Malaysian Journal of Mathematical Sciences **1(2)**: 193 - 204 (2007)

Effect of Non-uniform Suction or Injection on Mixed Convection Flow Over a Vertical Cylinder Embedded in a Porous Medium

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ABSTRACT

The effect of steady non-uniform suction or injection on mixed convection boundary layer flow over a vertical heated or cooled permeable cylinder, which is embedded in a fluid-saturated porous medium, is studied numerically using the Darcy law approximation. Both assisting and opposing flow cases are considered. Using suitable transformations, the coupled governing boundary layer equations are transformed into a form suitable for a numerical solution. The effects of the suction or injection, transverse curvature and mixed convection parameters on the local Nusselt number and temperature profiles are studied. The obtain results are presented graphically and discussed in details.

Keywords: Boundary layer, heat transfer, mixed convection, porous medium, suction/ injection, vertical cylinder

INTRODUCTION

Convective heat transfer in fluid-saturated porous media has been studied quite extensively during the last few decades. This has been motivated by its importance in many natural and industrial problems. Prominent among these are the utilization of geothermal energy, chemical engineering, thermal insulation systems, nuclear waste management, grain storage, fruits and vegetables, migration of moisture through air contained in fibrous insulation, food processing and storage, contaminant transport in ground water and many others. A detailed review of the subject of convective flows in porous media, including exhaustive list of references, was done by Nield and Bejan (2006), Ingham and Pop (2002, 2005), Vafai (2000, 2005), Pop and Ingham (2001), Ingham *et al.* (2004) and Bejan *et al.* (2004).

Free and mixed convection from vertical or horizontal cylinder embedded in a porous medium is the principal mode of heat transfer in numerous applications such as in connection with oil/gas lines, insulation of horizontal pipes, cryogenics as well as in the context of water distribution lines, underground electrical power transmission lines and burial of nuclear waste, to name just a few applications. The case of free and mixed convection flow from vertical cylinder placed in a porous medium has been studied extensively both analytically and numerically. It appears that Minkowycz and Cheng (1976) were the first to present a numerical solution of the problem of free convective boundary layer flow induced by a heated vertical cylinder embedded in a fluid-saturated porous medium when the surface temperature of the cylinder is taken to be proportional to x^m, where x is the distance from the

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leading edge/base of the cylinder and m is a constant. The results were obtained for various values of between 0 and 1. Similarity and local nonsimilarity methods of solution were used. The problem was later extended by Merkin (1986), Kumari et al. (1986), Ingham and Pop (1986), Merkin and Pop (1987), Kumari and Nath (1986), Yücel (1990), Chen *et al.* (1992), Hossain and Nakayama (1993), Bassom and Rees (1996), Pop and Na (1998), Yih (1998), Chen and Horn (1999) and Magyari *et al.* (2005).

In this paper we consider the problem of the effect of non-uniform suction or injection on steady mixed convection boundary layer flow along a vertical permeable slender cylinder embedded in a porous medium. Following Datta *et al.* (2006), we assume that the suction or injection velocity is variable and that the mainstream velocity is constant. It is also assumed that both the wall and ambient temperatures are constant. Both the assisting and opposing flow cases are considered.

GOVERNING EQUATIONS

Consider the Darcy steady mixed convection boundary layer flow of a viscous and incompressible fluid over a vertical permeable cylinder of radius, r_0, that is embedded in a fluid-saturated porous medium with a prescribed non-uniform transversal velocity of suction or injection, $v_w(x)$ as shown in *Fig. 1*, where x is the axial coordinate and r is the radial coordinate. It is assumed that the surface of the cylinder has the constant temperature, T_w, while the ambient fluid has the uniform temperature T_∞, also the mainstream velocity is U_∞. Under these assumptions, the equations of continuity, the Darcy equation with Boussinesq approximation, and the energy equation, can be written by using the usual boundary-layer approximation as, see Merkin and Pop (1987)

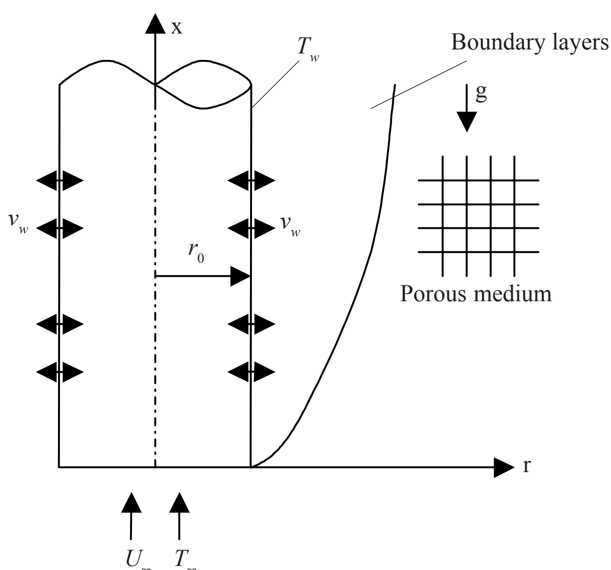

Fig. 1: Physical model and coordinate system

$$\frac{\partial}{\partial x}(r\,u)+\frac{\partial}{\partial r}(r\,v)=0 \tag{1}$$

$$u=U_\infty+\frac{g\beta K}{\upsilon}(T-T_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\frac{\alpha m}{r}\frac{\partial}{\partial}\left(r\frac{\partial T}{\partial r}\right) \tag{3}$$

where u and v are the velocity components along the x– and r– axis, T is the fluid temperature, g is the acceleration due to gravity, K is the permeability of the porous medium, υ is the kinematic viscosity and α_m is the effective thermal diffusivity of the porous medium. The boundary conditions to be applied to Eqs. (1)-(3) are

$$\left.\begin{array}{l} v=v_w(x) \quad \text{for } x_0\le x\le x_0^* \quad v=0 \quad \text{for } x<x_0 \quad \text{and} \quad x>x_0^* \\ T=T_w \end{array}\right\}\text{at } r=r_0 \tag{4}$$
$$u\to U_\infty,\quad T\to T_\infty \qquad \text{as} \qquad r\to\infty$$

where, $v_w(x)$ is the velocity of suction ($v_w<0$) or injection ($v_w>0$), and x_0 and x_0* are constants. We introduce now the following new variables, see Merkin and Pop (1987):

$$\psi=r_0\left(2\alpha_m U_\infty x\right)^{1/2} f(\xi,\eta),\quad \theta(\xi,\eta)=(T-T_\infty)/(T_w-T_\infty)$$
$$\eta=\left(\frac{2U_\infty}{\alpha_m x}\right)^{1/2}\left[\frac{r^2-r_0^2}{4r_0}\right],\quad \xi=\left(\frac{4}{r_0}\right)\left(\frac{\alpha_m x}{2U_\infty}\right)^{1/2} \tag{5}$$

where ψ is the stream function defined in the usual way as

$$u=\frac{1}{r}\frac{\partial\psi}{\partial r},\quad v=-\frac{1}{r}\frac{\partial\psi}{\partial x} \tag{6}$$

and expressions (6) automatically satisfy Eq. (1). Using (5), we get

$$u=U_\infty f',\quad v=-\frac{r_0}{r}\left(\frac{\alpha_m U_\infty}{2x}\right)^{1/2}\left(f+\xi\frac{\partial f}{\partial\xi}-\eta f'\right). \tag{7}$$

where primes denote partial differentiation with respect to η. Substituting (5) and (7) into Eqs. (2) and (3), we get

$$F=1+\lambda\theta \tag{8}$$

$$\left[(1+\xi\eta)\theta'\right]'+f\theta'=\xi\left(F\frac{\partial\theta}{\partial\xi}-\theta'\frac{\partial f}{\partial\xi}\right) \tag{9}$$

subject to the boundary conditions

$$f(\xi,0)=f_w(\xi) \quad \text{for } \xi_0 \le \xi \le \xi_0^*, \quad f(\xi,0)=0 \quad \text{for} \quad \xi<\xi_0 \text{ and } \xi>\xi_0^* \\ \theta(\xi,0)=1, \quad \theta(\xi,\infty)=0 \tag{10}$$

where ξ_0 and ξ_0^* are constants. The function f can be expressed as

$$f=\int_0^\eta F d\eta + f_w(\xi) \tag{11}$$

and $f_w(\xi)$ is given by

$$f_w(\xi)=-\frac{r_0}{2\alpha_m \xi}\int_{\xi_0}^{\xi} \xi v_w(\xi)\,d\xi, \quad v_w(\xi)=-\left(\frac{\alpha_m U_\infty}{2x}\right)^{1/2}\left(f+\xi\frac{\partial f}{\partial \xi}\right)_w. \tag{12}$$

The quantity $\lambda = Ra/Pe$ in Eq. (8) is the mixed convection parameter, $Ra = g\beta K(T_w - T_\infty)\, r_0/a_m v$ is the Rayleigh number and $Pe = U_\infty r_0/a_m$ is the Péclet number. It should be noticed that $\lambda > 0 (T_w > T_\infty)$ corresponds to the assisting flow, $\lambda < 0 (T_w < T_\infty)$ corresponds to the opposing flow and $\lambda = 0$ corresponds to the forced convection flow.

Following Datta *et al.* (2006), we assume that has the expression

$$v_w(\xi)=\begin{cases} -A\left(\dfrac{2\alpha_m}{r_0}\right)\sigma \sin\left[\sigma(\xi-\xi_0)\right] & \text{for} \quad \xi_0 \le \xi \le \xi_0^* \\ 0 \quad \text{for} \quad \xi<\xi_0 \quad \text{and} \quad \xi>\xi_0^* \end{cases} \tag{13}$$

where A and σ are non-dimensional constants with $A > 0$ for suction and $A < 0$ for injection. The function $v_w(\xi)$ is continuous for all values of ξ (curvature parameter) in $\left[\xi_0, \xi_0^*\right]$. The reason for taking such type of function is that it allows the mass transfer (suction or injection) to change slowly near the bottom and the top of the cylinder. Using (13), $f_w(\xi)$ becomes

$$f_w(\xi)=\begin{cases} \left(\dfrac{A}{\xi}\right)\Phi(\xi_0,\xi) & \text{for} \quad \xi_0 \le \xi \le \xi_0^* \\ 0 \quad \text{for} \quad \xi<\xi_0 \quad \text{and} \quad \xi>\xi_0^* \end{cases} \tag{14}$$

where

$$\Phi(\xi_0,\xi)=\xi_0-\xi\cos\left[\sigma(\xi-\xi_0)\right]+\frac{1}{\sigma}\sin\left[\sigma(\xi-\xi_0)\right]. \tag{15}$$

The parameter of physical interest is the local Nusselt number, which is defined as

$$Nu=\frac{x}{T_w-T_x}\left(-\frac{\partial T}{\partial r}\right)_{r=r_0}. \tag{16}$$

On using variables (5), we get

$$Nu Pe_x^{-1/2} = -\theta'(\xi, 0) \tag{17}$$

where $Pe_x = U_\infty x/\alpha_m$ is the local Péclet number.

RESULTS AND DISCUSSION

Equations (8) and (9), subject to the boundary conditions (10), are solved numerically using an implicit finite-difference scheme similar to that proposed by Blottner (1970). The procedure of this method is similar to the implicit scheme of the Crank-Nicolson type, except that the difference equations are written such that only one dependent variable appears in each equation. Therefore, the resulting implicit difference equations are solved for each dependent variable separately. This method is very well described in the paper by Blottner (1970) and has also been used by Kumari and Nath (1986) for the problem of mixed convection boundary layer flow over a thin vertical cylinder with localized injection/suction and cooling/heating placed in a viscous and incompressible fluid (non-porous media). Various computations were carried out to solve equations (8) and (9), subjected to the boundary conditions (10), for different values of the governing parameters: the mixed convection parameter λ, suction or injection parameter A and curvature parameter ξ. We have taken $\xi_{i0} < \xi_i < \xi_{i0}$ as $0.5 < \xi_i < 0.8$, $1.2 < \xi_i < 1.5$ and $2.2 < \xi_i < 2.5$, $\eta_\infty = 6$, $\Delta\eta = 0.015$ and $\Delta\xi_i = 0.01$. It is possible, however, to consider other intervals for ξ_i but we limit our analysis to the above mentioned range of it. The parameter A is in the range $-0.6 \le A \le 0.4$, the constant σ is taken as one ($\sigma = 1$) and $0 \le \xi \le 3$. The cases assisting ($\lambda > 0$), opposing ($\lambda < 0$) and forced convection flow ($\lambda = 0$) are considered. There is no problem in solving Eqs. (8) and (9) for $\lambda = 0$ and $\lambda > 0$. However, for opposing flow ($\lambda < 0$), the values of λ are limited because it is possible that for $\lambda > \lambda_s$ (< 0) the boundary layer separates from the cylinder. Here, λ_s, is the value of λ for which the boundary layer separates. Thus, for an impermeable flat plate ($\xi = 0$), Eqs. (8) and (9) reduce to ordinary differential equations, which can be combined into the following equation:

$$f''' + f\, f'' = 0 \tag{18}$$

subject to the boundary conditions

$$f(0) = 0, \quad f'(0) = 1 + \lambda, \quad f'(\infty) = 1. \tag{19}$$

Merkin (1980) has shown that for $\lambda < 0$ (opposing flow), Eq. (18) subject to (19) has solutions only in the range $-1.354 \le \lambda \le 0$ and for λ in the range $-1.354 < \lambda < -1$ the solution is not unique, there being dual solutions, and for λ in the range $-1 \le \lambda \le 0$ there is only one solution. On the other hand, the values of the local Nusselt number are given in Table 1 for $\xi = 0$ and different values of λ.

TABLE 1
Values of the local Nusselt number for $\xi = 0$

λ	-0.75	-0.50	-0.25	0	0.25	0.50	0.75	1.00
$-\lambda'(0)$	0.5826	0.6630	0.7348	0.7981	0.8603	0.9166	0.9695	1.0196

Further, we notice that for λ = 0, ξ = 0 and A = 0, Eqs. (8), (9) and the boundary conditions (10) reduce to

$$\begin{gathered} \theta'' + \eta\theta' = 0 \\ \theta(0) = 1, \quad \theta(\infty) = 0 \end{gathered} \tag{20}$$

This problem has the following analytical solution:

$$\theta(\eta) = erfc\left(\eta / \sqrt{2}\right) \tag{21}$$

where *erfc*(.) is the complementary error function.

Figs. 2 to *4* show the temperature profiles for some values of the parameters A, λ and ξ. It is seen from *Figs. 2* and *3*, that for λ = 0 (forced convection flow) and λ > 0 (assisting flow) the boundary layer thickness is slightly higher for the case of injection (A < 0) than for that of suction (A > 0). However, for opposing flow (λ < 0), the convergence of Eq. (8)-(9) subject to the boundary conditions (10) is rather weak. This is because, in the particular case ξ = 0, Eqs. (8) and (9) reduce to ordinary differential equation (18) which for λ = –0.25 has a unique solution, as it can be seen in *Fig. 4*.

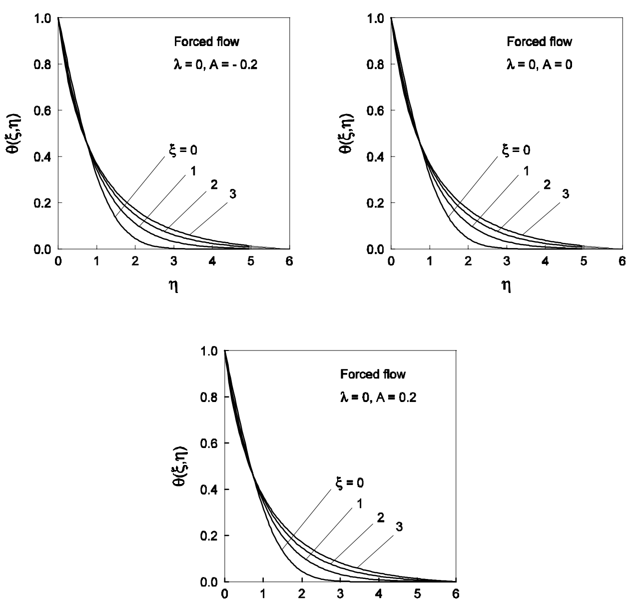

Fig. 2: Temperature profiles for different values of A and ξ for forced convection

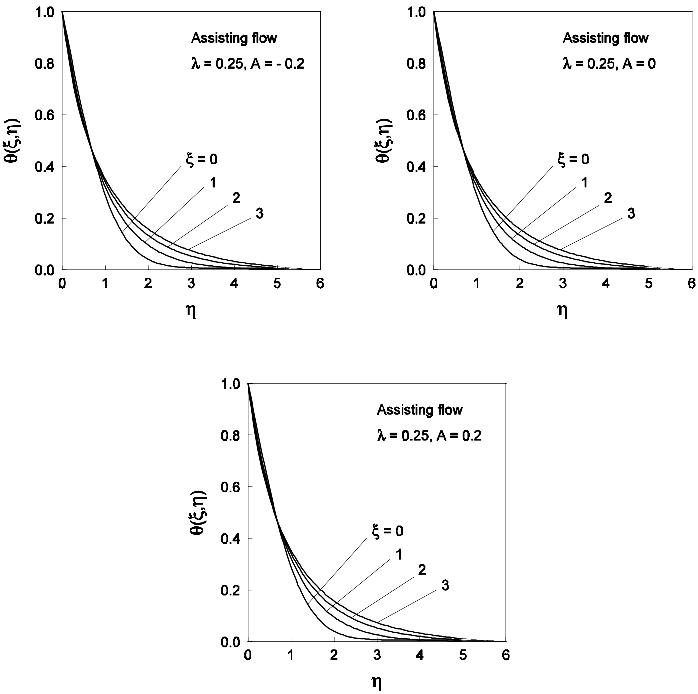

Fig. 3: Temperature profiles for different values of A and ξ for assisting flow ($\lambda > 0$)

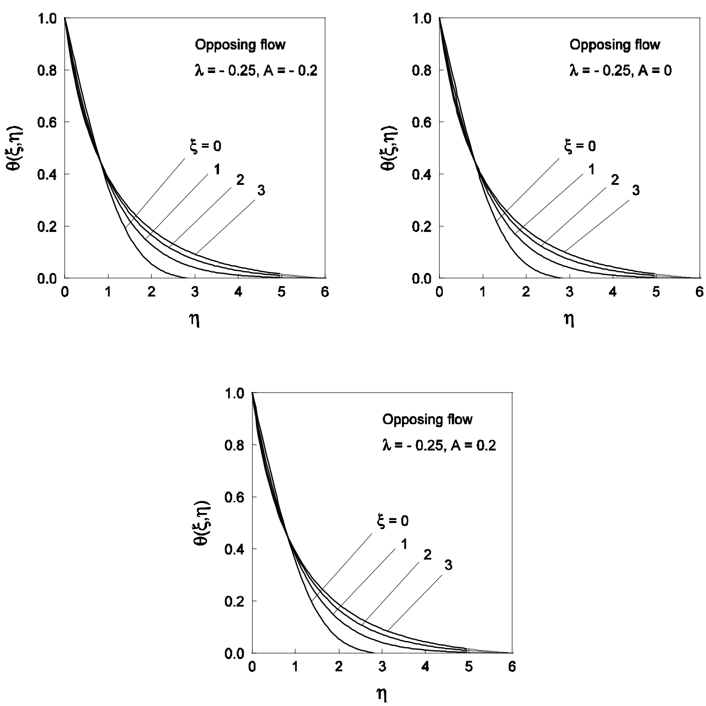

Fig. 4: Temperature profiles for different values of A and ξ for opposing flow ($\lambda < 0$)

The variation with ξ of the local Nusselt number $NuPe_x^{-1/2}$ given by Eq. (17) is illustrated in Figs. 5 to 7 for some values of the parameters A and λ, for the cases of forced convection, assisting and opposing flows, respectively. It is seen from these figures show that for $\xi = 0$, the numerical values of the local Nusselt number are in very good agreement with those given in Table 1. We notice also, that there is no any discontinuity in the Nusselt number close to $\xi = 0$. It appears, however, there is a singularity close to $\xi = 0$ but it is due to steps length, $\Delta\xi$, not enough small considered. Further, we can see from these figures that in the case of suction ($A > 0$), the heat transfer parameter increases and attains a maximum value in the middle of the interval of ξ considered. Then it decreases from its maximum value to its minimum value at the end of the interval of suction considered. These figures also show that the effect of the non-uniform injection ($A < 0$) is just the opposite relative to the value $A = 0$ (impermeable cylinder). Hence, the non-uniform suction helps to reduce the heat transfer coefficients at a particular streamwise location on the slender cylinder. This conclusion is in agreement with the results reported by Datta *et al.* (2006) for the case of forced convection boundary layer flow of a viscous and incompressible fluid past a permeable horizontal cylinder with non-uniform slot suction or injection.

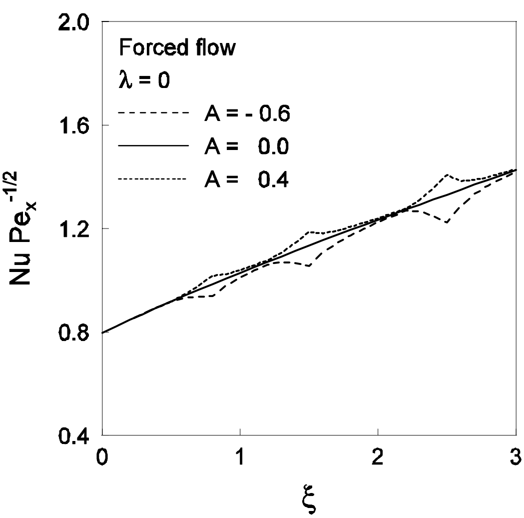

Fig. 5: Variation of the local Nusselt number with ξ for suction ($A > 0$) or injection ($A < 0$) when the flow is forced ($\lambda = 0$)

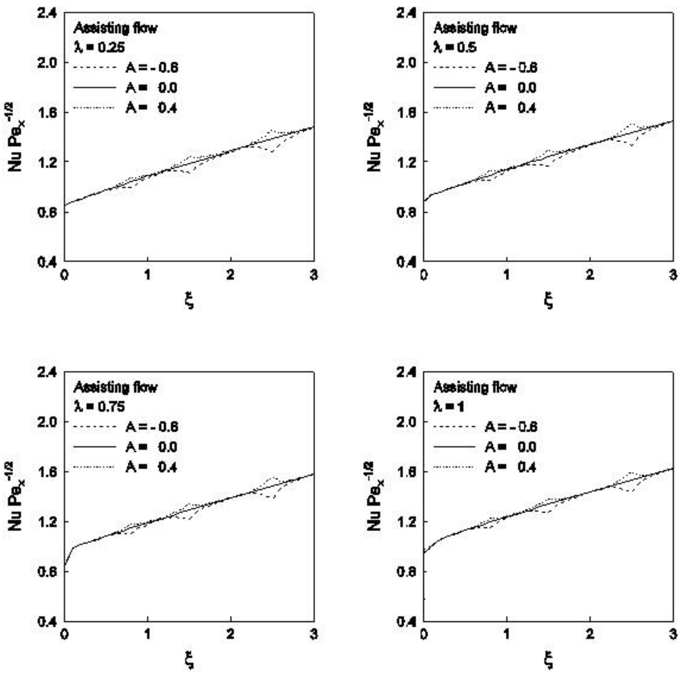

Fig. 6: Variation of the local Nusselt number with ξ for suction ($A > 0$) or injection ($A < 0$) when the flow is assisting ($\lambda > 0$)

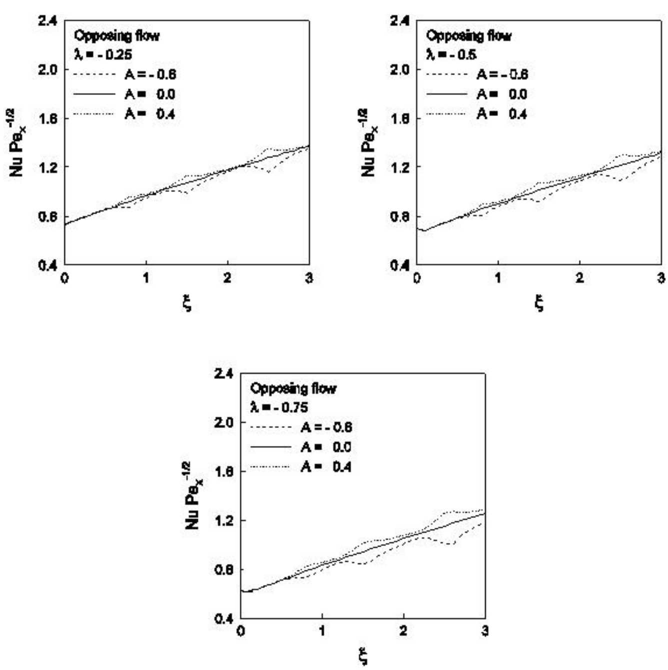

Fig. 7: Variation of the local Nusselt number with ξ for suction ($A > 0$) or injection ($A < 0$) when the flow is opposing ($\lambda < 0$)

CONCLUSIONS

Numerical solutions for the mixed convection boundary layer flow over a permeable vertical slender cylinder embedded in a fluid-saturated porous medium with non-uniform suction or injection have been obtained. It is shown that the heat transfer coefficient is significantly altered by the non-uniform suction or injection. We have also found that in some particular cases, the numerical results are in very good agreement with the analytical solution.

ACKNOWLEDGEMENT

The authors wish to express their very sincerely thanks to the reviewers for the very good constructive comments and suggestions.

NOMENCLATURE

A	suction or injection parameter
f	dimensionless stream function
g	gravitational acceleration
K	permeability of the porous medium
Pe	Péclet number for a porous medium
r	radial coordinate
r_0	radius of the cylinder
Ra	Rayleigh number for a porous medium
T	fluid temperature
u, v	velocity components in the and directions
$v_w(x)$	velocity of suction or injection
U_∞	mainstream velocity in the axial direction
x	axial coordinate

Greek symbols

α_m	effective thermal diffusivity
β	coefficient of thermal expansion
η	pseudo-similarity variable
θ	dimensionless temperature
λ	mixed convection parameter
υ	kinematic viscosity
ξ	curvature parameter
σ	constant
ψ	stream function

Subscripts

w	condition at the wall
∞	condition in the ambient fluid

Superscript

$'$ partial differentiation with respect to η

REFERENCES

BASSOM, A.P. and D.A.S. REES. 1996. Free convection from a heated vertical cylinder embedded in a fluid-saturated porous medium. Acta Mechanica 116: 139-151.

BEJAN, A., I. DINCER, S. LORENTE, A.F. MIGUEL and A.H. REIS. 2004. Porous and Complex Flow Structures in Modern Technologies. Springer, New York.

BLOTTNER, F.C. 1970. Finite-difference methods of solution of the boundary-layer equations. - AIAA J. 8: 193-205.

CHEN, C.-H. and J.-H. HORNG. 1999. Natural convection from a vertical cylinder in a thermally stratified porous medium. Heat Mass Transfer 34: 423-428.

CHEN, C.-K., C.-H. CHEN, W.J. MINKOWYCZ and U.S. GILL. 1992. Non-Darcian effects on mixed convection about a vertical cylinder embedded in a saturated porous medium. Int. J. Heat Mass Transfer 35: 3041-3046.

DATTA, P., D. ANILKUMAR, S. ROY and N.C. MAHANTI. 2006. Effect of non-uniform slot injection (suction) on a forced flow over a slender cylinder. Int. J. Heat Mass Transfer 49: 2366-2371.

HOSSAIN, M.A. and A. NAKAYAMA. 1993. Non-Darcy free convective flow along a vertical cylinder embedded in a porous medium with surface mass flux. Int. J. Heat Fluid Flow 14: 385-390.

INGHAM, D.B. and I. POP (eds.). 2005. Transport Phenomena in Porous Media III. Elsevier, Oxford.

INGHAM, D.B. and I. POP 1986. A horizontal flow past a partially heated infinite vertical cylinder embedded in a porous medium. Int. J. Engng. Sci. 24: 1351-1363.

INGHAM, D.B. and I. POP (eds.). 2002. Transport Phenomena in Porous Media. Pergamon, Oxford,1998, Vol. II.

INGHAM, D.B., A. BEJAN, E. MAMUT and I. POP (eds.) 2004. Emerging Technologies and Techniques in Porous Media. Kluwer, Dordrecht.

KUMARI, M. and G. NATH. 1986. Non-Darcy mixed convection boundary layer flow on a vertical cylinder in a saturated porous medium. Int. J. Heat Mass Transfer 32: 183-187.

KUMARI, M., I. POP and G. NATH. 1986. Non-Darcy free convective flow along a vertical cylinder in a saturated porous medium. Wärme- und Stoffübertr. 20: 33-37.

MAGYARI, E., I. POP and B. KELLER. 2005. Exact solutions for a mixed convection problem over a permeable vertical cylinder in a porous medium. Int. J. Heat Mass Transfer 48: 3435-3442.

MERKIN, J.H. 1980. Mixed convection boundary-layer flow on a vertical surface in a aturated porous medium. J. Engng. Math. 14: 301-313.

MERKIN, J.H. 1986. Free-convection from a vertical cylinder embedded in a saturated porous medium. Acta Mechanica 62: 19-28.

MERKIN, J.H. and I. POP. 1987. Mixed convection boundary-layer on a vertical cylinder embedded in a saturated porous medium. Acta Mechanica 66, 251-262.

MINKOWYCZ, W.J. and P. CHENG. 1976. Free convection about a vertical cylinder embedded in a porous medium. Int. J. Heat Mass Transfer 19, 805-813.

NIELD, D.A. and A. BEJAN. 1999. Convection in Porous Media (second edition). Springer, New York.

POP, I. and D.B. INGHAM. 2001. Convective Heat Transfer: Mathematical and Computational Modelling of Viscous Fluids and Porous Media. Pergamon, Oxford.

POP, I. and T.-Y. NA. 1998. Darcian mixed convection along slender vertical cylinders with variable surface heat flux embedded in a porous medium. Int. Comm. Heat Mass Transfer 25, 251-260.

VAFAI, K. (ed.). 2000. Handbook of Porous Media. Marcel Dekker, New York.

VAFAI, K. (ed.). 2005. Handbook of Porous Media (2nd edition). Taylor & Francis, New York.

YIH, K.A. 1998. Coupled heat and mass transfer in mixed convection about a vertical cylinder in a porous medium: the entire regime. Mechanics Res. Comm. 25, 623-630.

YÜCEL, A. 1990. Natural convection heat and mass transfer along a vertical cylinder in a porous medium. Int. J. Heat Mass Transfer 33, 2265-2274.