1,0,0,-4,\lambda,\lambda^2)$	$F(\mathbf{X})=\frac{f_5^{14}(\mathbf{X})\Theta_9^8}{f_9^{14}(\mathbf{X})}.$
U_{36}	$L\left(0,0,0,1,1,\lambda_1,\lambda_2,\lambda_3\right)$	$F_1(\mathbf{X})=\frac{f_6(\mathbf{X})f_8(\mathbf{X})}{f_7^2(\mathbf{X})}, F_2(\mathbf{X})=\frac{f_6^2(\mathbf{X})f_9(\mathbf{X})}{f_7^3(\mathbf{X})}, F_3(\mathbf{X})=\frac{f_6^7(\mathbf{X})\Theta_9}{f_7^7(\mathbf{X})}.$
U_{37}	$L\left(0,0,0,1,0,\lambda_1,\lambda_1,\lambda_2\right)$	$F_1(\mathbf{X})=\frac{f_8^3(\mathbf{X})}{f_6(\mathbf{X})f_9^2(\mathbf{X})}, F_2(\mathbf{X})=\frac{\Theta_9f_8^7(\mathbf{X})}{f_9^7(\mathbf{X})}.$
U_{38}	$L\left(0,0,0,1,0,\lambda,0,\lambda^3\right)$	$F(\mathbf{X})=\frac{f_8^7(\mathbf{X})}{f_6^7(\mathbf{X})\Theta_9^2}.$
U_{39}	$L\left(0,0,0,1,0,0,\lambda,\lambda^2\right)$	$F(\mathbf{X})=\frac{f_9^7(\mathbf{X})}{f_6^7(\mathbf{X})\Theta_9^3}.$
U_{40}	$L\left(0,0,0,0,1,1,\lambda_1,\lambda_2\right)$	$F_1(\mathbf{X})=\frac{f_7(\mathbf{X})f_9(\mathbf{X})}{f_8^2(\mathbf{X})}, F_2(\mathbf{X})=\frac{f_7^7(\mathbf{X})\Theta_9}{f_8^7(\mathbf{X})}.$
U_{41}	$L\left(0,0,0,0,1,0,\lambda,\lambda^3\right)$	$F(\mathbf{X})=\frac{f_9^7(\mathbf{X})}{f_7^7(\mathbf{X})\Theta_9^2}.$
U_{42}	$L\left(0,0,0,0,0,1,1,\lambda\right)$	$F(\mathbf{X})=\frac{\Theta_9f_8^7(\mathbf{X})}{f_9^7(\mathbf{X})}.$

where $\mathbf{X}=(x_3,x_4,x_5,x_6,x_7,x_8,x_9)$.

Remark. *In the table above*

- *The first column represents disjoint subsets of* FLb_{10};
- *The second column gives the canonical representatives of the disjoint subsets;*
- *In the third column we represent the list of invariant functions to state the isomorphism criterion for the corresponding disjoint subsets (see Proposition 3.1).*

Table 2: Single Orbits of FLb_{10}.

U_i	Representative of orbits	U_i	Representative of orbits
U_{43}	$L(1,0,0,0,1,-8,45,0)$	U_{57}	$L(0,0,0,1,0,1,0,0)$
U_{44}	$L(1,0,0,0,0,1,-9,0)$	U_{58}	$L(0,0,0,1,0,0,1,0)$
U_{45}	$L(1,0,0,0,0,0,1,0)$	U_{59}	$L(0,0,0,1,0,0,0,1)$
U_{46}	$L(1,0,0,0,0,0,0,1)$	U_{60}	$L(0,0,0,1,0,0,0,0)$
U_{47}	$L(1,0,0,0,0,0,0,0)$	U_{61}	$L(0,0,0,0,1,0,1,0)$
U_{48}	$L(0,1,0,-3,1,12,-9,0)$	U_{62}	$L(0,0,0,0,1,0,0,1)$
U_{49}	$L(0,1,0,-3,0,13,0,0)$	U_{63}	$L(0,0,0,0,1,0,0,0)$
U_{50}	$L(0,1,0,-3,0,12,1,0)$	U_{64}	$L(0,0,0,0,0,1,0,1)$
U_{51}	$L(0,1,0,-3,0,12,0,1)$	U_{65}	$L(0,0,0,0,0,1,0,0)$
U_{52}	$L(0,1,0,-3,0,12,0,0)$	U_{66}	$L(0,0,0,0,0,0,1,1)$
U_{53}	$L(0,0,1,0,0,-4,0,0)$	U_{67}	$L(0,0,0,0,0,0,1,0)$
U_{54}	$L(0,0,1,0,0,-4,0,1)$	U_{68}	$L(0,0,0,0,0,0,0,1)$
U_{55}	$L(0,0,1,0,0,-3,0,0)$	U_{69}	$L(0,0,0,0,0,0,0,0)$
U_{56}	$L(0,0,1,0,1,-4,0,0)$		

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