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A New Type of Weak Open Sets via Idealization in Bitopological Spaces

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Abstract

The main objective of this article is to define and study a kind of weak open sets called weakly bI-open sets with respect to an ideal in bitopological spaces. Additionally, we introduce some basic features of this notion.

Keywords: $(\Im_i, \Im_j)$-bI-open sets; $(\Im_i, \Im_j)$-WbI-open sets; $(\Im_i, \Im_j)$-WbI-closed sets.

1 Introduction and Preliminary

Kelly (1963) [9] established the theory of bitopological spaces $(X, \Im_1, \Im_2)$, to deal with two topologies $\Im_1$ and $\Im_2$ equipped with a non-empty set X. Since then many aspects of bitopology were investigated by several mathematician. Ideals and local functions with topologies were introduced by Kuratowski (1966) [10], Vaidyanathaswamy (1944) [13], Jankovic and Hamlett (1990) [7], Abd El-Monsef et al. (1992) [5] and many others. An ideal I is a family of all subsets X which will satisfy (i) $P \in I$ and $Q \subset P$ implies $Q \in I$ and (ii) $P \in I$ and $Q \in I$ implies $P \cup Q \in I$. We call $(X, \Im_1, \Im_2, I)$ as an ideal bitopological space. The concept of b-open sets has established by Andrijevic (1996) [2] and with this concept together with an ideal, another open set called bI-open sets has been introduced by Caksu Guler and Aslim (2005) [6] in topological spaces. Further this notion has been studied by Akdag (2007) [1] and Ekici (2012) [4]. After that Sarma (2015) [12] has introduced bI-open sets in an ideal bitopological spaces. Several characterisations, properties and the connection between this and other corresponding notions are studied. Recently , Mustafa et al. (2013) [11] has defined weakly bI-open sets with the help of ideals in topological spaces and established several characterizations.

For more convenient, we can specify the interior (respectively, closure) of a subset P in an ideal bitopological spaces $(X, \Im_1, \Im_2, I)$ due to the topology $\Im_i$ by $\Im_i$-$Int(P)$ (respectively, $\Im_i$-$Cl(P)$), for $i, j = 1, 2$ in which $i \neq j$.

Now, we recall some known definitions those will be used in this article.

If $P(X)$ denote the power set of X in $(X, \Im, I)$, then the operator $(.)^* : P(X) \longrightarrow P(X)$ is defined as a local function (1966) of a subset Q due to the topology $\Im$ and an ideal I which is defined as

$$Q^*(\Im, I) = \{y \in X : R \cap Q \notin I \,, R \in \Im(y)\},$$

for $Q \subset X$ and $\Im(y) = \{R \in \Im : y \in R\}$. Instead of $Q^*(\Im, I)$, we can simply write Q^* in such cases where there is no chance for confusion. For $\Im^*(I)$ finer than $\Im$, a Kuratowski closure operator is defined by $Cl^*(Q) = Q \cup Q^*$. Also, $\Im_i$-$Int^*(Q)$ denotes the interior of Q in $\Im_i^*(I)$ and $\Im_i$-$Int(Q_j^*)$ denotes the interior of Q_j^* due to the topology $\Im_i$, where

$$Q_j^* = \{y \in X : R \cap Q \notin I \text{ , for every } R \in \Im_j(y)\}.$$

Definition 1.1. *Sarma(2015) [12] A subset P in $(X, \Im_1, \Im_2, I)$ is called $(\Im_i, \Im_j)$-bI-open if $P \subset \Im_i$-$Int(\Im_j$-$Cl^*(P)) \cup \Im_j$-$Cl^*(\Im_i$-$Int(P))$.*

Definition 1.2. *Jelic (1990) [8] A subset P in $(X, \Im_1, \Im_2)$ is called $(\Im_i, \Im_j)$-preopen if $P \subset \Im_i$-$Int(\Im_j$-$Cl(P))$.*

2 $(\Im_i, \Im_j)$-Weakly bI-Open Sets

Definition 2.1. *A subset P in $(X, \Im_1, \Im_2, I)$ is called $(\Im_i, \Im_j)$-weakly bI-open(in short, $(\Im_i, \Im_j)$-WbI-open) if $P \subset \Im_j\text{-}Cl(\Im_i\text{-}Int(\Im_j\text{-}Cl^*(P))) \cup \Im_j\text{-}Cl^*(\Im_i\text{-}Int(\Im_j\text{-}Cl(P)))$.*
Here we shall denote the family of all $(\Im_i, \Im_j)$-WbI-open sets in $(X, \Im_1, \Im_2, I)$ by $(\Im_i, \Im_j)$-$WBIO(X)$.

Remark 2.1. *If I and J are any two ideals in $(X, \Im_1, \Im_2, I)$ in which $I \subset J$, then $(\Im_i, \Im_j)$-$WBJO(X) \subset (\Im_i, \Im_j)$-$WBIO(X)$.*

Remark 2.2. *Each $(\Im_i, \Im_j)$-bI-open set in $(X, \Im_1, \Im_2, I)$ is $(\Im_i, \Im_j)$-WbI-open. The converse may not be true as discussed below.*

Example 2.1. *Suppose $X = \{r, s, t\}$, $\Im_1 = \{\emptyset, \{r\}, X\}$, $\Im_2 = \{\emptyset, \{r\}, \{r, s\}, X\}$ and $I = \{\emptyset, \{r\}\}$. Then $\{r, t\}$ is $(\Im_1, \Im_2)$-WbI-open but not $(\Im_1, \Im_2)$-bI-open.*

Remark 2.3. *Intersection of any two $(\Im_i, \Im_j)$-WbI-open sets of $(X, \Im_1, \Im_2, I)$ may not be $(\Im_i, \Im_j)$-WbI-open set as discussed below.*

Example 2.2. *Suppose $X = \{u, v, w, s\}$, $\Im_1 = \{\emptyset, \{u\}, \{v\}, \{u, v\}, \{u, v, w\}, X\}$, $\Im_2 = \{\emptyset, X\}$ and $I = \{\emptyset, \{w\}, \{s\}, \{w, s\}\}$. Then the sets $\{u, w\}$ and $\{v, w\}$ are $(\Im_1, \Im_2)$-WbI-open sets but $\{u, w\} \cap \{v, w\} = \{w\}$ is not $(\Im_1, \Im_2)$-WbI-open set.*

Theorem 2.1. *Let $P, Q \subset X$. If P is $(\Im_i, \Im_j)$-WbI-open and $Q \in \Im_1 \cap \Im_2$, then $P \cap Q$ is also $(\Im_i, \Im_j)$-WbI-open.*

Proof. Let P be a $(\Im_i, \Im_j)$-WbI-open set. So,

$$P \subset \Im_j - Cl(\Im_i - Int(\Im_j - Cl^*(P))) \cup \Im_j - Cl^*(\Im_i - Int(\Im_j - Cl(P))).$$

Now $P \cap Q \subset \{\Im_j\text{-}Cl(\Im_i\text{-}Int(\Im_j\text{-}Cl^*(P))) \cup \Im_j\text{-}Cl^*(\Im_i\text{-}Int(\Im_j\text{-}Cl(P)))\} \cap Q$.

$\subset \{\Im_j\text{-}Cl(\Im_i\text{-}Int((P \cap Q) \cup (P_j^* \cap Q)))\} \cup \{(\Im_i\text{-}Int(\Im_j\text{-}Cl(P)) \cap Q) \cup (\Im_i\text{-}Int(\Im_j\text{-}Cl(P)) \cap Q)_j^*\}$

$\subset \{\Im_j\text{-}Cl(\Im_i\text{-}Int((P \cap Q) \cup (P \cap Q)_j^*))\} \cup \{(\Im_i\text{-}Int(\Im_j\text{-}Cl(P \cap Q)) \cup (\Im_i\text{-}Int(\Im_j\text{-}Cl(P \cap Q))_j^*\}$

$$= \Im_j\text{-}Cl(\Im_i\text{-}Int(\Im_j\text{-}Cl^*(P \cap Q))) \cup \Im_j\text{-}Cl^*(\Im_i\text{-}Int(\Im_j\text{-}Cl(P \cap Q))).$$

Hence $P \cap Q$ is $(\Im_i, \Im_j)$-WbI-open. □

Theorem 2.2. *Let $(X, \Im_1, \Im_2, I)$ is an ideal bitopological space. If $P_\delta \in (\Im_i, \Im_j)$-$WBIO(X)$ for each $\delta \in \wedge$, then $\bigcup\{P_\delta : \delta \in \wedge\} \in (\Im_i, \Im_j)$-$WBIO(X)$ where $\wedge$ an index set.*

Proof. Let $P_\delta \in (\Im_i, \Im_j)$-$WBIO(X)$. Then,

$$P_\delta \subset \Im_j - Cl(\Im_i - Int(\Im_j - Cl^*(P_\delta))) \cup \Im_j - Cl^*(\Im_i - Int(\Im_j - Cl(P_\delta))),$$

for each $\delta \in \wedge$.

Thus,

$$\begin{aligned}\bigcup_{\delta\in\wedge} P_\delta \subset & \bigcup_{\delta\in\wedge}\{\Im_j - Cl(\Im_i - Int(\Im_j - Cl^*(P_\delta))) \cup \Im_j - Cl^*(\Im_i - Int(\Im_j - Cl(P_\delta)))\} \\ \subset & \bigcup_{\delta\in\wedge}\{\Im_j - Cl(\Im_i - Int(P_\delta \cup (P_\delta)^*_j)) \\ & \cup\{(\Im_i - Int(\Im_j - Cl(P_\delta))) \cup (\Im_i - Int(\Im_j - Cl(P_\delta)))^*_j\} \\ \subset & \{\Im_j - Cl(\Im_i - Int((\bigcup_{\delta\in\wedge} P_\delta) \cup (\bigcup_{\delta\in\wedge} P_\delta)^*_j))\} \\ & \cup\{\Im_i - Int(\Im_j - Cl(\bigcup_{\delta\in\wedge} P_\delta)) \cup (\Im_i - Int(\Im_j - Cl(\bigcup_{\delta\in\wedge} P_\delta))^*_j)\} \\ \subset & \Im_j - Cl(\Im_i - Int(\Im_j - Cl^*(\bigcup_{\delta\in\wedge} P_\delta))) \cup \Im_j - Cl^*(\Im_i - Int(\Im_j - Cl(\bigcup_{\delta\in\wedge} P_\delta))).\end{aligned}$$

Hence, $\bigcup_{\delta\in\wedge} P_\delta$ is $(\Im_i, \Im_j)$-WbI-open. □

Lemma 2.1. *Sarma(2015) [12] Let $P, Q \subset X$ in which $Q \subset P$. Then, $Q^*_i(\Im_i|_P, I|_P) = Q^*_i(\Im_i, I) \cap P$, for $i = 1, 2$.*

For any subset P of $(X, \Im_1, \Im_2, I)$, $\Im_i|_P$ denote the relative topology on the subset P in which $i = 1, 2$. Also, $I|_P = \{P \cap I : I \in I\}$ be an ideal on P.

Theorem 2.3. *If $P \in (\Im_i, \Im_j)$-$WBIO(X)$ in $(X, \Im_1, \Im_2, I)$ and $Q \in \Im_1 \cap \Im_2$, then*

$$P \cap Q \in WBIO(Q, \Im_1|_Q, \Im_2|_Q, I|_Q).$$

Proof. Since $Q \in \Im_1 \cap \Im_2$, therefore $\Im_i$-$Int_Q(W) = \Im_i$-$Int(W)$, where W is a subset of Q and $i = 1, 2$. Then, by using Lemma 2.1, we get

$P \cap Q \subset \{\Im_j\text{-}Cl(\Im_i\text{-}Int(\Im_j\text{-}Cl^*(P))) \cup \Im_j\text{-}Cl^*(\Im_i\text{-}Int(\Im_j\text{-}Cl(P)))\} \cap Q \quad \subset \{((\Im_j\text{-}Cl(\Im_i\text{-}Int(P \cup P^*_j))) \cap Q) \cap Q\} \cup \{((\Im_i\text{-}Int(\Im_j\text{-}Cl(P))) \cap Q) \cup ((\Im_i\text{-}Int(\Im_j\text{-}Cl(P)))^*_j \cap Q)) \cap Q\}$

$\subset \{((\Im_j\text{-}Cl(\Im_i\text{-}Int(P \cup P^*_j))) \cap Q) \cap Q\} \cup \{((\Im_i\text{-}Int(\Im_j\text{-}Cl(P))) \cap Q) \cup ((\Im_i\text{-}Int(\Im_j\text{-}Cl(P))) \cap Q)^*_j) \cap Q\}$

$\subset \{((\Im_j\text{-}Cl(\Im_i\text{-}Int((P \cap Q) \cup (P \cap Q)^*_j))) \cap Q)\} \cup \{((\Im_i\text{-}Int_Q(\Im_j\text{-}Cl_Q(P \cap Q))) \cap Q) \cup ((\Im_i\text{-}Int_Q(\Im_j\text{-}Cl_Q(P \cap Q)^*_j)) \cap Q\}$

$\subset \{((\Im_j\text{-}Cl(\Im_i\text{-}Int((P \cap Q) \cup (P \cap Q)^*_j))) \cap Q)\} \cup \{((\Im_i\text{-}Int_Q(\Im_j\text{-}Cl_Q(P \cap Q))) \cup (\Im_i\text{-}Int_Q(\Im_j\text{-}Cl_Q(P \cap Q))^*_j))_{(\Im_1|Q, \Im_2|Q, I|Q)}\}$

$$= \{\Im_j\text{-}Cl_Q(\Im_i\text{-}Int_Q(\Im_j\text{-}Cl^*_Q(P \cap Q))\} \cup \{\Im_j\text{-}Cl^*_Q(\Im_i\text{-}Int_Q(\Im_j\text{-}Cl_Q(P \cap Q)))\}.$$

Hence $P \cap Q \in WBIO(Q, \Im_1|_Q, \Im_2|_Q, I|_Q)$. □

Theorem 2.4. *A subset $P \subset X$ is $(\Im_i, \Im_j)$-$WBIO(X)$ iff for all $y \in X$, there exists $Q \in (\Im_i, \Im_j)$-$WBIO(X)$ such that $y \in Q \subset P$.*

Proof. Obvious. □

Theorem 2.5. *If $P \in (\Im_i, \Im_j)$-$WBIO(X)$, then $\Im_j$-$Cl(P) = \Im_j$-$Cl(\Im_i$-$Int(\Im_j$-$Cl(P)))$.*

Proof. Since $P \in (\Im_i, \Im_j)$-$WBIO(X)$, so we have

$$\begin{aligned} P &\subset \Im_j - Cl(\Im_i - Int(\Im_j - Cl^*(P))) \cup \Im_j - Cl^*(\Im_i - Int(\Im_j - Cl(P))) \\ &\subset \Im_j - Cl(\Im_i - Int(\Im_j - Cl(P))) \cup \Im_j - Cl^*(\Im_i - Int(\Im_j - Cl(P))) \\ &= \Im_j - Cl(\Im_i - Int(\Im_j - Cl(P))). \end{aligned}$$

Therefore, $P \subset \Im_j$-$Cl(\Im_i$-$Int(\Im_j$-$Cl(P)))$.

Now,

$$\begin{aligned} \Im_j - Cl(P) &\subset \Im_j - Cl(\Im_j - Cl(\Im_i - Int(\Im_j - Cl(P)))) \\ &= \Im_j - Cl(\Im_i - Int(\Im_j - Cl(P))) \\ &\subset \Im_j - Cl(\Im_j - Cl(P)) = \Im_j - Cl(P). \end{aligned}$$

Hence $\Im_j$-$Cl(P) = \Im_j$-$Cl(\Im_i$-$Int(\Im_j$-$Cl(P)))$. □

Theorem 2.6. *Let $P, Q \subset X$. If $P \in (\Im_i, \Im_j)$-$WBIO(X)$ and $P \subset Q \subset \Im_j$-$Cl^*(P)$, then $Q \in (\Im_i, \Im_j)$-$WBIO(X)$.*

Proof. Since P is $(\Im_i, \Im_j)$-WbI-open set of X, therefore

$$P \subset \Im_j - Cl(\Im_i - Int(\Im_j - Cl^*(P))) \cup \Im_j - Cl^*(\Im_i - Int(\Im_j - Cl(P))).$$

Also,

$$\begin{aligned} Q \subset \Im_j - Cl^*(P) &\subset \Im_j - Cl^*\{\Im_j - Cl(\Im_i - Int(\Im_j - Cl^*(P))) \cup \Im_j - Cl^*(\Im_i - Int(\Im_j - Cl(P)))\} \\ &\subset \Im_j - Cl^*(\Im_j - Cl(\Im_i - Int(\Im_j - Cl^*(P)))) \cup \Im_j - Cl^*(\Im_j - Cl^*(\Im_i - Int(\Im_j - Cl(P)))) \\ &= \Im_j - Cl(\Im_i - Int(\Im_j - Cl^*(P))) \cup \Im_j - Cl^*(\Im_i - Int(\Im_j - Cl(P))). \end{aligned}$$

Hence,

$$Q \subset \Im_j - Cl(\Im_i - Int(\Im_j - Cl^*(Q))) \cup \Im_j - Cl^*(\Im_i - Int(\Im_j - Cl(Q)))$$

and consequently Q is $(\Im_i, \Im_j)$-WbI-open. □

Theorem 2.7. *Let $(X, \Im_1, \Im_2, I)$ be an ideal bitopological space such that $P \subset Y \subset X$ and $Y \in \Im_1 \cap \Im_2$. If $P \in (\Im_i, \Im_j)$-$WBIO(Y)$, then $P \in (\Im_i, \Im_j)$-$WBIO(X)$.*

Proof. Since $P \in (\Im_i, \Im_j)$-$WBIO(Y)$, therefore

$$\begin{aligned} P &\subset \Im_j - Cl_Y(\Im_i - Int_Y(\Im_j - Cl^*_Y(P))) \cup \Im_j - Cl^*_Y(\Im_i - Int_Y(\Im_j - Cl_Y(P))) \\ &= [\Im_j - Cl(\Im_i - Int_Y(\Im_j - Cl^*_Y(P))) \cup \Im_j - Cl^*(\Im_i - Int_Y(\Im_j - Cl_Y(P)))] \cap Y \\ &= [\Im_j - Cl(\Im_i - Int_Y(\Im_j - Cl^*_Y(P))) \cap Y] \cup [\Im_j - Cl^*(\Im_i - Int_Y(\Im_j - Cl_Y(P))) \cap Y] \\ &\subset \Im_j - Cl(\Im_i - Int(\Im_j - Cl^*_Y(P))) \cup \Im_j - Cl^*(\Im_i - Int(\Im_j - Cl_Y(P))) \\ &= \Im_j - Cl(\Im_i - Int(\Im_j - Cl^*(P) \cap Y)) \cup \Im_j - Cl^*(\Im_i - Int(\Im_j - Cl(P) \cap Y)) \\ &\subset \Im_j - Cl(\Im_i - Int(\Im_j - Cl^*(P))) \cup \Im_j - Cl^*(\Im_i - Int(\Im_j - Cl(P))). \end{aligned}$$

Hence, $P \in (\Im_i, \Im_j)$-$WBIO(X)$. □

Definition 2.2. *Balasubramanian (1991) [3] A space $(X, \Im_1, \Im_2)$ is said to be pairwise extremally disconnected if $\Im_j$-closure of every $\Im_i$-open set in X is $\Im_i$-open.*

Theorem 2.8. *If P is $(\Im_i, \Im_j)$-WbI-open set in $(X, \Im_1, \Im_2, I)$, where X is pairwise extremally disconnected then P is $(\Im_i, \Im_j)$-pre open.*

Proof. Since X is pairwise extremally disconnected, therefore for $P \in \Im_i$ we have $\Im_j$-$Cl(P) \in \Im_i$. So $\Im_i$-$Int(\Im_j$-$Cl(P)) \in \Im_i$ and hence $\Im_j$-$Cl(\Im_i$-$Int(\Im_j$-$Cl(P))) \in \Im_i$.

Also, since P is $(\Im_i, \Im_j)$-WbI-open, therefore we have

$$\begin{aligned} P &\subset \Im_j\text{-}Cl(\Im_i\text{-}Int(\Im_j\text{-}Cl^*(P))) \cup \Im_j\text{-}Cl^*(\Im_i\text{-}Int(\Im_j\text{-}Cl(P))) \\ &\subset \Im_j\text{-}Cl(\Im_i\text{-}Int(\Im_j\text{-}Cl(P))) \cup \Im_j\text{-}Cl(\Im_i\text{-}Int(\Im_j\text{-}Cl(P))) \\ &= \Im_j\text{-}Cl(\Im_i\text{-}Int(\Im_j\text{-}Cl(P))) \\ &\subset \Im_i\text{-}Int(\Im_j\text{-}Cl(\Im_j\text{-}Cl(P))) = \Im_i\text{-}Int(\Im_j\text{-}Cl(P)). \end{aligned}$$

Hence, P is $(\Im_i, \Im_j)$-pre open. □

Definition 2.3. *A subset P in $(X, \Im_1, \Im_2, I)$ is called $(\Im_i, \Im_j)$-WbI-closed if $X \setminus P$ is $(\Im_i, \Im_j)$-WbI-open.*

Theorem 2.9. *A subset $P \subset X$ is $(\Im_i, \Im_j)$-WbI-closed if $\Im_j$-$Int(\Im_i$-$Cl(\Im_j$-$Int^*(P))) \cap \Im_j$-$Int^*(\Im_i$-$Cl(\Im_j$-$Int(P))) \subset P$.*

Proof. Follows from the definition. □

Theorem 2.10. *If $P \subset X$ is $(\Im_i, \Im_j)$-WbI-closed, then $\Im_j$-$Int(\Im_i$-$Cl(\Im_j$-$Int(P)) \subset P$.*

Proof. Since P is $(\Im_i, \Im_j)$-WbI-closed, therefore $X \setminus P$ is $(\Im_i, \Im_j)$-WbI-open. So, by definition we have

$$\begin{aligned} X \setminus P &\subset \Im_j\text{-}Cl(\Im_i\text{-}Int(\Im_j\text{-}Cl^*(X \setminus P))) \cup \Im_j\text{-}Cl^*(\Im_i\text{-}Int(\Im_j\text{-}Cl(X \setminus P))) \\ &\subset \Im_j\text{-}Cl(\Im_i\text{-}Int(\Im_j\text{-}Cl(X \setminus P))) \cup \Im_j\text{-}Cl(\Im_i\text{-}Int(\Im_j\text{-}Cl(X \setminus P))) \\ &= X \setminus \Im_j\text{-}Int(\Im_i\text{-}Cl(\Im_j\text{-}Int(P))). \end{aligned}$$

Hence, $\Im_j$-$Int(\Im_i$-$Cl(\Im_j$-$Int(P)) \subset P$. □

Definition 2.4. *A subset N of $(X, \Im_1, \Im_2, I)$ is called a $(\Im_i, \Im_j)$-weakly bI-neighbourhood of a point y of X if there exists a $(\Im_i, \Im_j)$-WbI-open set P of X such that $y \in P \subset N$.*

Theorem 2.11. *A subset P of $(X, \Im_1, \Im_2, I)$ is $(\Im_i, \Im_j)$-WbI-open if and only if P is a $(\Im_i, \Im_j)$-weakly bI-neighbourhood of each of its points in X.*

Proof. First, suppose that P be $(\Im_i, \Im_j)$-WbI-open. Now, for all $y \in P$, we have $y \in P \subset P$ and P is $(\Im_i, \Im_j)$-WbI-open. This implies that P is a $(\Im_i, \Im_j)$-weakly bI-neighbourhood for each points.

Conversely, let P be a $(\Im_i, \Im_j)$-weakly bI-neighbourhood for each points. Thus, for all $y \in P$, there exists a $(\Im_i, \Im_j)$-WbI-open set U_y of $(X, \Im_1, \Im_2, I)$ such that $y \in U_y \subset P$. Then $P = \bigcup\{U_y : y \in P\}$. So, by Theorem 2.2 we have P is $(\Im_i, \Im_j)$-WbI-open in X. □

Definition 2.5. *A point y in $(X, \Im_1, \Im_2, I)$ is called $(\Im_i, \Im_j)$-weakly bI-limit point of a subset P of X if for all $(\Im_i, \Im_j)$-WbI-open set V in X containing y such that $V \cap (P \setminus \{y\}) \neq \emptyset$.*

The set of every $(\Im_i, \Im_j)$-weakly bI-limit points of P is called $(\Im_i, \Im_j)$-weakly bI-derived set of P and we denote it by $(\Im_i, \Im_j)$-WbI-$D(P)$.

Theorem 2.12. *A subset $P \subset X$ is $(\Im_i, \Im_j)$-WbI-closed iff it contains all of its $(\Im_i, \Im_j)$-weakly bI-limit points .*

Proof. Let P be $(\Im_i, \Im_j)$-WbI-closed. Suppose if possible, y is $(\Im_i, \Im_j)$-weakly bI-limit point of P such that $y \in X \setminus P$. Then $X \setminus P$ is $(\Im_i, \Im_j)$-WbI-open in X containing y. So $P \cap (X \setminus P) \neq \emptyset$, a contradiction.

Conversely, suppose that P contains all of its $(\Im_i, \Im_j)$-weakly bI-limit points. Then, for all $y \in X \setminus P$, there exists a $(\Im_i, \Im_j)$-WbI-open set V containing y such that $P \cap V = \emptyset$. That is $y \in V \subset X \setminus P$. Therefore, by Theorem 2.4, $X \setminus P$ is $(\Im_i, \Im_j)$-WbI-open and hence P is $(\Im_i, \Im_j)$-WbI-closed set. □

Theorem 2.13. *For any two subsets P and Q of $(X, \Im_1, \Im_2, I)$, the conditions stated below are equivalent:*

(a) $(\Im_i, \Im_j)$-WbI-$D(\emptyset) = \emptyset$.

(b) If $P \subset Q$, then $(\Im_i, \Im_j)$-WbI-$D(P) \subset (\Im_i, \Im_j)$-$WbI$-$D(Q)$.

(c) $(\Im_i, \Im_j)$-WbI-$D(P \cap Q) \subset (\Im_i, \Im_j)$-$WbI$-$D(P) \cap (\Im_i, \Im_j)$-$WbI$-$D(Q)$.

(d) $(\Im_i, \Im_j)$-WbI-$D(P) \cup (\Im_i, \Im_j)$-WbI-$D(Q) \subset (\Im_i, \Im_j)$-$WbI$-$D(P \cup Q)$.

Proof. Proofs are easy, so omitted. □

Definition 2.6. *Let P be a subset of $(X, \Im_1, \Im_2, I)$. The $(\Im_i, \Im_j)$-WbI-interior of P is the union of every $(\Im_i, \Im_j)$-WbI-open sets contained in P and we denote it by $(\Im_i, \Im_j)$-WbI-$Int(P)$.*
The $(\Im_i, \Im_j)$-WbI-closure of P is the intersection of every $(\Im_i, \Im_j)$-WbI-closed sets containing P and we denote it by $(\Im_i, \Im_j)$-WbI-$Cl(P)$.

Theorem 2.14. *For a subset P of $(X, \Im_1, \Im_2, I)$,*

(a) $(\Im_i, \Im_j)$-WbI-$Int(P)$ *is* $(\Im_i, \Im_j)$-WbI-*open.*

(b) $(\Im_i, \Im_j)$-WbI-$Cl(P)$ *is* $(\Im_i, \Im_j)$-WbI-*closed.*

(c) P *is* $(\Im_i, \Im_j)$*-*WbI*-open if and only if* $P = (\Im_i, \Im_j)$-WbI-$Int(P)$.

(d) P *is* $(\Im_i, \Im_j)$*-*WbI*-closed if and only if* $P = (\Im_i, \Im_j)$-WbI-$Cl(P)$.

Proof. Obvious. □

Theorem 2.15. *Let* $P \subset X$. *A point* $y \in (\Im_i, \Im_j)$-WbI-$Cl(P)$ *if and only if* $P \cap Q \neq \emptyset$, *for all* $(\Im_i, \Im_j)$-WbI*-open set* Q *of* X *containing* y.

Proof. Let $y \in (\Im_i, \Im_j)$-WbI-$Cl(P)$ and Q be $(\Im_i, \Im_j)$-WbI-open such that $y \in Q$. Also, suppose that $P \cap Q = \emptyset$. Therefore $P \subset X \setminus Q$ and $X \setminus Q$ is $(\Im_i, \Im_j)$-WbI-closed, which shows that

$$y \in (\Im_i, \Im_j) - WbI - Cl(P) \subset (\Im_i, \Im_j) - WbI - Cl(X \setminus Q) = X \setminus Q.$$

So, we have $y \in X \setminus Q$, a contradiction. Hence $P \cap Q \neq \emptyset$.

Conversely, let $P \cap Q \neq \emptyset$, for all $(\Im_i, \Im_j)$-WbI-open set Q in X containing y.

Again, let $y \notin (\Im_i, \Im_j)$-WbI-$Cl(P)$. So, there exists a $(\Im_i, \Im_j)$-WbI-closed set U of X such that $P \subset U$ and $y \notin U$. Now we have $y \in X \setminus U$, in which $X \setminus U$ is $(\Im_i, \Im_j)$-WbI-open and $(X \setminus U) \cap P = \emptyset$, a contradiction to our supposition. Hence, $y \in (\Im_i, \Im_j)$-WbI-$Cl(P)$. □

Theorem 2.16. *For a subset* P *of* $(X, \Im_1, \Im_2, I)$,

(a) $(\Im_i, \Im_j)$-WbI-$Cl(X \setminus P) = X \setminus (\Im_i, \Im_j)$-$WbI$-$Int(P)$.

(b) $(\Im_i, \Im_j)$-WbI-$Int(X \setminus P) = X \setminus (\Im_i, \Im_j)$-$WbI$-$Cl(P)$.

Proof. (a) Suppose $y \notin (\Im_i, \Im_j)$-WbI-$Cl(X \setminus P)$. Therefore, there exists a $(\Im_i, \Im_j)$-WbI-open set Q in X such that $y \in Q$ and $Q \cap (X \setminus P) = \emptyset$. Since $y \in Q$, so we have $y \notin X \setminus P$ and so $y \in P$. Thus, we get $y \in Q \subset P$ and so $y \in (\Im_i, \Im_j)$-WbI-$Int(P)$. This shows that $y \notin X \setminus (\Im_i, \Im_j)$-$WbI$-$Int(P)$. Consequently, $X \setminus (\Im_i, \Im_j)$-$WbI$-$Int(P) \subset (\Im_i, \Im_j)$-$WbI$-$Cl(X \setminus P)$.

Conversely, suppose that $y \notin X \setminus (\Im_i, \Im_j)$-$WbI$-$Int(P)$. Then we have $y \in (\Im_i, \Im_j)$-WbI-$Int(P)$ and therefore there exists a $(\Im_i, \Im_j)$-WbI-open set Q in X containing y such that $y \in Q \subset P$. Thus, we get $Q \cap (X \setminus P) = \emptyset$ and $y \notin (\Im_i, \Im_j)$-WbI-$Cl(X \setminus P)$. Thus $(\Im_i, \Im_j)$-WbI-$Cl(X \setminus P) \subset X \setminus (\Im_i, \Im_j)$-$WbI$-$Int(P)$. Hence $(\Im_i, \Im_j)$-WbI-$Cl(X \setminus P) = X \setminus (\Im_i, \Im_j)$-$WbI$-$Int(P)$.

(b) It is similar to the proof of (a). □

3 Conclusion

The bitopological spaces was first introduced by Kelly in [9], then later was studied by many researchers. The ideals and local functions with topologies were also introduced and related open sets were defined and several related properties and characterization studied by several mathematicians. In this present work we extended the work in the literature by defining a new weak open sets and called them weakly bI-open sets with respect to an ideal in bitopological spaces. Further, we introduced and proved some basic related properties.

Conflicts of Interest The authors declare that there is no conflict of interest in this article.

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