Malaysian Journal of Mathematical Sciences 2(2): 147-159 (2008)

Stability Analysis and Maximum Profit of Predator – Prey Population Model with Time Delay and Constant Effort of Harvesting

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ABSTRACT

In this paper we present a deterministic and continuous model for predator - prey population model based on Lotka-Volterra model. The model is then developed by considering time delay and the two populations are subjected to constant effort of harvesting. We study analytically the necessary conditions of harvesting to ensure the existence of the equilibrium points and their stabilities. The methods used to analyze the stability are linearization and by investigation the eigenvalues of the Jacobian matrix. The results show that there exists a globally asymptotically stable equilibrium point in the positive quadrant for the model with and without harvesting. The time delay can induce instability and a Hopf bifurcation can occur. The stable equilibrium point for the model with harvesting is then related to profit function problem. We found that there exists a critical value of the effort that maximizes the profit and the equilibrium point also remains stable. This means that the predator and prey populations can live in coexistence and give maximum profit although the two populations are harvested with constant effort of harvesting.

Keywords: Predator-Prey, Time Delay, Jacobian Matrix, Eigenvalues, Effort of Harvesting, Profit.

INTRODUCTION

Predator-prey population model based on Lotka-Volterra model is one of the most popular models in mathematical ecology. Luckinbill (1973) has considered a predator-prey population model and the result showed that the prey and predator can coexist by reducing the frequency of contact between them. Danca et al., (1997) have analyzed a predator-prey model using analytical and numerical methods. They found that the system can exhibit a rich behavior and also determined the domain of the values of the parameters for which the system has stationary states or chaotic behavior.

Kar and Chaudhuri (2004) have studied the predator-prey model based on Lotka-Volterra model with harvesting. They discussed about the possibility of existence of bionomic equilibrium and optimal harvesting. The effect of constant quota and constant effort of harvesting has been studied by Holmberg (1995) and the results showed that the constant catch quota can lead to both oscillations and chaos and an increased risk for over exploitation.

A predator-prey model with Holling type using harvesting efforts as control has been presented by Srinivasu et al., (2001). They showed that with harvesting, it is possible to break the cyclic behavior of the system and introduces a globally stable limit cycle in the system. One predator-one prey system in Hogart et al., (1992) where both the predator and prey are harvested with constant yield has been considered and the stability at maximum sustainable yield is established.

Song and Chen (2002) have considered the exploitation of a predator-prey population with stage structure and harvesting for the prey and showed that the nonnegative equilibrium point is globally asymptotically stable under a certain condition, there exists a threshold of harvesting of prey population and the optimal time delay maximizes the total population. The effects on population size and yield of different levels of harvesting of a predator in a predator-prey system have been explored by Matsuda and Abrams (2004) and showed that the predator may increase in population size with increasing fishing effort.

In this paper we present a deterministic and continuous model for predator – prey population based on Lotka – Volterra model and then developed the model by incorporating time delay and constant effort of harvesting. The time delay is considered in the model under assumption that the growth rate of the population does not depend on the current size of population but also on the past size. The stable equilibrium point is then related to the profit maximum function. We will analyze the possible influence of time delay on the stability of the model and find the critical value of the effort of harvesting.

Predator-Prey Population Model

We consider a predator – prey model based on Lotka – Volterra model with one predator and one prey populations. The model for the rate of change of prey population (x) and predator population (y) is

$$\begin{aligned} \frac{dx}{dt} &= rx\left(1-\frac{x}{K}\right)-\alpha xy \\ \frac{dy}{dt} &= -cy+\beta xy . \end{aligned} \tag{1}$$

The model includes parameter K, the carrying capacity, for the prey population in the absence of the predator. Parameter r is the intrinsic growth rate of the prey, c is the mortality rate if the predator without prey, α measures the rate of consumption of prey by the predator, β measures the conversion of prey consumed into the predator reproduction rate.

The equilibrium points of model (1) are $E_0=(0,0)$, $E_1=(K,0)$ and $E^*=(x^*,y^*)=\left(\frac{c}{\beta},\frac{r(K\beta-c)}{\alpha\beta K}\right)$. In order to get a positive equilibrium point we assume that $K\beta-c>0$. The Jacobian matrix of model (1) takes the form

$$J=\begin{pmatrix} r-\frac{2rx}{K}-\alpha y & -\alpha x \\ \beta y & -c+\beta x \end{pmatrix},$$

and at the equilibrium point E^*, we have

$$J=\begin{pmatrix} -\frac{rc}{\beta K} & -\frac{\alpha c}{\beta} \\ \frac{r\beta K-rc}{\alpha K} & 0 \end{pmatrix}.$$

The characteristic equation of the Jacobian matrix J at this point is $f(\lambda)=\lambda^2+\frac{rc}{\beta K}\lambda+\frac{c}{\beta K}(r\beta K-rc)$. Let $P=\frac{rc}{\beta K}$ and $Q=\frac{c}{\beta K}(r\beta K-rc)$, then the eigenvalues of the Jacobian matrix are $\lambda_{1,2}=\frac{-P\pm\sqrt{P^2-4Q}}{2}$.

Since P and Q are both positive, then both eigenvalues have negative real parts. It means that the equilibrium points E^* is locally asymptotically stable. Furthermore, since $K\beta - c > 0$ then the equilibrium point E^* is also globally asymptotically stable, see Toaha et al., (2006) and Ho and Ou (2002).

Example 1. Consider model (1) with parameters $r = 0.5$, $K = 500$, $\alpha = 0.0001$, $c = 0.0455$, and $\beta = 0.0001$. The equilibrium point of the model in the positive quadrant is $E^* = (455, 450)$. The eigenvalues associated with the equilibrium point are -0.45045 and -0.00455, which reveals that the equilibrium point is asymptotically stable.

Predator-Prey Model with Time Delay and Constant Effort of Harvesting

In this section we consider a time delay into the predator-prey model. Starting from Hutchinson's delay logistic model, May (1974) has proposed the following system

$$\begin{aligned} \frac{dx(t)}{dt} &= rx(t)\left(1 - \frac{x(t-\tau)}{K}\right) - \alpha x(t)y(t) \\ \frac{dy(t)}{dt} &= -cy(t) + \beta x(t)y(t), \end{aligned} \tag{2}$$

where r, K, τ, α, c, and β are positive constants. Model (2) contains a single discrete delay. The term $(1 - x(t-\tau)/K)$ in model (2) denotes a density dependent feedback mechanism which takes τ units of time to respond to changes in the population density. If we think the gestation period of prey is τ, then the per capita growth rate function should carry a time delay τ.

We consider the predator and prey population model (2) where the two populations are subjected to constant effort of harvesting. The model with harvesting is as follows

$$\begin{aligned} \frac{dx(t)}{dt} &= rx(t)\left(1 - \frac{x(t-\tau)}{K}\right) - \alpha x(t)y(t) - q_x E_x x(t) \\ \frac{dy(t)}{dt} &= -cy(t) + \beta x(t)y(t) - q_y E_y y(t). \end{aligned} \tag{3}$$

Here, q_x and q_y are the cathability coefficients of the prey and predator population respectively and E_x and E_y are the efforts of harvesting for the prey and predator population. For analysis, we set $q_x = q_y = 1$.
We consider that the predator and prey populations are harvested by the same gear of fishing, then it is reasonable to assume that the efforts of harvesting are the same, i.e, $E = E_x = E_y$. Then model (3) becomes

$$\begin{aligned} \frac{dx(t)}{dt} &= r_1 x(t)\left(1 - \frac{x(t-\tau)}{K_1}\right) - \alpha x(t)y(t) \\ \frac{dy(t)}{dt} &= -c_1 y(t) + \beta x(t)y(t), \end{aligned} \qquad (4)$$

where $r_1 = r - E$, $K_1 = \frac{(r-E)K}{r}$, and $c_1 = c + E$.

We assume that $r > E$. This assumption is made to guarantee the intrinsic growth of prey population is greater then the effort of harvesting. The equilibrium point of model (4) is similar with the equilibrium point of model (1), they are; $E_0 = (0, 0)$, $E_1 = (K_1, 0)$ and $E^* = (x^*, y^*) = \left(\frac{c_1}{\beta}, \frac{r_1(K_1\beta - c_1)}{\alpha\beta K_1}\right)$. In order to have a positive equilibrium point we assume that $K_1\beta - c_1 > 0$, that is; $0 < E < \frac{r(K\beta - c)}{(K\beta + r)}$. Under this assumption the equilibrium point E^* is in the first quadrant.

In order to understand the locally asymptotically stability of the equilibrium point E^* in the model with time delay, we analyze the associated linearization model with perturbation. Let $u(t) = x(t) - x^*$ and $v(t) = y(t) - y^*$. Substituting into model (4) then simplifying and neglecting the product terms to get the linearized model

$$\begin{aligned} \frac{du(t)}{dt} &= -\frac{r_1}{K_1} x^* u(t-\tau) - \alpha x^* v(t) \\ \frac{dv(t)}{dt} &= \beta y^* u(t). \end{aligned}$$

Analyzing the local stability of the equilibrium point E^* in the model with time delay is equivalent to analyzing the stability of zero equilibrium point in the linearized model. From the linearized model we have the characteristic equation

$$\lambda^2 + \lambda P_1 e^{-\lambda\tau} + Q_1 = 0, \tag{5}$$

where $P_1 = \frac{r_1}{K_1} x^*$ and $Q_1 = \alpha\beta x^* y^*$. For $\tau = 0$ the characteristic equation becomes

$$\lambda^2 + \lambda P_1 + Q_1 = 0, \tag{6}$$

which has the roots

$$\lambda = \frac{-P_1 \pm \sqrt{P_1^2 - 4Q_1}}{2}. \tag{7}$$

Since P_1 and Q_1 are both positive, the characteristic equation (6) has negative real roots.

Theorem 1. Let $\tau = 0$ and $0 < E < \frac{r(K\beta - c)}{(K\beta + r)}$. Then the equilibrium point E^* for model (4) is asymptotically stable.

Proof. From the condition $0 < E < \frac{r(K\beta - c)}{(K\beta + r)}$ we get the equilibrium point E^* is in the positive quadrant. Since $\tau = 0$, then the characteristic equation as stated in equation (6) has negative eigenvalues. We conclude that the equilibrium point E^* is asymptotically stable.

We know that the equilibrium point E^* of model (1) is globally asymptotically stable. Since the equilibrium point E^* of model (4) is similar with the equilibrium point E^* of model (1), then we conclude that the equilibrium point E^* of model (4) is also globally asymptotically stable. □

Now for $\tau \neq 0$, if $\lambda = i\omega$, $\omega > 0$, is a root of the characteristic equation (5), then we have

$$-\omega^2 + iP_1\omega e^{-i\omega\tau} + Q_1 = 0,$$

$$-\omega^2 + iP_1\omega\cos(\omega\tau) + P_1\omega\sin(\omega\tau) + Q_1 = 0.$$

Separating the real and imaginary parts, we have

$$\begin{aligned} -\omega^2 + P_1\omega\sin(\omega\tau) + Q_1 &= 0, \\ P_1\omega\cos(\omega\tau) &= 0. \end{aligned} \tag{8}$$

Squaring both sides gives

$$P_1^2\omega^2\sin^2(\omega\tau) = \omega^4 - 2Q_1\omega^2 + Q_1^2$$
$$P_1^2\omega^2\cos^2(\omega\tau) = 0\,.$$

Adding both equations and regrouping by powers of ω, we obtain the following fourth degree polynomial

$$\omega^4 - \left(P_1^2 + 2Q_1\right)\omega^2 + Q_1^2 = 0\,, \tag{9}$$

from which we obtain

$$\omega_\pm^2 = \frac{1}{2}\left\{\left(P_1^2 + 2Q_1\right) \pm \sqrt{P_1^4 + 4P_1^2 Q_1}\right\}. \tag{10}$$

From (10) we can see that there are two positive solutions of $\omega_\pm^2$. We can now find the values of $\tau_j^\pm$ by substituting $\omega_\pm^2$ into equations (8) and solving for τ. We obtain

$$\tau_k^+ = \frac{\pi}{2\omega_+} + \frac{2k\pi}{\omega_+}, \quad \tau_k^- = \frac{3\pi}{2\omega_-} + \frac{2k\pi}{\omega_-}, \qquad k = 0, 1, 2, \cdots. \tag{11}$$

Theorem 2. Let $K\beta - c > 0$, $0 < E < \frac{r(K\beta - c)}{(K\beta + r)}$ and $\tau_k^\pm$ be defined in equation (11). Then there exists a positive integer *m* such that there are *m* switches from stability to instability and to stability. In other words, when $\tau \in [0, \tau_0^+) \cup (\tau_0^-, \tau_1^+) \cup \cdots \cup (\tau_{m-1}^-, \tau_m^+)$, the equilibrium point E^* for model (4) is stable, and when $\tau \in (\tau_0^+, \tau_0^-) \cup (\tau_1^+, \tau_1^-) \cup \cdots \cup (\tau_{m-1}^+, \tau_{m-1}^-)$, the equilibrium point E^* is unstable. Therefore, there are Hopf bifurcations at E^* for $\tau = \tau_k^\pm$, $k = 0, 1, 2, \cdots$.

Proof. From (7) we know that the equilibrium point E^* is stable for $\tau = 0$. Then to prove the theorem we need only to verify the transversability conditions, see Cushing (1977),

$$\left.\frac{d(\mathrm{Re}\,\lambda)}{d\tau}\right|_{\tau=\tau_k^+} > 0 \text{ and } \left.\frac{d(\mathrm{Re}\,\lambda)}{d\tau}\right|_{\tau=\tau_k^-} < 0\,.$$

Differentiating the equation (5) with respect to τ we obtain

$$2\lambda\frac{d\lambda}{d\tau}+P_1e^{-\lambda\tau}\frac{d\lambda}{d\tau}+\lambda P_1e^{-\lambda\tau}\left(-\tau\frac{d\lambda}{d\tau}-\lambda\right)=0,$$

$$\left(2\lambda+(1-\lambda\tau)P_1e^{-\lambda\tau}\right)\frac{d\lambda}{d\tau}=\lambda^2P_1e^{-\lambda\tau}.$$

For convenience, we study $\left(\frac{d\lambda}{d\tau}\right)^{-1}$ instead of $\frac{d\lambda}{d\tau}$. Then we have

$$\left(\frac{d\lambda}{d\tau}\right)^{-1}=\frac{2\lambda e^{\lambda\tau}+P_1(1-\lambda\tau)}{\lambda^2P_1}=\frac{2\lambda e^{\lambda\tau}+P_1}{\lambda^2P_1}-\frac{\tau}{\lambda}.$$

From the characteristic equation (5) we know that $e^{\lambda\tau}=\frac{-\lambda P_1}{\lambda^2+Q_1}$.

Then we have $\left(\frac{d\lambda}{d\tau}\right)^{-1}=\frac{-\lambda^2+Q_1}{\lambda^2(\lambda^2+Q_1)}-\frac{\tau}{\lambda}$. Therefore

$$\begin{aligned}\operatorname{sign}\left(\frac{d(\operatorname{Re}\lambda)}{d\tau}\right)_{\lambda=i\omega}&=\operatorname{sign}\left(\operatorname{Re}\left(\frac{d\lambda}{d\tau}\right)^{-1}\right)_{\lambda=i\omega}\\
&=\operatorname{sign}\left(\operatorname{Re}\left[\frac{-1}{\lambda^2+Q_1}\right]_{\lambda=i\omega}+\operatorname{Re}\left[\frac{Q_1}{\lambda^4+\lambda^2Q_1}\right]_{\lambda=i\omega}\right)\\
&=\operatorname{sign}\left(\frac{-1}{-\omega^2+Q_1}+\frac{Q_1}{\omega^4-\omega^2Q_1}\right)=\operatorname{sign}\left(\frac{\omega^4-Q_1{}^2}{\omega^2\left(\omega^2-Q_1\right)^2}\right)\\
&=\operatorname{sign}\left(\omega^4-Q_1{}^2\right).\end{aligned}$$

From equation (9) we know that $\omega^4-Q_1{}^2=2\omega^4-\left(P_1{}^2+2Q_1\right)\omega^2$. Then we have

$$\begin{aligned}\operatorname{sign}\left(\frac{d(\operatorname{Re}\lambda)}{d\tau}\right)_{\lambda=i\omega}&=\operatorname{sign}\left(2\omega^4-(P_1{}^2+2Q_1)\omega^2\right)\\
&=\operatorname{sign}\left(2\omega^2-(P_1{}^2+2Q_1)\right).\end{aligned}$$

By substituting the expression for $\omega_{\pm}^2$, it is easy to see that the sign is positive for ω_{+}^2 and the sign is negative for ω_{-}^2. Therefore, crossing from left to right with increasing τ occurs for values of τ corresponding to ω_+ and crossing from right to left occurs for values of τ corresponding to ω_-. From (10) and the last result, we can verify that the transversality conditions are satisfied. Therefore $\tau_k^{\pm}$ are bifurcation values. □

Example 2. Consider model (4) with parameters $r = 1.1$, $K = 110$, $\alpha = 0.2$, $c = 0.9$, $\beta = 0.1$, and $E = 0.1$. The equilibrium point of the model in the positive quadrant is $(10, 4.5)$. For $\tau = 0$, the Jacobian matrix of the model associated with the equilibrium point has eigenvalues negative real parts, i.e., $-0.05000 \pm 0.94736\, i$. This means that the equilibrium point of the model without time delay is stable. Further, we have $K_1\beta - c_1 = 9.0$, $\omega_+ = 1.0$, and $\tau_0^+ = 1.57080$. By Theorem 2, a Hopf bifurcation occurs when $\tau = 1.57080$. Following Theorem 2 we have $\tau_0^+ = 1.57080$, $\tau_0^- = 5.23599$, $\tau_1^+ = 7.85398$, $\tau_1^- = 12.21730$, $\tau_2^+ = 14.13717$, $\tau_2^- = 19.19862$, $\tau_3^+ = 20.42035$, $\tau_3^- = 26.17994$, $\tau_4^+ = 26.70354$, $\tau_4^- = 33.16126$, $\tau_5^+ = 32.98672$, and $\tau_5^- = 40.14257$. Then we have 4 stability switches and for $\tau > \tau_4^+$, the solution remains unstable.

Theorem 3. Given $\tau > 0$. The equilibrium point E^* for model (4) is stable if the effort of harvesting satisfies $E \in E_{\otimes} \cap E_{\oplus}$, where $E_{\otimes} = \left(0, \dfrac{r(K\beta - c)}{(K\beta + r)}\right)$ and $E_{\oplus}$ is a set of the solution of the inequality

$$f(E) = \frac{\pi^4}{4\tau^4} - \frac{\pi^2}{\tau^2} P_1^2 - \frac{2\pi^2}{\tau^2} Q_1 + 4Q_1^2 > 0.$$

Proof. In order the equilibrium point E^* exists in the positive quadrant we assume that $0 < E < \dfrac{r(K\beta - c)}{(K\beta + r)}$. From Theorem 2, we know that the equilibrium point E^* is stable when $\tau \in [0, \tau_0^+)$. From equation (10) we have $\omega_+ = \left[\dfrac{1}{2}\left\{\left(P_1^2 + 2Q_1\right) + \sqrt{P_1^4 + 4P_1^2 Q_1}\right\}\right]^{1/2}$. Since the time delay τ is given then we have to set a condition such that $\tau < \dfrac{\pi}{2\omega_+}$ or $\omega_+ < \dfrac{\pi}{2\tau}$. Then we have

$$\left[\frac{1}{2}\left\{\left(P_1^2+2Q_1\right)+\sqrt{P_1^4+4P_1^2Q_1}\right\}\right]^{1/2}<\frac{\pi}{2\tau},$$

$$\frac{1}{2}\left\{\left(P_1^2+2Q_1\right)+\sqrt{P_1^4+4P_1^2Q_1}\right\}<\frac{\pi^2}{4\tau^2},$$

$$\left\{\left(P_1^2+2Q_1\right)+\sqrt{P_1^4+4P_1^2Q_1}\right\}<\frac{\pi^2}{2\tau^2},\ \sqrt{P_1^4+4P_1^2Q_1}<\frac{\pi^2}{2\tau^2}-\left(P_1^2+2Q_1\right),$$

$$P_1^4+4P_1^2Q_1<\left(\frac{\pi^2}{2\tau^2}-\left(P_1^2+2Q_1\right)\right)^2,$$

$$P_1^4+4P_1^2Q_1<\frac{\pi^4}{4\tau^4}-\frac{\pi^2}{\tau^2}\left(P_1^2+2Q_1\right)+\left(P_1^2+2Q_1\right)^2.$$

After simplifying we get

$$\frac{\pi^4}{4\tau^4}-\frac{\pi^2}{\tau^2}P_1^2-\frac{2\pi^2}{\tau^2}Q_1+4Q_1^2>0.$$

Since both P_1 and Q_1 depend on E then the last inequality can be solved in terms of E. Further we conclude that the equilibrium point E^* is stable when the effort satisfies $E\in E_{\otimes}\cap E_{\oplus}$. □

Example 3. Consider model (4) with parameters $r=1$, $K=1000$, $\alpha=0.2$, $c=1$, and $\beta=0.1$. Take $\tau=1.8$. Then we have the equilibrium point of the model becomes $E^*=\left(10+10E,\ 4.96000-5.05000E\right)$, $E_{\otimes}=\left(0,\ 0.98020\right)$ and $f(E)=4.08040E^4+0.14544E^3-1.86110E^2-0.03379E+0.21214$. From this equation we have the positive solution, $E_{\oplus}=\left(-0.48196,\ 0.45550\right)\cup\left(0.48207,\ \infty\right)$. Further, the equilibrium point E^* is stable for the time delay $\tau=1.8$ when the effort of harvesting $E\in\left(0,\ 0.45550\right)\cup\left(0.48207,\ 0.98020\right)$.

Analysis of Profit Maximum

Now we relate the stable equilibrium point $E^*=\left(x^*,\ y^*\right)=\left(\frac{c_1}{\beta},\ \frac{r_1(K_1\beta-c_1)}{\alpha\beta K_1}\right)$ to the maximum profit problem. We define total cost, $TC=c_f+c_vE$ and total revenue, $TR=p_xx^*E+p_yy^*E$. Hence we have the profit function

$\pi = p_x x^* E + p_y y^* E - c_f - c_v E$. Substituting $x^* = \frac{c_1}{\beta}$ and $y^* = \frac{r_1(K_1\beta - c_1)}{\alpha\beta K_1}$ to obtain $\pi = \frac{-A_1}{\alpha\beta K}E^2 + \frac{B_1}{\alpha\beta K}E - c_f$ which has the critical point $E_c = \frac{B_1}{2A_1}$, where $A_1 = p_y K\beta + p_y r - \alpha p_x K$ which is assumed to be positive and $B_1 = \alpha p_x Kc - \alpha c_v K\beta - p_y cr + p_y K\beta r$. Under this assumption, the critical point $E_c = \frac{B_1}{2A_1}$ maximizes the profit function.

From the assumption $0 < E < \frac{r(K\beta - c)}{(K\beta + r)}$, then we need a condition so that the critical point $E_c \in \left(0, \frac{r(K\beta - c)}{(K\beta + r)}\right)$, i.e., $0 < \frac{B_1}{2A_1} < \frac{r(K\beta - c)}{(K\beta + r)}$. In the case of critical point $E_c > \frac{r(K\beta - c)}{(K\beta + r)}$, the profit function becomes maximum at $E = \frac{r(K\beta - c)}{(K\beta + r)}$. However, this situation leads to the extinction of the population *y*, since $y^* = 0$.

Example 4. Consider model (4) with parameters $r = 2$, $K = 1000$, $\alpha = 0.1$, $c = 1$, and $\beta = 0.15$. The equilibrium point of the model is $E^* = (x^*, y^*)$, where $x^* = 6.66667 + 6.66667E$ and $y^* = 19.86667 - 10.13333E$. Take $p_x = 0.5$, $p_y = 1.0$, $c_f = 2.0$, and $c_v = 1.5$. The profit function becomes $\pi = \pi(E) = -6.80000E^2 + 21.70000E - 2$ which has the critical point $E_c = 1.59559$. We can verify that the critical point $E_c < \frac{r(K\beta - c)}{K\beta + c} = 1.96053$. Then the critical point gives profit maximum, i.e., $\pi_{\max} = 15.31213$. By substituting $E_c = 1.59559$ we have the equilibrium point $E^* = (x^*, y^*) = (17.30392, 3.69804)$ and this equilibrium point is stable.

CONCLUSION

In the model without time delay and harvesting, the positive equilibrium occurs when $K\beta - c > 0$ and it is globally asymptotically stable. This means that the predator and prey populations can live in coexistence. For the model with time delay and harvesting, there exist time delay intervals of stability for the equilibrium point and also exists a critical value of the effort that maximizes the profit function. This means that under suitable values of the parameters, time delay and effort of harvesting, the predator and prey populations can remain in existence and give profit maximum.

ACKNOWLEDGEMENT

This research is supported by Universiti Putra Malaysia under the grant IRPA with project code 5422600.

REFERENCES

Cushing, J.M. 1977. *Integrodifferential equations and delay models in population dynamics*. Heidelberg: Springer-Verlag.

Danca, M., Codreanu, S. and Bako, B. 1997. Detailed analysis of a nonlinear prey-predator model. *Journal of Biological Physics* **23**:11-20.

Ho, C.P. and Ou, Y.L. 2002. Influence of time delay on local stability for a predator-prey system. *Journal of Tunghai Science* **4**:47-62.

Hogarth, W.L., Norbury, J., Cunning, I. and Sommers, K. 1992. Stability of a predator-prey model with harvesting. *Ecological Modelling* **62**:83-106.

Holmberg, J. 1995. Socio-ecological principles and indicators for sustainability, PhD Thesis, Goteborg University, Sweden.

Kar, T.K. and Chaudhuri, K.S. 2004. Harvesting in a two-prey one predator fishery: A bioeconomic model. *J. ANZIAM* **45**:443-456.

Luckinbill, L.S. 1973. Coexistence in laboratory populations of paramecium aurelia and its predator didinium nasutum. *Journal of Ecology* **54**(6):1320-1327.

Matsuda, H. and Abrams, P.A. 2004. Effects of predators-prey interaction and adaptive change on sustainable yield. *Can. J. Fish. Aquat. Sci./J. Can. Sci. Halieut. Aquat.* **61**(2):175-184.

May, R.M. 1974. *Stability and complexity of model ecosystems*. Princeton, New Jersey: Princeton University Press.

Song, X.Y. and Chen, L.S. 2002. Optimal harvesting and stability for a predator-prey system with stage structure. *Acta Mathematicae Applicatae Sinica* **18**(3):423-430.

Srinivasu, P.D., Ismail, S. and Naidu, C.R. 2001. Global dynamics and controllability of a harvested prey-predator system. *J. Biological Systems* **9**(1):67-79.

Toaha, S., Hassan, M.A., Ismail, F. and June, L.W. 2006. Stability analysis of prey-predator population model with harvesting, submitted to *J. IRSIAM.*