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Exploring Mathematical Values Through Mathematics Teachers' Beliefs and Instructional Practices

Wan Zah Wan Ali, Rohani Ahmad Tarmizi, Habsah Ismail, Ramlah Hamzah, Mohd. Majid Konting, Mat Rofa Ismail & Sharifah Kartini Said Husain
Institute for Mathematical Research
Universiti Putra Malaysia, 43400 UPM, Serdang, Selangor, Malaysia
E-mail: wanzah@educ.upm.edu.my

ABSTRACT

Mathematical values are deep affective qualities which education aims to foster through mathematics subjects in schools and are crucial components of the classroom affective environment. Mathematical values comprise teachers' beliefs, attitudes, and their instructional practices in mathematics classrooms. This study investigated Malaysian mathematics secondary school teachers' beliefs and their instructional practices based on four schools of philosophy of mathematics which are logicism, formalism, intuitionism and kuhnism. A quantitative research method with a survey design was used for assessment during this study. An instrument to measure the two constructs was developed based on the four mathematical philosophies mentioned earlier. The findings indicated that majority of mathematics teachers' beliefs were inclined towards kuhnism whilst their instructional practices were inclined towards formalism. These findings imply that in practice majority of mathematics teachers in secondary school emphasized on symbols and formulas in their teaching. However, it can also be concluded that the mathematics teachers' instructional beliefs were closely affiliated with their social norms and culture. Thus, these findings suggested that mathematics teachers' beliefs were not congruent with their practices. Although the teachers' beliefs were towards kuhnism, they did not portray these in their teachings, which seemed to emphasize on formalism.

Keywords: Mathematical values, mathematics teaching, mathematics education

INTRODUCTION

Worldwide and in Malaysia, mathematics is an important subject in school curriculums and is taught at all levels. Mathematics has generally been conceived to be uninteresting compared to other subjects such as language, literature and physical education (Aplin and Saunders 1996; Lee and Cockman 1995) and also science subjects (Allchin 1999; Proctor 1991). This is probably due to the fact that these non-mathematics subjects are directly applicable to everyday living and experiences; hence inculcation and discussion of values can take place in classroom teaching. In order to make teaching interesting, meaningful and applicable, the goals of the Malaysian school mathematics curriculum have stipulated that learners should realize that mathematics is relevant or applicable to everyday life. Hence to achieve the goals of teaching mathematics, emphasis should be given not only to mathematical content and procedural knowledge but also to the

mathematical values. As such, learners will be actively learning as well as applying mathematics in everyday life.

Mathematical values are deep affective qualities which education aims to foster through mathematics subjects in schools and are a crucial component of the classroom affective environment (Bishop *et al.*1999; Bishop 1996). Bishops and his colleagues propose that mathematical values should be fostered in mathematics teaching in order to ensure quality and mastery of mathematical knowledge among learners at all levels. In addition, mathematics education should give learners an appreciation and awareness of the nature and value of mathematics and its usefulness (Ernest 1991).

According to Bishop (2001), mathematical values held by teachers may represent teacher's internalization and 'cognitization' of affective variables such as interests, beliefs and attitudes in the context of their affective-cognitive personal system. Bishop, in his study focused on values which can be integrated in the mathematics classroom which was categorized into mathematical, mathematics educational, and general education. Mathematical values relate to the epistemology of mathematics as a discipline whilst mathematics educational values are specifically associated with the institutional norms within which school mathematics is taught. In addition, Bishop relates to the third category, the general educational values, as values which are generally expected to be taught or inculcated in students by their mathematics teachers. Mathematical values can be inculcated through the nature of mathematics and through one's experience in the mathematics classroom.

As Bishop *et al.* (1999) noted, there is little knowledge about what values teachers are teaching in mathematics classes. In addition, little do we know about how aware teachers are of their own value positions and about how these affect teachers' mathematics instructional practice. Also little do we know about how their teaching develops certain values in their students.

In this study, we aimed to explore the Malaysian scenario of mathematical values in secondary school teaching of mathematics. What mathematical values do Malaysian teachers teach in mathematics classrooms? What are mathematics teachers' instructional practices with regard to their own values position. Specifically this study focused on mathematics teachers' mathematical beliefs towards mathematics and their corresponding instructional practices in mathematics classrooms. In this study, mathematics teachers' mathematical beliefs towards mathematics are referred to as consisting of the teachers conscious and subconscious concepts, meanings, rules, mental images, preferences and values that teachers have regarding the discipline of mathematics and the process of teaching and learning mathematics. According to Pajares (1992), mathematical beliefs are personal principles, constructed from experience by individuals unconsciously, which are then interpreted into new experiences and information and to guide action.

Teachers' beliefs have always been considered important in developing approaches to the teaching of the subject matter or content. Teachers always model their values to their students through their teaching and interaction with their students. Therefore it has an important link to teachers' classroom instruction.

Research on teachers' beliefs has demonstrated that beliefs have a profound influence both on teachers' instructional practices and on their willingness to revise their classroom practices (Cooney and Shealy 1995; Fennema and Nelson 1997). Previous studies by Marcilo (1987), Barr (1988), Grouws and Cramer (1989), Peterson *et al.* (1989) and Sowder (1989) have also provided important information about the beliefs of students and teachers on how important the role of beliefs is in teaching and learning of mathematics. For this research, mathematics teachers' beliefs towards the mathematical values are investigated through their instructional practices.

Implication of Beliefs to Mathematical Values

Mathematics is often considered as a value free subject because it deals with abstract entities and ideas. Subjects such as languages, literature studies, physical education, (Aplin and Saunders 1996; Lee and Cockman 1995; Muray 1977) and the sciences (Allchin 1999; Proctor 1991; Tan 1997), deal more directly and explicitly with aspects of life experiences. Values are rarely considered in the teaching of mathematics. It is a widespread belief that mathematics is the most value-free of all school subjects, not just among teachers but also among parents, university mathematicians and employers (Bishop *et al.* 1999).

However, mathematics like other subjects can also be applied to real-life situations. Bishop *et al.* (1999) were convinced that teaching and learning of values does go on in mathematics classes. "School mathematics is mathematics as it is conceptualized, represented, structured and sequenced to share with the next generation through the formal schooling experience" (Schmidt *et al*. 1997). FitzSimons (1994) also stated that mathematics is a human and cultural knowledge as is any other field of knowledge where teachers inevitably teach values**.** As adults, teachers certainly express their feelings, beliefs and values about mathematics which clearly relate to the mathematics teaching they experienced at school.

Ernest (1989) suggested that teachers' beliefs about mathematics teaching encompass three components: the teachers' views of the nature of mathematics, teachers' view of the nature of mathematics teaching and teachers' view of the process of learning mathematics. Ernest emphasized the importance of mathematics teachers' beliefs by claiming that teachers' approaches to mathematics teaching depend basically on their systems of beliefs which center mainly on their conceptions of the nature of mathematics and on their mental models of teaching and learning mathematics.

According to the literature, there are two views of the nature of mathematics held by proponents of mathematical philosophies which are absolutist philosophies (value free) and fallibilists philosophies (value-laden). According to Lerman (1983), from an absolutist view, all of mathematics is based on universal, absolute foundations and is the paradigm of knowledge, certainty, absolutism, value free and abstract, with its connections to the real world perhaps of a "platonic nature". Ernest (1989) described the Platonist view which saw mathematics as a static but unified body of knowledge, a realm of interconnecting structures and truths that were discovered but not created. Ernest (1991) further stated that absolutists view mathematics as a body of knowledge

that is certain and without flaw. He added that absolutists also perceived attitude towards mathematics as having no relevance towards social interest. Based on the absolutist view (the value-free view), there emerged three major schools of thoughts in mathematics which were Logicism, Formalism and Intuitionism. These will be discussed later.

Another conception or view of mathematics that has been identified was the fallibilist view. In contrast to the absolutist view of mathematics, the fallibilists view mathematics as statements of mathematics that are potentially flawed and must be held open to revision and correction (Ernest 1991). Mathematics was seen as an extension of natural language and is acquired and developed through social interaction. According to Ernest (1992),

> Mathematics is a branch of knowledge which is indissolubly connected with other knowledge, through the web of language. Language functions by facilitating the formations of theories about social situations and physical reality. Dialogue with other persons and interactions with the physical world play a key role in refining these theories, which consequently are continually being revised to improve "fit". As a part of the web language, mathematics thus maintains contact with the theories describing social and physical reality (Ernest 1992: 94).

Lerman (1983) indicated that a fallibilist perspective of mathematics was mainly that mathematics develops through conjectures, proofs and refutations and uncertainty was accepted as inherent in the discipline. According to Kuhn (2006), mathematics can also be viewed as an abstract knowledge, but also connected with real life events which will enhance students' appreciation towards mathematics and experience enjoyment towards learning mathematics. Kuhn (2006) categorized this school of thought as Kuhnism which emphasized the integration of values in teaching mathematics hence this school of thought was classified as value loaded. A description of the four different schools of philosophy of mathematics, namely logicism, formalism, intuitionism and kuhnism and their relation to instructional practices followed.

(i) Mathematical Beliefs and Practices from the Logicism School of Philosophy

Logicism is the school of thought that believes mathematics can be expressed in purely logical terms and proven from logical principles alone. Logic is the proper foundation of mathematics and every mathematical statement is logical truth. According to logicists, all mathematical concepts can be reduced to abstract properties that can be derived through logical principles. According to Russell (1903), all mathematical truths can be proven by logic. In his book, *Principles of Mathematics*, he stated that "Mathematics is a logical symbol which is one of the important discoveries in this era" (Abdul Latif Samian 1997). Russell explained that mathematics theorems and proofs can be derived through logical deduction. From the instructional practices aspect, logicism focused on set theory as the foundation of mathematics.

According to Ernest (1989, 1991) teachers who practiced logicism are more likely to create a teacher centered instructional environment. These teachers also tend to explain the reasons for the rules and procedures used. Adhering to these mathematical beliefs,

students were asked to memorize rules and textbooks were considered important in the classroom.

(ii) Mathematical Beliefs and Practices from the Formalism School of Philosophy
From the formalism point of view, mathematics is a formal and systematic discipline which is represented by symbols. Pure mathematics can be expressed as a formal system in which the truths of mathematics are represented by formal theorems. Therefore, mathematical knowledge is brought about through the manipulation of symbols that operates by prescribed rules and formula. The deductive method is used to manipulate the symbols. According to this view, the focus in solving mathematics problems is based on using symbols.

Ernest (1989, 1991) stated that formalist teachers emphasize that students' mastery in the use of symbols is the main mathematical skills that should be achieved. Good instruction involves clear presentation of the steps used in any procedure followed by extensive drills to ensure memorization. Formalist teachers will teach in a teacher centered instructional environment.

(iii) Mathematical Beliefs and Practices from the Intuitionism School of Philosophy
Intuitionism view human mathematical activity as fundamental in the construction of proofs or mathematical objects and the creation of new knowledge. It also acknowledges the fact that the axioms of mathematical theory (and logic) are fundamentally incomplete. Hence, what is needed is the addition of a greater mathematical truth as revealed informally or by intuition (Brouwer 1927; Dummett 1977). Therefore, mathematics is conceived as an intellectual activity in which mathematical concepts are seen as mental constructions regulated by natural laws. These constructions are regarded as abstract objects that do not necessarily depend on proofs. Based on intuitionism, mathematical thought is a natural outgrowth of the human cognitive apparatus, which finds itself in our physical universe that is the foundation of mathematics. The effectiveness of mathematics was constructed by the brain in order to be effective in this universe.

(iv) Mathematical Beliefs and Practices from the Kuhnism School of Philosophy
Thomas Samuel Kuhn (1922-1996) presented the idea that science does not evolve gradually toward truth, but instead undergoes periodic revolutions, which is called paradigm shifts. Kuhn was responsible for popularizing the term paradigm, which he described as essentially a collection of beliefs shared by scientists, a set of agreements about how problems are to be understood. According to Kuhn, paradigms are essential to scientific inquiry, for "no natural history can be interpreted in the absence of at least some implicit body of intertwined theoretical and methodological belief that permits selection, evaluation, and criticism." Indeed, a paradigm guides the research efforts of scientific communities, and it is this criterion that most clearly identifies a field as a science.

Even though Kuhn only emphasized his work in relation to science, but his paradigm was accepted throughout other fields including mathematics. Kitcher (1984) supported

Kuhn's idea which discussed the form of Kuhn's revolution in mathematics with his examples. Besides Kitcher, Gladwell (1997) also supported Kuhn's idea and the thought that it can be applied to other fields such as philosophy, history, sociology, economics and religion. According to Gladwell,

> Kuhn will be remembered because he taught that the process of science was fundamentally human, that discoveries were the product not of some plodding, rational process but of human ingenuity intermingled with politics and personality…that science was, in the end, a social process (Gladwell 1997).

Shaharir (1992) and Shaharir and Samian (1987) showed the existence of Kuhn's idea in mathematics particularly in division operation, differential and optimization. They argued that the influence of culture on mathematics and knowledge in general has been accepted as the *a priori* truth.

From the instructional practices perspective, Kuhnism emphasized on real life situation in teaching mathematics such as using names of places instead of symbols such as x and y, which could be considered as integration of values in mathematics. Therefore, mathematics is not only seen as an abstract knowledge, but also connected with real life situations. This could develop students' appreciation towards the culture of mathematics and enjoy learning mathematics. In addition, this could enhance student teacher interactions in the classroom where students are allowed to explore and investigate while teachers facilitate learning in the classrooms.

This study sought to examine secondary school mathematics teachers' perception on mathematical values and transmission of mathematical values in teaching among Malaysian mathematics secondary school teachers. Specifically the objectives of this study were to:

- Describe teachers' beliefs of mathematical values related to mathematics teaching based on the four philosophy of mathematics namely logicism, formalism, intuitionism and kuhnism;
- Describe teachers' instructional practices in mathematics teaching based on philosophy of mathematics namely logicism, formalism, intuitionism and kuhnism.

METHODOLOGY

The following discussion will cover methodological aspect of the research such as research design, population and sampling, instrumentation, reliability and validity, collection of data and data analysis.

(i) Research Design

A quantitative research method with a survey design was used in order to assess secondary mathematics teachers' mathematical beliefs and their instructional practices. This design is most suitable because this study sought to describe the Malaysian secondary mathematics teachers' beliefs and practices. According to Fraenkel and Wallen (2006),

the survey research is used when the researcher want to describe and find out some aspects or characteristics of the population of which that group is a part.

(ii) Population and Samples of Study

The target population of this study was secondary school mathematics teachers from various schools in Malaysia. The population selection was based on six geographical zones in Malaysia which are north, south, east, west, Sabah and Sarawak zones. An additional factor was considered in this selection which then included population from the states of Melaka and Pulau Pinang which were among the early states ruled by the British. These two states are believed to be influenced by the difference in culture and values adopted from the then British rulers. Hence the accessible population of this study consisted of teachers from seven states in Malaysia, which were Perlis, Melaka, Kelantan, Perak, Pulau Pinang, Sabah and Sarawak.

The participants for the research are mathematics teachers teaching at randomly selected secondary schools in three districts each from the states studied. In addition the selection encompassed teachers from the four types of secondary schools in Malaysia, which were daily, boarding, religious and vocational and technical schools.

Two categories of daily schools namely the urban and rural schools were considered during the selection. Therefore the daily schools from urban and rural areas were from the three districts of the previously selected states each. However, the other schools such as boarding, religious and vocational and technical schools were not sub-categorized into urban or rural schools. There were two types of boarding schools considered in this study namely the government boarding school and the '*Maktab Rendah Sains Mara*' (MRSM).

A total of 183 schools from the 7 states chosen for the research were of the following categories:

Daily Schools	=	53 Urban Schools, 84 Rural Schools
Boarding Schools	=	11 Government Boarding Schools
	=	12 MRSM
Religious Schools	=	12
Technical Schools	=	12
		183

INSTRUMENTATION

The instrument of this study consisted of a set of Likert-scale questionnaires which comprised of 45 items, both positively and negatively worded, which were related to mathematics teachers' beliefs and instructional practices in their mathematics teaching.

The instrument was developed by the researchers based on the literature. The instrument was divided into two parts. Part I contained 13 items on the demography aspects of the teachers such as respondents' ethnic background, years of teaching experience, qualifications, etc. For these items, respondents were asked to fill in the blanks provided.

Part II consisted of 45 items which were to obtain mathematics teachers' mathematical beliefs and their mathematics instructional practices based on mathematics philosophy. For each item, teachers were to respond by circling the agreement level towards the item; which were ranked as 'strongly disagree', 'disagree', 'fairly agree', 'agree' and 'strongly agree'. Based on these responses a composite score for both mathematical beliefs and practices were computed for each respondent.

A set of questionnaires was distributed to the participants for respondents. The sets of questionnaires, a cover letter briefing about the research and instructions to the headmasters and self addressed envelopes were posted to the headmasters. The headmasters were responsible to distribute the questionnaire to all mathematics teachers in their school. The mathematics teachers were then asked to return the completed questionnaire direct to the researcher using the envelope provided to them. A reminder was sent to the headmasters for those teachers who delayed in responding to the questionnaire two weeks after the dateline indicated in the cover letter.

VALIDITY AND RELIABILITY

To ascertain the validity of the instrument, a panel of three educational researchers (Professors in mathematics and mathematics education) were appointed to validate the content of the instrument. They were given a set of questionnaires consisting of items measuring constructs such as mathematics teachers' mathematical beliefs and their mathematics instructional practices based on mathematics philosophy. They were also given the objectives of the study and some guidelines to assist in the validation process. A month later, panel members were contacted by phone as a reminder. Written comments were given in due course. Verbal comments were also collected during discussions with the team of researchers. The comments were then discussed and taken into consideration by the team of researchers and were used in constructing the items to measure the required constructs in the questionnaires.

The reliability coefficient for the construct measuring mathematics teachers' mathematical beliefs, its sub-constructs (based on the four paradigms: logicism, formalism, intuitionism and kuhnism) and mathematics teachers' instructional practices were obtained using the Cronbach Alpha based on the findings from the pilot study. The

TABLE 1
Realibility of each construct on mathematics teachers' beliefs and instructional practices

Constructs	Alpha Coefficient	
	Beliefs	Instructional Practices
Logicism	0.50	0.73
Formalism	0.62	0.66
Intuitionism	0.75	0.61
Kuhnism	0.75	0.77
Overall	0.81	0.83

reliability index for each construct ranges from 0.50 to 0.77, which isconsidered rather high (based on Fraenkel and Wallen, 2006). The overall reliability index obtained for beliefs in mathematical values was r = 0.81 and for instructional practices was r = 0.83.

FINDINGS AND DISCUSSION

A total of 1,786 secondary mathematics teachers were given questionnaires of the study. However only 1,560 questionnaires were returned and 58 questionnaires of these were incomplete. Therefore a total of 1,502 complete questionnaires were used for data analysis, which comprised 84% of the total sample size determined during the sampling procedures.

From the total of 1,502 respondents, 986 (65%) were females and 516 (35%) were males. From Table 2, majority of the samples were Malay teachers (1,041; 66.7%) followed by Chinese teachers (363; 23.3%) and other ethnic categories (72; 4.6%) whilst only 26 teachers (1.7%) were Indians.

TABLE 2
Profile of respondents

Profile of Respondents	No. of Respondents	Percentage of Respondents
Gender		
Female	986	65
Male	516	35
Ethnic Group		
Malay	1041	66.7
Chinese	363	23.3
Indians	26	1.7
Others	72	4.6
Age		
Below 25 yrs old	118	7.6
26 – 30 yrs	263	16.9
31 – 35 yrs	329	21.1
36 – 40 yrs	303	19.4
41 – 45 yrs	218	14
46 – 50 yrs	174	11.2
Above 51 yrs	65	4.2
Teaching Experiences		
Less 3 years	309	19.8
4-7 years	301	19.3
8-11 years	251	16.1
12-15 years	168	10.8
16-19 years	97	6.2
Above 20 years	246	15.8

The respondents were between the ages 23 to 54 years. Three hundred and twenty nine (21.1%) of the teachers were aged between 31 to 35 years. This was followed by 303 (19.4) teachers in the 36-40 years age bracket. In general, majority of the teachers were aged were between 25 to 40 years.

Findings showed that 81.2% (1063) had over three years teaching experience. The majority of the teachers (19.6%) had four to seven years teaching experience and 16.8% had eight to eleven years teaching experience. About a quarter (15.8%) of the teachers had over twenty years teaching experience.

Teachers' Beliefs of Mathematical Values Based on Philosophy of Mathematics

The findings are discussed based on four different teachers' beliefs on mathematics, which are 1) Logicism, 2) Formalism, 3) Intuitionism and 4) Kuhnism. The main focus of this study was to describe teachers' belief on mathematics from the different perspectives. A composite score for teachers' mathematical beliefs were calculated for each perspective. The score ranged from one to five with one indicated low mathematical beliefs whilst five indicating high mathematical beliefs. Further interpretations were based on the general rule provided by Kubiszyn and Borich (1996), which stated that the cut-off point of the mean rating is 3.0 and that scores higher that 3.0 is regarded as high in mathematical beliefs whilst the contrary is regarded as low. In addition, according to Nugent, Sieppert, and Hudson (2001) these scores can be conceived as reflecting a magnitude continuum. In this study, scores ranging from 3.00 to 4.00, was indicated as moderate level of teachers' mathematical beliefs hence moderately inclined towards value free beliefs, whilst scores 4.00 to 5.00 indicated a high level of teachers' mathematical beliefs and therefore highly inclined towards value free beliefs. On the other continuum, scores ranging from 2.99 to 1.00 indicated low level of mathematical beliefs which therefore inclined towards value loaded beliefs. This scoring and interpretation was used for the beliefs based on logicism, formalism and intuitionism. However for kuhnism, the interpretation of the mean scores was in reverse wherein high scores indicated low mathematical beliefs therefore inclining towards value loaded beliefs.

Teachers' Mathematical Belief Based on Logicism Perspective

This construct was measured using 5 items. Teachers' responses (mean composite scores) ranged from 2.75 to 4.44 with an overall mean response of 3.68. This showed that the respondents had moderately high beliefs on mathematics based on logicism perspective. This finding indicated that teachers' mathematical beliefs on this perspective were inclined towards the value free paradigm.

Analysis based on individual items indicated that mathematics teachers had a strong belief in mathematics as knowledge of logic with a mean of 4.44 (s=0.61). The majority of teachers totaling 1,442 (92.4%), agreed with this statement. There were also a few teachers who felt that mathematics was not knowledge of logic. However, there were 51 respondents (3.3%) who fairly agreed and only 10 respondents (0.7%) disagreed. Item 3 as shown in Table 3 indicated that most of the teachers did not agree with this statement

TABLE 3

Frequencies, means and standard deviations of teachers' beliefs on mathematics based on logicism

Item	Disagree	Fairly Agree	Agree	Mean	SD
1. Mathematics is a logical knowledge	10 (0.7%)	51 (3.3%)	1442 (92.4%)	4.44	0.61
2. Mathematics is full of rules for problem solving	20 (2.2%)	123 (8.2%)	1409 (90.4%)	4.30	0.68
3. Mathematics is a field of knowledge with less emotions	527 (35.3%)	684 (45.8%)	283 (18.9%)	2.75	0.97
4. The last answer of a solution is the most important in problem solving	377 (25%)	567 (37.7%)	559 (37.2%)	3.15	1.11
5. All mathematics solutions can be obtained by thinking logically	119 (7.9%)	382 (24.5%)	1001 (66.7%)	3.76	0.88
n = 1560	$\overline{X}$ = 3.68	SD = 0.50			

hence obtaining the lowest mean of 2.75. This indicated that mathematics teachers' beliefs were value loaded. Details of the responses are shown in Table 3.

Teachers' Belief on Mathematics Based on Formalism Perspective

This section also consists of 5 items. Findings showed that teachers' beliefs towards mathematics as formalism were moderately high with a mean of 3.97 and a standard deviation of 0.48. Most teachers agreed (86.5%) that mathematics is an exceptionally unique knowledge compared to other knowledge whilst only 34 respondents (2.2%) disagreed, hence the mean score of 4.25. The least agreeable item in this section was, 'Mathematics is an abstract knowledge' which obtained the lowest mean of 3.49 and a standard deviation of 0.92. Only 766 respondents (51%) agreed with the item, followed

TABLE 4

Frequencies, means and standard deviations of teachers' beliefs towards mathematics based on formalism

Item	Disagree	Fairly	Agree	Mean	SD
1. Mathematics is solving problems by symbols	61 (4.1%)	313 (20.9%)	1127 (75.1%)	3.85	0.72
2. Mathematics is exceptionally unique knowledge compared to other knowledge	34 (2.2%)	169 (11.2%)	1301 (86.5%)	4.24	0.77

Table 4 cont'd

3. Mathematics is an abstract knowledge	182 (12.1%0	554 (36.9%)	766 (51%)	3.49	0.92
4. The truth of mathematics is absolute	25 (1.6%)	152 (10.1%)	1322 (88.2%)	4.18	0.70
5. Mathematics is a knowledge with all sorts of formula manipulations	39 (2.6%)	176 (11.7%)	1291 (85.7%)	4.06	0.68

n = 1560 $\overline{X}$ = 3.97 SD = 0.48

by 554 respondents who fairly agreed (36.9%) and 182 respondents (12.1%) who disagreed. The results obtained are shown in Table 4.

Teachers' Belief on Mathematics Based on Intuitionism Perspective

A total of 5 items were solicited and an overall mean response of 3.74 was obtained. This indicated that mathematics teachers' beliefs were moderately high hence inclined towards value free. The highest mean response of 4.08 was obtained from the item 'Laws of mathematics can be derived through literate human'. Most of the respondents agreed with this statement with 1,336 respondents (85.6%). There were also some respondents who fairly agreed and only a handful disagreed with this statement, comprising of 132 (8.5%) and 37 (2.4%) teachers respectively. The findings also indicated that as many as 1,266 (81.2%) agreed with the item 'Mathematics is a knowledge which

TABLE 5

Frequencies, mean and standard deviation of teachers' beliefs towards mathematics based on intuitionism

Item	Disagree	Fairly Agree	Agree	Mean	SD
1. Mathematical activities are based on construction of mental and intuition capabilities	83 (5.3%)	300 (19.2%)	1118 (71.7%)	3.89	0.82
2. Mathematics is a knowledge which involves intuition	235 (15.1%)	685 (43.9%)	581 (37.3%)	3.25	0.87
3. Laws of mathematics can be derived through literate human	37 (2.4%)	132 (8.5%)	1336 (85.6%)	4.08	0.66
4. Mathematical activities are activities which involves individual inner feelings	123 (7.9%)	527 (33.8%)	847 (54.3%)	3.55	0.77
5. Mathematics is an unlimited process in search for solution	42 (22.7%)	308 (19.7%)	1149 (73.7%)	3.93	0.72

n = 1560 $\overline{X}$ = 3.74 SD = 0.54

involves intuition' leading to a mean of 3.25. This indicated that mathematics teachers' beliefs were relatively high hence again inclining towards value free paradigm. Details of items from this sub-construct are shown in Table 5.

Teachers' Belief on Mathematics Based on Kuhnism Perspective

Six items were used to investigate this perspective. The overall mean response obtained was 4.06. For this perspective high mean scores indicated high beliefs, which can be categorized as value loaded beliefs.

The findings further indicated that many mathematics teachers agreed with the item 'mathematics as a universal knowledge without any boundaries', hence the mean score obtained was 4.48 and a standard deviation of 0.60. A majority of the teachers agreed with this statement (1,444; 92.5%). There were also a few teachers who fairly agreed with the item (51; 3.3%) and only 11 respondents (0.7%) disagreed. On the other hand, the mean response for the item 'mathematics is a knowledge influenced by culture' was found to be low with a mean of 3.48 with a standard deviation of 0.82. A total of 781 respondents (50.1%) agreed with the statement followed by 577 respondents (37%) who fairly agreed and 145 respondents (9.3%) disagreeing. Details of the responses are shown in Table 6.

In general,the findings showed that between the four sub-constructs of beliefs toward mathematics, mean score of teachers' belief based on kuhnism was the highest. The

TABLE 6

Frequencies, mean and standard deviation of teachers' beliefs towards mathematics based on kuhnism

Item	Disagree	Fairly Agree	Agree	Mean	SD
1. Mathematics is a universal knowledge without any boundaries	11 (0.7%)	51 (3.3%)	1444 (92.5%)	4.48	0.61
2. Mathematics aimed at helping individuals to understand the universe	17 (1.1%)	181 (11.6%)	1306 (83.7%)	4.18	0.68
3. Mathematics is a part of art	67 (4.3%)	226 (14.5%)	1213 (77.7%)	3.99	0.79
4. Mathematical developments are influenced by culture of the society	24 (1.5%)	116 (7.4%)	1362 (87.3%)	4.21	0.66
5. The laws of nature can be derived through mathematics	42 (2.7%)	246 (15.8)	1215 (77.9%)	3.99	0.73
6. Mathematics is a knowledge influenced by culture	145 (9.3%)	577 (37%)	781 (50.1%)	3.48	0.82

n = 1560 $\overline{X}$ = 4.06 SD = 0.48

mean response was 4.06 and standard deviation of 0.48. This finding showed that mathematics teachers' beliefs were inclined towards value loaded. This means that

TABLE 7
Mean and standard deviation of five sections of mathematics teachers' beliefs towards mathematics

Sub Parts	Mean	Standard Deviation
Logicism	3.68	0.50
Formalism	3.97	0.48
Intuitionism	3.74	0.54
Kuhnism	4.06	0.48

mathematics teachers' beliefs are associated with culture, society and community which are integral part of values.

Table 7 shows the means and standard deviation of the four sub-constructs of the mathematics teachers' beliefs towards mathematics based on philosophy of mathematics.

Mathematics Teachers' Instructional Practices Based on Mathematics Philosophy
A total of 20 items were solicited from the teachers. These items aimed at assessing instructional practices during mathematics teaching and learning. The items were then divided into four sub-constructs based on mathematics philosophy such as logicism, formalism, intuitionism and kuhnism. A composite score for teachers' instructional practices were calculated for each perspective. The score ranged from one to five with one indicating value loaded mathematical practices whilst five indicating value free mathematical practices. In this study, mean scores of 4.00 to 5.00 indicated a high value free instructional practices whilst scores ranging from 3.00 to 4.00, indicated a moderate level. On the other continuum, scores ranging from 2.99 to 1.00 indicated value loaded instructional practices. This scoring and interpretation will be used for the practices based on logicism, formalism and intuitionism. However for kuhnism perspectives, the interpretation of the mean scores will be in the reverse direction wherein high scores indicates value loaded practices.

Teachers Instructional Practices in Logicism Perspectives
Five items associated with instructional practices based on the logicism were posed to mathematics teachers. The overall mean response of 3.76 was obtained with a standard deviation of 0.45. This showed that mathematics teachers had a moderate value free instructional practice. Based on logicism perspective, the findings indicated that most of the teachers always emphasized on improving students' mistakes (mean of 4.09). A total of 1,377 teachers (88.3%) agreed that they always focused on improving students' mistakes and there were also 107 (6.9%) teachers who fairly agreed with the statement. Based on logicism perspective, the least agreeable item was 'teachers asked students to present solution based on set theory' which obtained a mean response of 3.57 and a

TABLE 8
Frequencies, means and standard deviations of teachers' instructional practices based on logicism

Item	Disagree	Fairly Agree	Agree	Mean	SD
1. Use terms of theory set in explaining mathematical concept	55 (3.6%)	398 (25.5%)	1032 (66.1%)	3.73	0.66
2. Ask students to present solution based on set theory	74 (4.7%)	536 (34.4%)	870 (55.8%)	3.57	0.68
3. Use mathematical reasoning questions	65 (4.1%)	525 (33.7%)	904 (58%)	3.61	0.67
4. Write mathematical solution in systematic form	43 (2.7%)	360 (23.1%)	1086 (72.3%)	3.80	0.67
5. Focus on correcting students mistakes from logical perspective	13 (0.8%)	107 (6.9%)	1377 (88.3%)	4.09	0.54

n = 1560 $\overline{X}$ = 3.76 SD = 0.45

standard deviation of 0.68. Table 8 shows that 870 respondents (55.8%) agreed with the statement followed by 536 respondents (34.4%) who fairly agreed. Table 8 provides detailed frequencies, means and standard deviations.

Teachers Instructional Practices Based on Formalism Perspectives

Five items associated with instructional practices based on formalism were posed to mathematics teachers. An overall mean response of 4.02 was obtained with a standard deviation of 0.45. This showed that mathematics teachers' instructional practices were inclined towards value free.

The results also indicate that teachers' greatest emphasis was on giving confidence to students to derive solution for any mathematical problem with a mean response of 4.40 and a standard deviation of 0.58. Majority of mathematics teachers; 1,443 respondents (92.5%) agreed that they always gave confidence to students to derive a solution for mathematical problems. This was followed by 50 respondents (3.2%) who

TABLE 9
Responses on instructional practices based on formalism

Item	Disagree	Fairly Agree	Agree	Mean	SD
1. Present mathematical definitions during lesson induction	33 (2.1%)	271 (17.4%)	1195 (76.6%)	4.00	0.71
2. Use mathematical symbols throughout the lesson	41 (2.6 %)	367 (23.5%)	1091 (69.9%)	3.88	0.71
3. Present lesson conclusion in mathematical form	30 (2.0%)	340 (21.8%)	1127 (72.2%)	3.86	0.66

4. Give confidence to students to derivesolution for any mathematical problems	8 (0.5%)	50 (3.2%)	1443 (92.5%)	4.40	0.58
5. Encourage students to memorize formulas	61 (3.9%)	304 (19.5%)	1136 (72.9%)	3.94	0.80

n = 1560 $\overline{X}$ = 4.02 SD = 0.45

fairly agreed and only 8 respondents (0.5%) who disagreed with the statement. Responses to the item 'present lesson conclusion in mathematical form' were found to be very high (94.0%). This indicated that mathematics teachers were inclined towards value free practices in the classrooms. The details of the responses are shown in Table 9.

Teachers Instructional Practices Based on Intuitionism Perspectives

The overall mean response obtained for the 5 items for this sub-construct was 3.94. This showed that teachers' instructional practices were inclined towards value free beliefs. This means that teachers' instructional practices in the classrooms were not related to values and that mathematics thinking was seen as mental constructions regulated by natural laws. Mathematics teachers showed positive agreement towards the item 'arrange

TABLE 10

Responses on instructional practices based on intuitionism view

Item	Disagree	Fairly Agree	Agree	Mean	SD
1. Arrange teaching materials in hierarchy form	17 (10%)	107 (6.9%)	1377 (88.3%)	4.16	0.61
2."There is no limit to problem solving" is my principle then teaching mathematics	22 (1.4%)	240 (15.4%)	1223 (78.4%)	4.01	0.65
3. Do not rely on textbooks during teaching	40 (2.6%)	152 (9.7%)	1306 (83.7%)	4.15	0.73
4. Present mathematical explanations during teaching based on my own thinking	52 (3.3%)	354 (22.7%)	1091 (69.9%)	3.83	0.71
5. Present questions that involved students to think	89 (5.7%)	614 (39.4%)	790 (50.7%)	3.54	0.75

n = 1560 $\overline{X}$ = 3.94 SD = 0.43

teaching materials in hierarchy form' with a mean score of 4.16. On the other hand, the mean response towards the item 'present questions that involved students to think' was 3.54 and a standard deviation of 0.75. This indicated that teachers were again inclined towards value free beliefs. Details of items related to instructional practices based on the intuitionism pertspective are illustrated in Table 10.

Teachers Instructional Practices Based on Kuhnism Perspectives

The findings as cited in Table 11 indicate that the mean response of mathematics teachers' instructional practices based on Kuhnism view was 3.67 and a standard deviation of 0.54. This may suggest that mathematics teachers were inclined towards value loaded

TABLE 11

Responses on instructional practices based on kuhnism view

Item	Disagree	Fairly Agree	Agree	Mean	SD
1. Explain mathematical concepts in relation to real life situation to students	11 (0.7%)	129 (8.3%)	1357 (87%)	4.09	0.55
2. Present meaning of mathematical symbol in relation to local culture	82 (5.2%)	602 38.6%)	808 (41.8%)	3.54	0.70
3. Integrate historical perspective in teaching	114 (7.3%)	501 (32.1%)	878 (56.3%)	3.57	0.91
4. Relate mathematics to arts in teaching	159 (10.2%)	600 (38.5%)	736 (47.1%)	3.42	0.82
5. Present nature's rule in explaining mathematical concept	71 (4.6%)	404 (25.9%)	1014 (65%)	3.71	0.71

n = 1560 $\overline{X}$ = 3.67 SD = 0.54

beliefs, which also indicated that they practiced integration of values in their teaching and learning. The item with the highest mean (4.09) was 'teachers emphasized on explaining mathematical concepts in relation to real life situation'. Majority of the teachers (1,357; 87%) agreed while only 129 (8.3%) and 11 (0.7%) teachers who fairly agreed and disagreed with this statement respectively. Detail of the responses are shown in Table 11.

The findings show that mathematics teachers' instructional practices in mathematics classroom centered on formalism. The mean response obtained from the teachers was high with a score of 4.02 and a standard deviation of 0.45. This finding showed that mathematics teachers' practices emphasized mathematics formulas and solving mathematics problems using symbols. On the contrary, mathematics teachers' instructional practices based on kuhnism was found to be the lowest amongst the other three school of philosophy of mathematics with a mean response of 3.67 and a standard deviation of 0.54. This indicates that among mathematics teachers, less emphasis was given on real life materials and students experiences during teaching and learning on

TABLE 12
Mean and standard deviation of four sub parts of mathematics teachers' instructional practices

Sub Parts	Mean	Standard Deviation
Logicism	3.76	0.45
Formalism	4.02	0.45
Intuitionism	3.94	0.43
Kuhnism	3.67	0.54

mathematics. Table 12 below shows the means and standard deviation of the four sub-construct of mathematics teachers' instructional practice.

CONCLUSION

This study focused on teachers' belief and practices based on philosophy of mathematics namely logicism, formalism, intuitionism and kuhnism. These classification were also divided into two namely the absolutist and fallibilist. The absolutists adopt the value free paradigm in which mathematical knowledge is certain, absolute and without flaw. The fallibilists however adopts the value load paradigm in which mathematics is not only seen as an abtract knowledge but also connected to real life.

Based on the findings of the research, it can be concluded that the majority of mathematics teachers' beliefs were mainly based on Kuhn's philosophy of mathematics. Mathematics teachers believed that mathematics is influenced by culture and society and which aimed at helping students and individuals understand the universal principles upon which the mathematical rules and concepts are derived.

In relation to instructional practices, mathematics teachers adopted the formalism perspective. Thus, it can be concluded that the mathematics teachers' instructional practices were closely affiliated with their social norms and culture. This suggests that mathematics teachers' beliefs were not congruent with their practices. Teachers' beliefs relates to kuhnism whilst practices relates to formalism. This also suggest that even though they belief on kuhnism, they did not portray it in their teaching which is seen to emphasize formalism.

With the view on kuhnism which relates mathematics to culture and society, the findings of the study reveal that teachers' belief based on this view is rather high. This can be attributed to the fact that mathematics teachers may hold this belief but the constraints and demands to fulfill the examination needs and syllabus may refrain the teachers from imparting this value in their mathematics teaching'. The fact that there is not much difference in mathematics teachers' belief based on the four views as shown by the findings of the study indicates that the teachers in the study in general, do not quite understand the distinct principles inherent in each view.

Previous studies have shown that values are important elements in teaching and learning mathematics (Bishop *et al.*1999; Bishop 1996). Similarly, the Malaysian mathematics teachers' beliefs seemed to emphasize the importance of values. However, the extent of integrating values in teaching and learning were minimal. Thus efforts towards improvement such as providing training to these teachers may enhance their beliefs and practices.

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