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Symmetric and Asymmetric Information Modeling in Economic Growth

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ABSTRACT

This study presents a model which can be used to improve our understanding of relationships involving asymmetric information. The results show that when there is asymmetric information it is profitable to make some of the salary dependent on the outcome (to give the agent an incentive to work hard). Yet, not all the salary is outcome dependent, since the agent is risk-averse. In short, the optimal contract is the result of a tradeoff between optimal risk-allocation and optimal incentive mechanisms. The effort (and, as a consequence, the amount produced) will be lower under asymmetric information than symmetric information.

Keywords: A principal-agent problem, economic growth, symmetric and asymmetric information

INTRODUCTION

The typical example of asymmetric information involves a landlord and his tenant, although the relationship is generalizable to many situations (such as between doctors and patients). The essential point is that these relationships are often characterized by asymmetric information, that is, one person knows more than the other about some important variable. For instance, the tenant knows how hard he works, but the landlord may only observe the actual harvest. Since a good harvest is only partially correlated with high effort (variables other than effort influence the harvest, such as the weather conditions), the principal cannot infer the effort of the agent directly from the outcome. Given this situation of asymmetric information, we might ask what kind of salary the landlord should offer the principal in order to maximize his profit.

MODEL

It is instructive to see what would happen in a model of symmetric information. We may then compare the social optimality of this contract to the contract that will emerge under asymmetric information.

Symmetric Information

The principal wants to maximize his expected profit, which can be written as:

$$r = px - w \tag{1}$$

r is profit, p is the price of the product, x is the amount produced and w is the wage paid to the agent. Assume that the following relationships hold:

$$x = de + u \tag{2}$$

e is effort, de is first derivation of e, and u is a random variable. So here, x is the amount produced is a function of effort (e) and a random variable (u). Assume, further, that $E(u) = 0$ and $Var(u) > 0$ (but constant). The agent's expected utility can be written symbolically as:

$$EU(w) = a + b.de - hb^2Var(u) - \frac{ce^2}{2} \tag{3}$$

Or, in words: The first part [$a+b.de$] means that the agent receives a fixed sum (a) and a sum depending on his effort (de), his wage is: $w = a + bx$. However, the agents dislikes uncertainty, so a higher variance means less utility (which explains the $hb^2Var(u)$, where, h is simply a positive constant which indicates aversion to variations in income). Moreover, since we are assuming a "lazy" agent, hard work decreases his utility ($\frac{ce^2}{2}$, where c is a positive constant measuring his "aversion" to work). Lastly, assume that the agent can receive $\widehat{U}$ if he decides not to work for his landlord (e.g. $\widehat{U}$ is the utility from unemployment benefits).

The principal's problem is to maximize profit subject to the constraint that he has to offer a contract that gives the agent at least the same utility he could get from unemployment benefits. That is:

$$a + b.de - hb^2Var(u) - \frac{ce^2}{2} = \widehat{U} \tag{3a}$$

or, rearranged we have: $a + b.de = \widehat{U} + hb^2Var(u) - \frac{ce^2}{2}$

Now, the principal will, as mentioned, maximize his expected profit (1). We know that expected production is: $x = de$. We also know that $w = a + b.de$, so:

$$r = p.de - [a + b.de] \tag{4}$$

Before we maximize, the constraint we substitute into the function. That is, we have an expression for [$a+b.de$] which can be substituted into the profit function:

$$r = p.de - \widehat{U} - hb^2Var(u) - \frac{ce^2}{2} \tag{5}$$

To find the optimal wage contract (that is the optimal values for a and b), we differentiate with respect to b:

$$\frac{dr}{db} = -2hbVar(u) = 0 \tag{6}$$

$$\frac{d^2r}{db^2} = -2hVar(u) < 0 \qquad \text{when } h > 0$$
$$\Rightarrow b = 0 \qquad \text{is the optimal} \tag{6a}$$

This implies that in the case of symmetric information,

$$b^* = 0 \tag{7}$$

(since we have assumed that h and $Var(u)$ both are positive constants, b has to be zero to make the whole expression zero).

If b is zero, a has to be large enough to make the agent's utility at least as high as the utility from unemployment benefits (otherwise the agent cannot make the agent work for him). This means that we have:

$$a^* = \hat{U} + \frac{ce^2}{2} \tag{8}$$

Optimal work effort (from the principal's point of view) is:

$$\frac{dr}{de} = pd - ce = 0 \tag{9}$$

or,

$$e^* = \frac{pd}{c} \tag{9a}$$

If c positive
Insert this into a^* to get:

$$a^* = \hat{U} + \frac{p^2d^2}{2c} \tag{8a}$$

In short, under symmetric information we have the following three optimality conditions (8 a), (7), and (9 a).

I interpreted this means that the agent, or the tenant to return to our original example, will be offered a contract with a specified level of effort (e^*), a fixed wage (a^*), and no incentive to supply more effort (since b^*=0). We have assumed that the agent is risk-averse, while the principal is risk-neutral. Thus, it is profitable for both parties if the principal takes the gains and losses from variations, while the risk-averse agent receive a steady income.

Asymmetric Information

Assume, now that the agent knows his own effort, but the principal does not know this effort (he only observes the final outcome). This means that it is difficult to make a contract in which the effort level is specified since there is not way of proving that the agent does not supply the agreed effort (by definition effort is unknown to others except for the agent who, of course, will say that he worked as hard as he could). Thus, when offering the agent a contract, the principal has to–first-offer him a contract that the agent is willing to take (i.e. a utility higher than $\widehat{U}$). Second, the agent has to consider the incentives of the agent – how much effort the agent will supply for various wage systems. The first of these may be called the participation constraint, the second can be called the incentive constraint. Within is constraints the principal will maximize his profit.

We already know that when we substitute the participation constraint is substituted into the profit function, we get (5). Now, to find the agent's optimal level of effort for a given wage system, we simply find $\frac{dEU(w)}{de}$. Recall that (3), so $\frac{dEU(w)}{de} = 0$ is:

$$\frac{dEU(w)}{de} = bd - ce = 0 \tag{10}$$

or,

$$e* = \frac{bd}{c} \tag{10a}$$

If we substitute this expression into the profit function, we have:

$$r = pd\left(\frac{bd}{c}\right) - \widehat{U} - hb^2Var(u) - \frac{c\left(\frac{bd}{c}\right)^2}{2} \tag{11}$$

To find the optimal wage contract, we take the differential with respect to *b*:

$$\frac{dr}{db} = \frac{pd^2}{c} - 2hbVar(u) - \frac{bd^2}{c} = 0 \tag{12}$$

Hence, solving for *b,* we have:

$$b* = \frac{pd^2}{\left[2hcVar(u) + d^2\right]} \tag{12a}$$

Once again, when we know *e** and *b*, a** we can find by substituting *b** and *e** into the participant constraint (*a** must be large enough to make the agent work). We find that:

$$a* = \widehat{U} + h(b*)2Var(u) + \frac{c(e*)^2}{2c} - b*de* \tag{13}$$

In short, we have the following optimal values: (13), (12 a) and (10 a).

COMPARISON

There are at least two interesting differences between the results above and the optimal values when there was symmetric information. First, when there is asymmetric information it is profitable to make some of the salary dependent on the outcome (to give the agent an incentive to work hard). This follows from the fact that $b^* > 0$ in the case of asymmetric information). Yet, not all the salary is outcome dependent, since the agent is risk-averse. In short, the optimal contract is the result of a tradeoff between optimal risk-allocation and optimal incentive mechanisms. Second, the effort (and, as a consequence, the amount produced) will be lower under asymmetric information than symmetric information. Compare (9 a) and (10 a).

Under which conditions will the effort supplied under asymmetric information be the same as the effort supplied under symmetric information? Recall that (12 a). if we have for asymmetric information $b^* = \frac{pd^2}{d^2}$, then $e^* = \frac{pd}{c}$ the same as e^* in symmetric information (9 a).

There are three possibilities:

1. $h = 0$ (no risk aversion)
2. $c = 0$ (no laziness, no aversion to work)
3. $Var(u) = 0$ (no variation in the outcome)

Since these conditions are unlikely to hold, we can concluded that the optimal contract between the principal and the agent in the case of asymmetric information is not socially optimal (most people dislike to work, dislike fluctuations in income and in most cases random variables affect the outcome). If possible both the agent and the principal would prefer the symmetric information contract.

CONCLUSION

We have presented a very condensed version of a model which could have included many more variables and assumptions. Yet, it is interesting to speculate on the wider implications of the results. For instance, an economic system which eliminates private property rights will eventually run into many principal-agent problems since they will have to hire managers (agents) who do not own the factories themselves. In effect, private property reduces the number and seriousness of principal-agent type relationships since with private property, there is less incentive to shirk. Of course, the argument is more complicated and principal-agent models alone do not determine whether private property is a good thing.

Moreover, there are many principal-agent relationships under capitalism (managers-stock owners, is one). Nevertheless, it is impossible to answer all questions, and we must advance by examining one argument at a time. Given this limited aim, the model can show the mechanisms and variables that are involved (the degree of fluctuation, risk aversion, work aversion and so on).

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