Malaysian Journal of Mathematical Sciences 15(1): 79–90 (2021)

MALAYSIAN JOURNAL OF MATHEMATICAL SCIENCES

Journal homepage: http://einspem.upm.edu.my/journal

Edge Irregular k-labeling for Disjoint Union of Cycles and Generalized Prisms

Hasni, R.*[1], Tarawneh, I.[2], Siddiqui, M. K.[3], Raheem, A.[4], and Asim, M. A.[5]

[1]*Faculty of Ocean Engineering Technology and Informatics, Universiti Malaysia Terengganu, Malaysia*
[2]*Khalid lbn Al-Walid School, Karak, Jordan*
[3]*Department of Mathematics, COMSATS University of Islamabad, Lahore Campus, Pakistan*
[4]*Department of Higher Education, Govt. Postgraduate College Asghar Mall, Rawalpindi, Pakistan*
[5]*College of Computer Science and Information Tehnology, Jazan University, Jazan, KSA*

E-mail: hroslan@umt.edu.my
* *Corresponding author*

Received: 22 January 2020
Accepted: 29 November 2020

ABSTRACT

For a simple graph G, a vertex labeling $\phi : V(G) \to \{1, 2, \ldots, k\}$ is called k-labeling. The weight of an edge xy in G, denoted by $w_\pi(xy)$, is the sum of the labels of end vertices x and y, i.e. $w_\phi(xy) = \phi(x) + \phi(y)$. A vertex k-labeling is defined to be an edge irregular k-labeling of the graph G if for every two different edges e and f, there is $w_\phi(e) \neq w_\phi(f)$. The minimum k for which the graph G has an edge irregular k-labeling is called the edge irregularity strength of G, denoted by $es(G)$. In this paper, we determine the exact value of edge irregularity strength of disjoint union of cycles and generalized prisms. These results proved the upper bound for edge irregularity strength for such graphs and can be used in future for addressing schemes in different applications.

Keywords: Irregular assignment, edge irregularity strength, disjoint union, cycle, generalized prism.

1. Introduction

A graph G consists of finite nonempty set of vertices $V(G)$ and edges $E(G)$ is a useful discrete structures due to its wide range applications. A graph can be classified in different ways due its architecture as directed or undirected, connected or disconnected similarly cyclic or acyclic. A graph is often built from another graph by applying certain graph operations like addition or deletion of vertices or edges, transpose or complement, union or intersection. Two graphs are said to be an edge disjoint if they share no edges, and vertex disjoint if they share no vertices. Disjoint sub-graphs are obtained by decomposing a super-graph or sometime they are unite to compose one super-graph. Disjoint graphs can be used to represent inter-networks and cluster networks and labeling for such graphs can improve these fields.

Graph labeling is an assignment of integers to the vertices, edges or both so the labeling is called vertex labeling, edge labeling or total labeling respectively. Chartrand et al. (1988) defined irregular labeling as an assignment to the edges of G such that the sum of all labels of all incident edges at each vertex are different. The minimum value of the largest label of an edge from all existing irregular labelings is known as *irregularity strength* denoted as $s(G)$. In Chartrand et al. (1988), the authors determined the lower bound of irregularity strength for m isomorphic copies of complete graphs $K_n, n \geq 5$. Faudree et al. (1989) determined the upper bound for m isomorphic copies of complete graphs $K_n, n \geq 5$. Kalkowski et al. (2011) determined the best upper bound is $s(G) \leq 6\lceil \frac{n}{\delta} \rceil$. Przybylo (2008),Cammack et al. (1991) and Aigner and Triesch (1990) added some results on irregularity strength $s(G)$ for trees and forests.

Motivated by the work of Chartrand et al. (1988), Bača et al. (2007) introduced two types of total k-labelings as edge irregular total k-labeling and vertex irregular total k-labeling. Total k-labeling is called *edge irregular total k-labeling* if for every two distinct pair of edges weights are distinct $wt_\phi(e) \neq wt_\phi(e')$, where $wt_\phi(e) = \phi(x) + \phi(xy) + \phi(y)$. The minimum k for which graph has an edge irregular total k-labeling is known as total edge irregularity strength of G and denoted by $tes(G)$. Total k-labeling is called *vertex irregular total k-labeling* if for every two distinct pair of vertices weights are distinct $wt_\phi(x) \neq wt_\phi(x')$, where $wt_\phi(x) = \phi(x) + \sum_{y \in N(x)} \phi(xy)$. The minimum k for which G has a vertex irregular total k-labeling is known as total vertex irregularity strength, denoted by $tvs(G)$. Some results on total edge irregularity strength and total vertex irregularity strength can be found in Ahmad et al. (2012a,b, 2014b), Baskoro et al. (2010), Jendroľ et al. (2010).

Ahmad et al. (2014a) introduced the edge irregular k-labeling as $\phi : V \to \{1, 2, \ldots, k\}$. In this labeling vertices are labeled with a condition that edge weight for every two edges must be unique. While weight of any edge $e = xy \in E(G)$ is calculated as $w_\phi(xy) = \phi(x) + \phi(y)$. The minimum k for which the graph G has an edge irregular k-labeling is called *edge irregularity strength* of G, denoted by $es(G)$.

The following theorem established the lower bound of the edge irregularity strength for any graph G.

Theorem 1.1. *(Ahmad et al. (2014a)) Let* $G = (V, E)$ *be a simple graph with maximum degree* $\Delta = \Delta(G)$. *Then*

$$es(G) \geq \max\left\{ \left\lceil \frac{|E(G)| + 1}{2} \right\rceil, \Delta(G) \right\}.$$

In Ahmad et al. (2014a), the values of $es(G)$ for path, cycle, star, double star, and Cartesian product of two paths are calculated. Tarawneh et al. (2018, 2019) determined the irregularity strength of disjoint union of star graph, certain families of grid graphs and zigzag graphs. Tarawneh et al. (2016a,b) calculated the edge irregularity strength of corona product of graphs with paths and corona product of cycle with isolated vertices. Imran et al. (2017) determined the exact value of the edge irregularity strength of caterpillars, n-star graphs, (n, t)-kite graphs, cycle chains and friendship graphs. Asim et al. (2018) computed the improved upper-bound for complete graphs using algorithmic approach. Ahmad et al. (2018) computed edge irregularity strength of complete m-ary trees using algorithmic approach. Most of the above articles are compiled recently in Gallian (2018), as dynamic survey of graph labeling.

In this paper, we determined the exact values of edge irregularity strength of disjoint union of cycles and generalized prisms. Bača and Siddiqui (2014) has already proved total edge irregularity strength of generalized prism.

2. Edge irregular k-labeling of disjoint union of cycles

Let C_n be a cycle with $n \geq 3$ edges. In the following, we will consider the exact value of edge irregularity strength of disjoint union of two cycles C_{n_1} and C_{n_2} where $n_1, n_2 \geq 3$.

Theorem 2.1. *For any integer $n_1, n_2 \geq 4$, $n_1 \equiv 0\,(mod 4)$ and $n_2 \equiv 0, 1, 2\,(mod 4)$, then*

$$es(C_{n_1} \cup C_{n_2}) = \lceil \frac{n_1 + n_2 + 1}{2} \rceil.$$

Proof. Let $V(C_{n_1}) = \{x_i, 1 \leq i \leq n_1\}$, $E(C_{n_1}) = \{x_i x_{i+1}, 1 \leq i \leq n_1 - 1\} \cup \{x_{n_1} - 1\}$, $V(C_{n_2}) = \{y_j, 1 \leq j \leq n_2\}$ and $E(C_{n_2}) = \{y_j y_{j+1}, 1 \leq j \leq n_2 - 1\} \cup \{x_{n_2} - 1\}$. According to Theorem 1.1, we have $es(C_{n_1} \cup C_{n_2}) \geq \lceil \frac{n_1+n_2+1}{2} \rceil$.

To prove the equality, we define $\phi_1 : V(C_{n_1} \cup C_{n_2}) \rightarrow \{1, 2, \ldots, \lceil \frac{n_1+n_2+1}{2} \rceil\}$ be the vertex labeling such that

$$\phi_1(x_i) = \begin{cases} 2\lfloor \frac{i-1}{2} \rfloor + 1, & \text{if } 1 \leq i \leq \frac{n_i}{2} + 1, \\ n_1 - i + 2, & \text{if } \frac{n_i}{2} + 2 \leq i \leq n_1. \end{cases}$$

$$\phi_1(y_i) = \begin{cases} \frac{n_1}{2} + 2\lfloor \frac{j-1}{2} \rfloor + 1, & \text{if } 1 \leq j \leq \lfloor \frac{n_2}{2} \rfloor, \\ & \text{and } j = \lceil \frac{n_2}{2} + 1 \rceil \text{ if } n_2 \equiv 1, 2\,(\text{mod}\,4), \\ \frac{n_1+n_2+1}{2}, & \text{if } \frac{n_j}{2} + 1 \leq j \leq \frac{n_j}{2} + 2, \text{ and } n_2 \equiv 2\,(\text{mod}\,4), \\ \frac{n_1+2n_2+1}{4}, & \text{if } j = \lceil \frac{n_2}{2} \rceil,\ n_2 \equiv 0, 1\,(\text{mod}\,4), \\ & \text{and } \lceil \frac{n_1}{2} + 3 \leq j \leq n_2. \end{cases}$$

The weight of the edges are as follows:

$$w_{\phi_1}(x_i x_{i+1}) = \begin{cases} 2i, & \text{if } 1 \leq i \leq \frac{n_1}{2}, \\ 2(n_1 - i) + 3, & \text{if } \frac{n_1}{2} + 1 \leq i \leq n_1 - 1. \end{cases}$$

$$w_{\phi_1}(x_{n_1} x_1) = 3.$$

$$w_{\phi_1}(y_iy_{j+1}) = \begin{cases} n_1+2j, & 1 \le j \le \lceil \frac{n_2}{2} \rceil - 1 \\ & \text{and } j = \lceil \frac{n_2}{2} \rceil \text{if } n_2 \equiv 0,1 \,(\text{mod}\, 4), \\ \frac{2n_1+n_2+4}{2} + 2\lfloor \frac{j-1}{2} \rfloor, & \text{if } \frac{n_2}{2} \le j \le \frac{n_2}{2}+1 \\ & \text{and } n_2 \equiv 2 \,(\text{mod}\, 4), \\ n_1+n_2+2\lfloor \frac{j-1}{2} \rfloor - j + 3, & j = \lceil \frac{n_2}{2} \rceil + 1 \text{ if } n_2 \equiv 0,1 \,(\text{mod}\, 4), \\ n_1 + \frac{3n_2}{2} + 2\lfloor \frac{j-1}{2} \rfloor - j + 4, & j = \frac{n_2}{2} + 2 \text{ if } n_2 \equiv 2 \,(\text{mod}\, 4), \\ n_1+n_2+1, & j = \frac{n_2}{2} + 1 \text{ if } n_2 \equiv 2 \,(\text{mod}\, 4), \\ n_1+2n_2-2j+3, & j = \lceil \frac{n_2}{2} \rceil + 2 \text{ if } n_2 \equiv 0,1 \,(\text{mod}\, 4) \\ & \lceil \frac{n_2}{2} + 3 \le j \le n_2 - 1. \end{cases}$$

It is easy to verify that the edge weights are distinct for all pairs of distinct edges. Thus, the vertex labeling ϕ_1 is the desired edge irregular k-labeling. This completes the proof. □

The disconnected graph $\cup_{j=1}^{p} C_{n_j}$ consists of the vertex set and edge set as follows:

$$V(\cup_{j=1}^{p} C_{n_j}) = \cup_{j=1}^{p} \cup_{i=1}^{p_j} \{v_i^j\},$$
$$E(\cup_{j=1}^{p} C_{n_j}) = \cup_{j=1}^{p} \cup_{i=1}^{p_j} \{v_i^j v_{i+1}^j\}.$$

In the following theorem, we determine the exact value of edge irregularity strength of disjoint union of cycles $\cup_{j=1}^{p} C_{n_j}$ where $n_j \ge 3$ and $p \ge 3$.

Theorem 2.2. *For $n_j \ge 4$, $n_j \equiv 0\,(mod 4)$ and $p \ge 3$, $es(\cup_{j=1}^{p} C_{n_j}) = \lceil \frac{n_j p+1}{2} \rceil$.*

Proof. Let $\cup_{j=1}^{p} C_{n_j}$ be a graph with vertex set $V(\cup_{j=1}^{p} C_{n_j})$ and edge set $E(\cup_{j=1}^{p} C_{n_j})$. According to Theorem 1.1, we have that $es(\cup_{j=1}^{p} C_{n_j}) \ge \lceil \frac{n_j p+1}{2} \rceil$.

To prove the equality, we define $\phi_2 : V(\cup_{j=1}^{p} C_{n_j}) \to \{1, 2, \dots, \lceil \frac{n_j p+1}{2} \rceil\}$ be the vertex labeling such that

$$\phi_2(x_{i,j}) = \begin{cases} \frac{n_j(j-1)}{2} + 2\lfloor \frac{i-1}{2} \rfloor + 1, & \text{if } 1 \le i \le \frac{n_j}{2} + 1, 1 \le j \le p \\ \frac{n_j(j+1)}{2} - i + 2, & \text{if } \frac{n_j}{2} + 2 \le i \le n_j, 1 \le j \le p \end{cases}$$

The weight of the edges are as follows:

$$w_{\phi_2}(x_{i,j}x_{i+1,j}) = \begin{cases} n_j(j-1)+2i, & \text{if } 1 \le i \le \frac{n_j}{2}, 1 \le j \le p \\ n_j j+1, & \text{if } i = \frac{n_j}{2}+1, 1 \le j \le p \\ n_j(j+1)-2i+3, & \text{if } \frac{n_j}{2}+2 \le i \le n_j-1, 1 \le j \le p \end{cases}$$

$$w_{\phi_1}(x_{n_j,j}x_{1,j}) = n_j(j-1)+3.$$

It is easy to verify that the edge weights are distinct for all pairs of distinct edges. Thus, the vertex labeling ϕ_2 is the desired edge irregular k-labeling. This completes the proof. □

In the following figure, we present the labeling as in the proof of Theorem 2.2 for the case $n = 8$, $p = 4$.

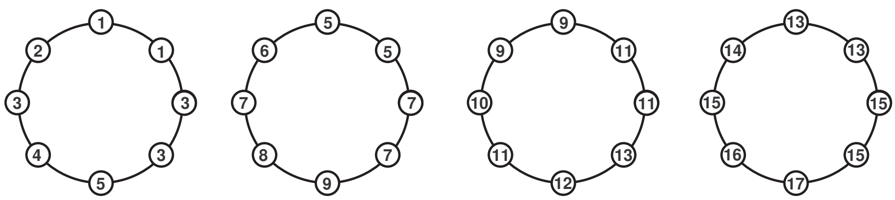

Figure 1: An edge irregular 17-labeling for $\cup_{j=1}^{4} C_{n_j}$, $n_j = 8$

3. Edge irregular k-labeling of disjoint generalized prism

In Theorem 2.2, we determined the exact value of edge irregularity strength $es(\cup_{j=1}^{p} C_{n_j})$ for $n_j \equiv 0 \,(\mathrm{mod}\, 4)$. We try to find $es(\cup_{j=1}^{p} C_{n_j})$ for all values of n_j but so far without success. So we conclude the following open problem.

Problem 1 Determine the exact value of $es(\cup_{j=1}^{p} C_{n_j})$ for all values of n_j.

The generalized prism $D_{n_j}^{m_j}$ is defined as the Cartesian product $C_{n_j} \times P_{m_j}$ of a cycle on n_j vertices with a path on m_j vertices. We shall determine the exact value of edge irregularity strength of disjoint union of generalized prisms, denoted by $\cup_{j=1}^{p} D_{n_j}^{m_j}$.

The vertex set and edge set of $\cup_{j=1}^{p} D_{n_j}^{m_j}$ are defined as follows: $V(\cup_{j=1}^{p} D_{n_j}^{m_j}) = \{x_{i,s}^{j} : 1 \leq i \leq n_j, 1 \leq s \leq m_j \,\text{and}\, 1 \leq j \leq p\}$ and $E(\cup_{j=1}^{p} D_{n_j}^{m_j}) = \{x_{i,s}^{j} x_{i+1,s}^{j} : 1 \leq i \leq n_j - 1, 1 \leq s \leq m_j \,\text{and}\, 1 \leq j \leq p\} \cup \{x_{n_j,s}^{j} x_{1,s}^{j} : 1 \leq i \leq m_j, 1 \leq j \leq p\} \cup \{x_{i,s}^{j} x_{i,s+1}^{j} : 1 \leq i \leq n_j, 1 \leq s \leq m_j - 1 \,\text{and}\, 1 \leq j \leq p\}$.

In the following theorem, we consider the exact value of edge irregularity strength of disjoint union of generalized prisms, denoted by $\cup_{j=1}^{p} D_{n_j}^{m_j}$. where $m_j \geq 2$, $n_j \geq 4$ and $n_j \equiv 0 \,(\text{mod}\, 4)$.

Theorem 3.1. *If $m_j \geq 2$, $n_j \geq 4$ and $n_j \equiv 0 \,(mod 4)$, then*

$$es(\cup_{j=1}^{p} D_{n_j}^{m_j}) = \lceil \frac{(2m_j - 1)n_j + 1}{2} \rceil.$$

Proof. Let $\cup_{j=1}^{p} D_{n_j}^{m_j}$ be a graph with vertex set $V(\cup_{j=1}^{p} D_{n_j}^{m_j})$ and edge set $E(\cup_{j=1}^{p} D_{n_j}^{m_j})$. According to Theorem 1.1, we have that $es(\cup_{j=1}^{p} D_{n_j}^{m_j}) \geq \lceil \frac{(2m_j-1)n_j+1}{2} \rceil$.

To prove the equality, we define $\phi_3 : V(\cup_{j=1}^{p} D_{n_j}^{m_j}) \to \{1, 2, \ldots, \lceil \frac{(2m_j-1)n_j+1}{2} \rceil\}$ be the vertex labeling such that

$$\phi_3(x^j_{i,s}) = \begin{cases} n_j(s-1)+2\lfloor\frac{i-1}{2}\rfloor+1, & 1\le i\le \frac{n_j}{2}+1, \\ & 1\le s\le m_j, s \text{ is odd}, \\ & 1\le j\le p, \\ n_js-i+2, & \frac{n_j}{2}+2\le i\le n_j, \\ & 1\le s\le m_j, s \text{ is odd}, \\ & 1\le j\le p, \\ n_j(s-1)+2\lfloor\frac{i}{2}\rfloor+1, & 1\le i\le \frac{n_j}{2}, \\ & 1\le s\le m_j, s \text{ is even}, \\ & 1\le j\le p, \\ n_js-i+1, & \frac{n_j}{2}+1\le i\le n_j, \\ & 1\le s\le m_j, s \text{ is even}, \\ & 1\le j\le p. \end{cases}$$

The weight of the edges are as follows:

$$w_{\phi_3}(x^j_{i,s}x^j_{i,s+1}) = \begin{cases} n_j(2s-1)+2i, & 1\le i\le \frac{n_j}{2}, 1\le s\le m_j \\ & \text{and } 1\le j\le p, \\ n_j(2s-1)+1, & i=\frac{n_j}{2}+1, 1\le s\le m_j \\ & \text{and } 1\le j\le p, \\ 2(n_js-i)+3, & \frac{n_j}{2}+2\le i\le n_j, 1\le s\le m_j \\ & \text{and } 1\le j\le p. \end{cases}$$

$$w_{\phi_3}(x^j_{i,s}x^j_{i+1,s}) = \begin{cases} 2n_j(s-1)+2i, & 1 \leq i \leq \frac{n_j}{2}, 1 \leq s \leq m_j \\ & \text{and } s \text{ is odd}, 1 \leq j \leq p, \\ n_j(2s-1)+1, & i = \frac{n_j}{2}+1, 1 \leq s \leq m_j \\ & \text{and } s \text{ is odd}, 1 \leq j \leq p, \\ 2(n_js-i)+3, & \frac{n_j}{2}+2 \leq i \leq n_j-1, 1 \leq s \leq m_j \\ & \text{and } s \text{ is odd}, 1 \leq j \leq p, \\ 2n_j(s-1)+2i+2, & 1 \leq i \leq \frac{n_j}{2}-1, 1 \leq s \leq m_j \\ & \text{and } s \text{ is even}, 1 \leq j \leq p, \\ n_j(2s-1)+1, & i = \frac{n_j}{2}, 1 \leq s \leq m_j \\ & \text{and } s \text{ is even}, 1 \leq j \leq p, \\ 2(n_js-i)+1, & \frac{n_j}{2}+1 \leq i \leq n_j-1, 1 \leq s \leq m_j \\ & \text{and } s \text{ is even}, 1 \leq j \leq p, \end{cases}$$

$$w_{\phi_3}(x^j_{n_j,s}x^j_{1,s}) = \begin{cases} 2n_j(s-1)+3, & 1 \leq s \leq m_j, 1 \leq j \leq p \\ & \text{and } s \text{ is odd}, \\ 2n_j(s-1)+2, & 1 \leq s \leq m_j, 1 \leq j \leq p \\ & \text{and } s \text{ is even}. \end{cases}$$

It is easy to verify that the edge weights are distinct for all pairs of distinct edges. Thus, the vertex labeling ϕ_3 is the desired edge irregular k-labeling.

This completes the proof. □

In Theorem 3.1, we determined the exact value of edge irregularity strength $es(\cup_{j=1}^{p} D_{n_j}^{m_j})$ for $n_j \equiv 0 \pmod 4$ and $m_j \geq 2$. We try to find $es(\cup_{j=1}^{p} D_{n_j}^{m_j})$ for all values of n_j but so far without success. So we conclude the following open problem.

Problem 2 Determine the exact value of $es(\cup_{j=1}^{p} D_{n_j}^{m_j})$ for $m_j \geq 2$ and all values of n_j.

In the next corollary, we determine the exact value of edge irregularity strength of disjoint union of prisms $\cup_{j=1}^{p} D_{n_j} = \cup_{j=1}^{p} D_{n_j}^{1}$.

Corollary 3.1. *If* $n_j \geq 4$, $n_j \equiv 0\,(mod 4)$ *and* $p \geq 2$, *then*

$$es(\cup_{j=1}^{p} D_{n_j}) = \lceil \frac{3n_j + 1}{2} \rceil.$$

Proof. The proof follows from Theorem 3.1. □

In the following figure, we present the labeling as in the proof of Corollary 3.1 for the case $n = 8$, $p = 2$.

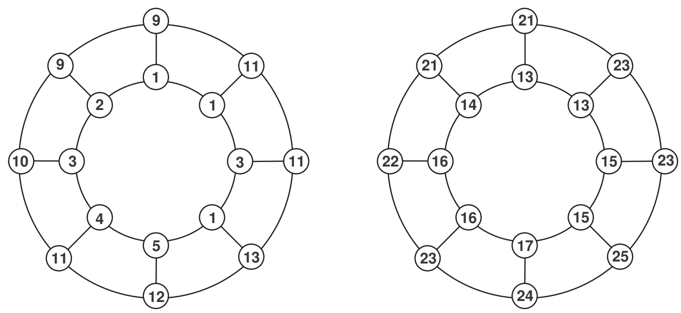

Figure 2: An edge irregular 24-labeling for $\cup_{j=1}^{2} D_{n_j}$, $n_j = 8$

4. Conclusion

According to definition of edge irregularity strength labels of vertices may repeat to attain a smallest value of $es(G)$ but weights of edges must be unique. It is very interesting and challenging at the same time to find the exact value for the edge irregularity strength for new families of graphs. The problems discussed in this paper proved the upper bound of edge irregularity strength for disjoint union of cycles, prisms and generalized prisms. These results can be used in future for addressing schemes in different applications.

Acknowledgement

The authors would like to thank the referees for their valuable and constructive comments that improve the paper. This research is supported by the Fundamental Research Grant Scheme (FRGS), Phase 1/2020, Ministry of Higher

Education, Malaysia with Reference Number FRGS/1/2020/STG06/UMT/02/1 (Grant Vot. 59609).

References

Ahmad, A., Al-Mushayt, O., and Bača, M. (2014a). On edge irregularity strength of graphs. *Applied Mathematics and Computation*, 243:607–610.

Ahmad, A., Baskoro, E., and Imran, M. (2012a). Total vertex irregularity strength of disjoint union of helm graphs. *Discussiones Mathematicae Graph Theory*, 32(3):427–434.

Ahmad, A., Bača, M., Asim, M., and Hasni, R. (2018). Computing edge irregularity strength of complete m-ary trees using algorithmic approach. *UPB Scientific Bulletin, Series A*, 80(3):145–152.

Ahmad, A., Bača, M., Bashir, Y., and Siddiqui, M. (2012b). Total edge irregularity strength of strong product of two paths. *Ars Combinatoria*, 106:449–459.

Ahmad, A., Bača, M., and Siddiqui, M. (2014b). On edge irregular total labeling of categorical product of two cycles. *Theory of Computing Systems*, 54(1):1–12.

Aigner, M. and Triesch, E. (1990). Irregular assignments of trees and forests. *SIAM Journal on Discrete Mathematics*, 3(4):439–449.

Asim, M., Ali, A., and Hasni, R. (2018). Iterative algorithm for computing irregularity strength of complete graph. *Ars Combinatoria*, 138:17–24.

Baskoro, E. T., Salman, A. N. M., and Gaos, N. N. (2010). On the total vertex irregularity strength of trees. *Discrete Mathematics*, 310(21):3043–3048.

Bača, M., Jendroľ, S., Miller, M., and Ryan, J. (2007). On irregular total labelings. *Discrete Mathematics*, 307:1378–1388.

Bača, M. and Siddiqui, M. (2014). Total edge irregularity strength of generalized prism. *Applied Mathematics and Computation*, 235:168–173.

Cammack, L. A., Schelp, R. H., and Schrag, G. C. (1991). Irregularity strength of full d-ary trees. *Congressus Numerantium*, 81:113–120.

Chartrand, G., Jacobson, M., Lehel, J., Oellermann, O., Ruiz, S., and Saba, F. (1988). Irregular networks. *Congressus Numerantium*, 64:187–192.

Faudree, R., Jacobson, M., Lehel, J., and Schelp, R. (1989). Irregular networks, regular graphs and integer matrices with distinct row and column sums. *Discrete Mathematics*, 76(3):223–240.

Gallian, J. A. (2018). A dynamic survey of graph labeling. *Electronic Journal of Combinatorics*, 1. https://doi.org/10.37236/27.

Imran, M., Aslam, A., Zafar, S., and Nazeer, W. (2017). Further results on the edge irregularity strength of graphs. *Indonesian Journal of Combinatorics*, 1(2):36–45.

Jendroľ, S., Miškuf, J., and Soták, R. (2010). Total edge irregularity strength of complete graphs and complete bipartite graphs. *Discrete Mathematics*, 310(3):400–407.

Kalkowski, M., Karonski, M., and Pfender, F. (2011). A new upper bound for the irregularity strength of graphs. *SIAM Journal on Discrete Mathematics*, 25(3):1319–1321.

Przybylo, J. (2008). Irregularity strength of regular graphs. *The Electronic Journal of Combinatorics*, 15(1):82–92.

Tarawneh, I., Hasni, R., and Ahmad, A. (2016a). On the edge irregularity strength of corona product of cycle with isolated vertices. *AKCE International Journal of Graphs and Combinatorics*, 13(3):213–217.

Tarawneh, I., Hasni, R., and Ahmad, A. (2016b). On the edge irregularity strength of corona product of graphs with paths. *Applied Mathematics E - Notes*, 16:80–87.

Tarawneh, I., Hasni, R., and Asim, M. (2018). On the edge irregularity strength of disjoint union of star graph and subdivision of star graph. *Ars Combinatoria*, 141:93–100.

Tarawneh, I., Hasni, R., Siddiqui, M., and Asim, M. (2019). On the edge irregularity strength of disjoint union of graphs. *Ars Combinatoria*, 142:239–249.