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Spatial Modelling of Peak Frequencies of Brain Signals

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ABSTRACT

Spatial modelling of various phenomena has been undertaken in many diversified fields. In this project, we concentrate on the modelling of the peak frequencies of brain signals and the objective is to fit and illustrate spatial regression with Simultaneous Autoregressive *(SAR)* covariance structure. We found that the peak frequencies can be modelled appropriately as, $Y_i = \beta_{00} + \beta_{11}x_1x_2 + \beta_{21}x_1^2x_2 + \varepsilon_i$, with a simultaneous autoregressive correlation structure.

INTRODUCTION

Spatial modelling of various phenomena has been undertaken in many diversified fields. For instance, spatial modelling of rainfall (Smith, 1994), spatial regression of relative humidity (Mahendran Shitan, 2004), trend surface analysis for agricultural land value data 1977-8 in Iowa (Cliff and Ord, 1981) and forest landscape patterns (Jin-Ping, Guo and Yang, Xiao, 1999), etc. Whenever we deal with spatial data, it is vital to be thoughtful of spatial correlation amongst the neighbouring sites and this feature has to be taken into consideration in the modelling process.

In this project, we concentrate on the modelling of the peak frequencies of brain signals and the objective is to fit and illustrate spatial regression with Simultaneous Autoregressive *(SAR)* covariance structure. In section 2 regression with simultaneous autoregressive errors is briefly described and the methodology is in section 3. The results are presented in section 4 and finally the conclusions are drawn in section 5.

REGRESSION WITH SIMULTANEOUS AUTOREGRESSIVE *(SAR)* ERRORS

In this study we fit regression model with Simultaneous Autoregressive *(SAR)* covariance structure and hence we briefly discuss the *SAR* model. The *SAR* model was first proposed by Whittle in 1954 where a given set of observations observed on a lattice are modelled as functions of the neighbouring sites. That is given a set of observations, say $\{X_i\}$, the obsevations are modelled as follows,

$$X_i = \sum_{\substack{j=1 \\ j \neq i}}^{n} g_{ij} X_j + \{\varepsilon_i\}, \quad i = 1, 2, ..., n, \tag{1}$$

where $\{g_{ij}\}$ is a sequence of constants, $\{\varepsilon_i\}$ is a sequence uncorrelated errors with $E(\varepsilon_i) = 0$ and $Var(\varepsilon_i) = \sigma^2$. A detailed account of the *SAR* model can be found in Cliff and Ord, 1981.

A class of models that incorporates correlation reflecting the spatial structure is of the form, $Y_i = \mu_i + \varepsilon_i$, where Y_i is the random variable at site i, μ_i is the mean at site i which is modelled in terms of the covariates and ε_i is the random error terms. Further, we could allow ε_i to be a function of the neighbouring sites as,

$$\varepsilon_i = \sum_{\substack{j=1 \\ j \neq i}}^{n} g_{ij} \varepsilon_j + \delta_i, \quad i = 1, 2, \ldots, n, \tag{2}$$

where $\{g_{ij}\}$ is a sequence of constants, $\{\delta_i\}$ is a sequence uncorrelated errors with $E(\delta_i) = 0$ and $Var(\delta_i) = \sigma^2$. This is what we call as Regression with Simultaneous Autoregressive *(SAR)* Errors.

This model can be written in matrix forms as, $\boldsymbol{\varepsilon} = \mathbf{G}\boldsymbol{\varepsilon} + \boldsymbol{\delta}$, where the vector $\boldsymbol{\varepsilon}^T = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)$, vector $\boldsymbol{\delta}^T = (\delta_1, \delta_2, \ldots, \delta_n)$,

$\boldsymbol{\varepsilon} \sim MVN(\mathbf{0}, \mathbf{\Sigma})$, $\boldsymbol{\delta} \sim MVN(\mathbf{0}, \sigma^2\mathbf{I})$ and the matrix $\mathbf{G}$ is given as follows,

$$\mathbf{G} = \begin{bmatrix} 0 & g_{12} & g_{13} & \cdots & g_{1n} \\ g_{21} & 0 & g_{23} & \cdots & g_{2n} \\ g_{31} & g_{32} & 0 & \cdots & g_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ g_{n1} & g_{n2} & \cdots & \cdots & 0 \end{bmatrix}.$$

Since g_{ij} are constants that need estimation and there are too many of them to be estimated, some simplification can be made by allowing $\mathbf{G} = \rho\mathbf{W}$, where ρ is an unknown constant that can be estimated for a given data set and

$$\mathbf{W} = \begin{bmatrix} 0 & w_{12} & w_{13} & \cdots & w_{1n} \\ w_{21} & 0 & w_{23} & \cdots & w_{2n} \\ w_{31} & w_{32} & 0 & \cdots & w_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & \cdots & \cdots & 0 \end{bmatrix},$$

is a matrix of known weights. The covariance matrix, $\mathbf{\Sigma}$ would then be given as $\sigma^2(\mathbf{I} - \rho\mathbf{W})^{-1}(\mathbf{I} - \rho\mathbf{W}^T)^{-1}$ for the *SAR* model.

An application of regression model with *SAR* covariance structure has been applied to the Sudden Infant Death Syndrome (SIDS) data set for North Carolina Counties (see Kaluzny, *et. al.,* 1998).

METHODOLOGY

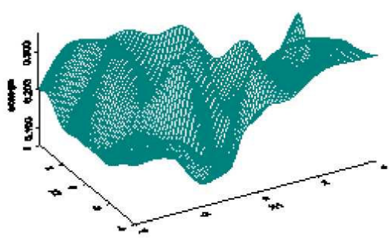
In this section the data set used in this study and the model fitting are described.

Data Set

The primary data set consisted of the event-related optical (EROS) signals observed over time at 81 spatial locations (9×9 grid) over the cortical surface of the brain. The center of the brain surface is referenced by the co-ordinate (0, 0). The *x* and *y* axes each stretch from −4 to 4. Each time series had a length of *n* = 125.

A periodogram of the time series at each spatial location was then obtained (see Brockwell and Davis, 2002 for details). Thereafter, we produced a smoothed periodogram at each spatial location over the cortical surface of the brain. The frequency at which the peak of the smoothed periodogram occurred was noted at each spatial location.

Figure 1 shows three dimensional plots of the peak frequency values over the cortical surface of the brain. The plot clearly suggest fitting a regression surface and in this paper we model the peak frequencies as functions of the location co-ordinates together with a spatially correlated error structure.



<u>Rotation</u>
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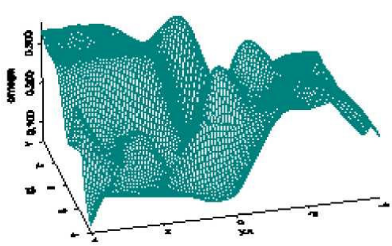
Angle to x axis : 240
Distance : 25



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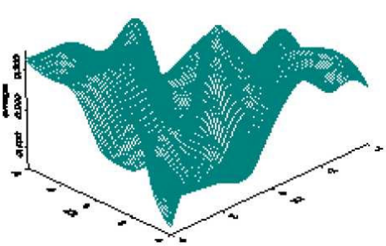

<u>Rotation</u>
Angle to a axis : 65
Angle to x axis : 45
Distance : 25

Figure 1: 3D plot over the cortical surface

Model Fitting

To apply the method described in section 2 and to obtain the weights, the researcher first needs to ascertain or define which are the neighbouring sites and then work out the weights. For this study the neighbours for a given spatial location has been defined as all spatial locations located within a unit in scale from the point of interest. The weights, $w_{ij} = 1$ if point *i* and *j* are neighbours and $w_{ij} = 0$, otherwise. The neighbours of the eighty one spatial locations considered in this study are listed out in Table 1.

Various models of increasing complexity as discussed in the results section (see Section 3), were fitted to the data set and the modelling process was done using S-plus Spatial Statistics Module (Kaluzny, *et. al.,* 1998).

To evaluate between competing models, the test statistic (Cressie, 1993) used in this study is,

$$U^2 = 2\left(\frac{n-p-r}{n}\right)\left(L_p - L_{p+r}\right) \sim \chi^2(r), \tag{3}$$

where n is the number of data points, p is the number of parameters estimated, r is the additional number of parameters estimated, L_p is the negative log likelihood for the smaller model and L_{p+r} is the negative log likelihood for the larger model. The log likelihood function for the *SAR* model is given by

$$-\frac{n}{2}\log(2\pi) - \frac{n}{2}\log(\sigma^2) + \log\left|\mathbf{I} - \rho\mathbf{W}\right| - \frac{1}{2\sigma^2}\boldsymbol{\varepsilon}^T(\mathbf{I} - \rho\mathbf{W}^T)(\mathbf{I} - \rho\mathbf{W})\boldsymbol{\varepsilon}. \tag{4}$$

To determine whether any of the coefficients of the covariates were significant or not, we used the Likelihood Ratio Test given as – $-2\log\lambda \sim \chi^2(k)$ (see Maddala, 1989), where

$$\lambda = \frac{\text{Maximum of Likelihood under restrictions}}{\text{Maximum of Likelihood without restrictions}} \tag{5}$$

TABLE 1: Peak frequencies and the neighbours of 81 spatial locations on the brain

Row	Spatial Location	X_1 (*x*-axis)	Y_2 (*y*-axis)	Peak frequencies	Neighbours
1	(-4, -4)	-4	-4	0.20	2, 10
2	(-4, -3)	-4	-3	0.18	1,3,11
3	(-4,-2)	-4	-2	0.15	2,4,12
4	(-4,-1)	-4	-1	0.15	3,5,13
5	(-4, 0)	-4	0	0.15	4,6,14
6	(-4, 1)	-4	1	0.15	5,7,15
7	(-4, 2)	-4	2	0.15	6,8,16
8	(-4, 3)	-4	3	0.20	7,9,17
9	(-4, 4)	-4	4	0.20	8,18
10	(-3, -4)	-3	-4	0.20	1,11,19
11	(-3, -3)	-3	-3	0.20	2,10,12,20
12	(-3, -2)	-3	-2	0.20	3,11,13,21
13	(-3, -1)	-3	-1	0.19	4,12,14,22
14	(-3, 0)	-3	0	0.25	5,13,15,23
15	(-3, 1)	-3	1	0.31	6,14,16,24
16	(-3, 2)	-3	2	0.25	7,15,17,25
17	(-3, 3)	-3	3	0.30	8,16,18,26,
18	(-3, 4)	-3	4	0.30	9,17,27
19	(-2, -4)	-2	-4	0.16	10,20,28
20	(-2, -3)	-2	-3	0.19	11,19,21,29
21	(-2, -2)	-2	-2	0.14	12,20,22,30
22	(-2, -1)	-2	-1	0.27	13,21,23,31
23	(-2, 0)	-2	0	0.25	14,22,24,32
24	(-2, 1)	-2	1	0.30	15,23,25,33
25	(-2, 2)	-2	2	0.32	16,24,26,34
26	(-2, 3)	-2	3	0.31	17,25,27,35
27	(-2, 4)	-2	4	0.29	18,26,36
28	(-1, -4)	-1	-4	0.16	19,29,37
29	(-1, -3)	-1	-3	0.12	20,28,30,38
30	(-1, -2)	-1	-2	0.06	21,29,31,39
31	(-1, -1)	-1	-1	0.30	22,30,32,40
32	(-1, 0)	-1	0	0.27	23,31,33,41
33	(-1, 1)	-1	1	0.22	34,32,34,42
34	(-1, 2)	-1	2	0.16	25,33,35,43
35	(-1, 3)	-1	3	0.20	26,34,36,44
36	(-1, 4)	-1	4	0.30	27,35,45
37	(0, -4)	0	-4	0.26	28,38,46
38	(0, -3)	0	-3	0.32	29,37,39,47
39	(0, -2)	0	-2	0.08	30,38,40,48
40	(0, -1)	0	-1	0.06	31,39,41,49
41	(0, 0)	0	0	0.24	32,40,42,50
42	(0, 1)	0	1	0.29	33,41,43,51
43	(0, 2)	0	2	0.05	34,42,44,52
44	(0, 3)	0	3	0.09	35,43,45,53
45	(0, 4)	0	4	0.27	36,44,54
46	(1, -4)	1	-4	0.27	37,47,55
47	(1, -3)	1	-3	0.31	38,46,48,56
48	(1, -2)	1	-2	0.29	39,74,79,57
49	(1, -1)	1	-1	0.21	40,48,50,58

TABLE 1(continued): Peak frequencies and the neighbours of 81 spatial locations on the brain

Row	Spatial Location	X_1 (*x*-axis)	Y_2 (*y*-axis)	Peak frequencies	Neighbours
50	(1, 0)	1	0	0.11	41,49,51,59
51	(1, 1)	1	1	0.07	42,50,52,60
52	(1, 2)	1	2	0.09	43,51,53,61
53	(1, 3)	1	3	0.08	44,52,54,62
54	(1, 4)	1	4	0.27	45,53,63
55	(2, -4)	2	-4	0.32	46,56,64
56	(2, -3)	2	-3	0.29	47,55,57,65
57	(2, -2)	2	-2	0.30	48,56,58,66
58	(2, -1)	2	-1	0.30	49,57,59,67
59	(2, 0)	2	0	0.13	50,58,60,68
60	(2, 1)	2	1	0.19	51,59,61,69
61	(2, 2)	2	2	0.10	52,60,62,70
62	(2, 3)	2	3	0.09	53,61,63,71
63	(2, 4)	2	4	0.17	54,62,72
64	(3, -4)	3	-4	0.33	55,65,73
65	(3, -3)	3	-3	0.35	56,64,66,74
66	(3, -2)	3	-2	0.32	57,65,67,75
67	(3, -1)	3	-1	0.32	58,66,68,76
68	(3, 0)	3	0	0.20	59,67,69,77
69	(3, 1)	3	1	0.13	60,68,70,78
70	(3, 2)	3	2	0.16	61,69,71,79
71	(3, 3)	3	3	0.10	62,70,72,80
72	(3, 4)	3	4	0.20	63,71,81
73	(4, -4)	4	-4	0.33	64,74
74	(4, -3)	4	-3	0.31	65,73,75
75	(4, -2)	4	-2	0.31	66,74,76
76	(4, -1)	4	-1	0.29	67,75,77
77	(4, 0)	4	0	0.20	68,76,78
78	(4, 1)	4	1	0.19	69,77,79
79	(4, 2)	4	2	0.30	70,78,80
80	(4, 3)	4	3	0.15	71,79,81
81	(4, 4)	4	4	0.07	72,80

RESULTS

In this section the results of our study are presented.

We have seen that Figure 1 clearly suggests fitting a regression surface. However data values of the neighbouring points are likely to be correlated. As such tests for spatial correlation were conducted using the Moran and Geary Statistic. The Moran spatial correlation was found to be 0.5229 with a standard error of 0.0819. The computed z statistic value was 6.534 and had a p-value of 6.402×10^{-11}. The Geary spatial correlation value was 0.4997 with a standard error of 0.0827. The computed z statistic value was -6.047 and a p-value of

$1.477 \times 10^{-9.}$ Both these tests indicate that the observations were significantly spatially correlated due to the extremely small *p*-value thereby rejecting the null hypothesis of no spatial correlation. Hence, this prompted us to fit various models of increasing complexity with spatially correlated error structure.

Let Y_i represent the peak frequency value recorded at point i, x_1 be the coordinate of *x*-axis and x_2 be the coordinate of the *y*-axis over the cortical surface of the brain.

The eighteen (18) models considered in this study were,

$$Y_i = \beta_{00} + \varepsilon_i, \qquad \text{(Model 1)}$$

$$Y_i = \beta_{00} + \beta_{30}x_1^3 + \varepsilon_i, \qquad \text{(Model 2)}$$

$$Y_i = \beta_{00} + \beta_{10}x_1 + \beta_{20}x_1^2 + \varepsilon_i, \qquad \text{(Model 3)}$$

$$Y_i = \beta_{00} + \beta_{30}x_1^3 + \beta_{12}x_1x_2^2 + \varepsilon_i, \qquad \text{(Model 4)}$$

$$Y_i = \beta_{00} + \beta_{11}x_1x_2 + \beta_{30}x_1^3 + \varepsilon_i, \qquad \text{(Model 5)}$$

$$Y_i = \beta_{00} + \beta_{11}x_1x_2 + \beta_{21}x_1^2x_2 + \varepsilon_i, \qquad \text{(Model 6)}$$

$$Y_i = \beta_{00} + \beta_{10}x_1 + \beta_{11}x_1x_2 + \beta_{20}x_1^2 + \varepsilon_i, \qquad \text{(Model 7)}$$

$$Y_i = \beta_{00} + \beta_{20}x_1^2 + \beta_{30}x_1^3 + \beta_{12}x_1x_2^2 + \varepsilon_i, \qquad \text{(Model 8)}$$

$$Y_i = \beta_{00} + \beta_{11}x_1x_2 + \beta_{20}x_1^2 + \beta_{30}x_1^3 + \varepsilon_i, \qquad \text{(Model 9)}$$

$$Y_i = \beta_{00} + \beta_{11}x_1x_2 + \beta_{12}x_1x_2^2 + \beta_{30}x_1^3 + \varepsilon_i, \qquad \text{(Model 10)}$$

$$Y_i = \beta_{00} + \beta_{11}x_1x_2 + \beta_{21}x_1^2x_2 + \beta_{30}x_1^3 + \varepsilon_i, \qquad \text{(Model 11)}$$

$$Y_i = \beta_{00} + \beta_{10}x_1 + \beta_{11}x_1x_2 + \beta_{21}x_1^2x_2 + \varepsilon_i, \qquad \text{(Model 12)}$$

$$Y_i = \beta_{00} + \beta_{01}x_2 + \beta_{11}x_1x_2 + \beta_{21}x_1^2x_2 + \varepsilon_i, \qquad \text{(Model 13)}$$

$$Y_i = \beta_{00} + \beta_{10}x_1 + \beta_{01}x_2 + \beta_{11}x_1x_2 + \beta_{20}x_1^2 + \varepsilon_i, \qquad \text{(Model 14)}$$

$$Y_i = \beta_{00} + \beta_{10}x_1 + \beta_{01}x_2 + \beta_{11}x_1x_2 + \beta_{21}x_1^2x_2 + \varepsilon_i, \qquad \text{(Model 15)}$$

$$Y_i = \beta_{00} + \beta_{10}x_1 + \beta_{11}x_1x_2 + \beta_{21}x_1^2x_2 + \beta_{30}x_1^3 + \varepsilon_i, \qquad \text{(Model 16)}$$

$$Y_i = \beta_{00} + \beta_{11}x_1x_2 + \beta_{12}x_1x_2^2 + \beta_{21}x_1^2x_2 + \beta_{30}x_1^3 + \varepsilon_i, \qquad \text{(Model 17)}$$

$$Y_i = \beta_{00} + \beta_{10}x_1 + \beta_{01}x_2 + \beta_{11}x_1x_2 + \beta_{20}x_1^2 + \beta_{30}x_1^3 + \varepsilon_i, \qquad \text{(Model 18)}$$

The parameter estimates of our fitted models are contained in Table 2. Using equation (3), the test statistic U^2 were computed for the various models considered in this study and are also tabulated in Table 2 together with the p values.

From Table 2, we notice that the estimated parameter coefficients take on a wide variety of values both positive and negative. For every model considered in this study, the estimate for σ^2 was found to be 0.004.

The estimated value for ρ is in the range of 0.156 to 0.209. The log likelihood remains in the vicinity of 43.21 to 48.99 and U^2 does not exceed 10.561.

However, the most crucial thing that needs to be observed in Table 2 is the p value, which ranges from 0.019 to 0.373. The p value indicates whether or not a particular model differs from the null model (Model 1) significantly. Clearly then a smaller p value would assist us in the selection of a model. Of all the models considered in this study, two models namely model 6 and 11 had the smallest p values of 0.019 and hence they were significant at the 0.05 level. However since model 6 is the simpler model of the two models, it would seem reasonable to choose model 6 over model 11. The significance of the coefficients of the co-variates were established by the Likelihood Ratio Test which gave the value, $x^2 = 8.445$ with 2 degree of freedom and p value of 0.015.
To test for the significance of ρ the Likelihood Ratio Test gave a value, $x^2 = 18.318$ with 1 degree of freedom and p value of 1.870×10^{-5}. This is very highly significant at the 0.001 level.

TABLE 2: Results of fitted models

	Estimated parameter coefficients	$\hat{\sigma}^2$	$\hat{\rho}^2$	Log Likelihood	U^2	p- value
Model 1	$\hat{\beta}_{00} = 0.217$	0.004	0.201	43.21	-	-
Model 2	$\hat{\beta}_{00} = 0.217$ $\hat{\beta}_{30} = 0.001$	0.004	0.200	44.17	1.825	0.177
Model 3	$\hat{\beta}_{00} = 0.245$ $\hat{\beta}_{10} = 0.007$ $\hat{\beta}_{20} = -0.003$	0.004	0.209	44.26	1.970	0.373
Model 4	$\hat{\beta}_{00} = 0.217$ $\hat{\beta}_{30} = 0.001$ $\hat{\beta}_{12} = -0.001$	0.004	0.200	44.49	2.402	0.301
Model 5	$\hat{\beta}_{00} = 0.216$ $\hat{\beta}_{11} = -0.005$ $\hat{\beta}_{30} = 0.001$	0.004	0.170	46.93	6.981	0.030
Model 6	$\hat{\beta}_{00} = 0.216$ $\hat{\beta}_{11} = -0.005$ $\hat{\beta}_{21} = -0.001$	0.004	0.166	47.43	7.919	0.019
Model 7	$\hat{\beta}_{00} = 0.230$ $\hat{\beta}_{10} = 0.006$ $\hat{\beta}_{11} = -0.004$ $\hat{\beta}_{20} = -0.001$	0.004	0.181	46.61	6.296	0.098
Model 8	$\hat{\beta}_{00} = 0.243$ $\hat{\beta}_{20} = -0.002$ $\hat{\beta}_{30} = 0.001$ $\hat{\beta}_{12} = -0.001$	0.004	0.206	44.92	3.167	0.367
Model 9	$\hat{\beta}_{00} = 0.228$ $\hat{\beta}_{11} = -0.005$ $\hat{\beta}_{20} = -0.001$ $\hat{\beta}_{30} = 0.001$	0.004	0.176	47.12	7.241	0.065
Model 10	$\hat{\beta}_{00} = 0.216$ $\hat{\beta}_{11} = -0.005$ $\hat{\beta}_{12} = -0.001$ $\hat{\beta}_{30} = 0.001$	0.004	0.170	47.25	7.481	0.058
Model 11	$\hat{\beta}_{00} = 0.216$ $\hat{\beta}_{11} = -0.005$ $\hat{\beta}_{21} = -0.001$ $\hat{\beta}_{30} = 0.001$	0.004	0.163	48.61	10.000	0.019

TABLE 2: Results of fitted models (continued)

	Estimated parameter coefficients	$\hat{\sigma}^2$	$\hat{\rho}^2$	Log Likelihood	U^2	p- value
Model 12	$\hat{\beta}_{00} = 0.216$ $\hat{\beta}_{10} = 0.006$ $\hat{\beta}_{11} = -0.005$ $\hat{\beta}_{21} = -0.001$	0.004	0.168	48.02	8.907	0.031
Model 13	$\hat{\beta}_{00} = 0.216$ $\hat{\beta}_{01} = 0.003$ $\hat{\beta}_{11} = -0.005$ $\hat{\beta}_{21} = -0.001$	0.004	0.170	47.50	7.944	0.047
Model 14	$\hat{\beta}_{00} = 0.227$ $\hat{\beta}_{10} = 0.006$ $\hat{\beta}_{01} = -0.006$ $\hat{\beta}_{11} = -0.005$ $\hat{\beta}_{20} = -0.001$	0.004	0.172	47.13	7.162	0.128
Model 15	$\hat{\beta}_{00} = 0.216$ $\hat{\beta}_{10} = 0.006$ $\hat{\beta}_{01} = 0.003$ $\hat{\beta}_{11} = -0.005$ $\hat{\beta}_{21} = -0.001$	0.004	0.172	48.10	8.935	0.063
Model 16	$\hat{\beta}_{00} = 0.215$ $\hat{\beta}_{10} = -0.011$ $\hat{\beta}_{11} = -0.005$ $\hat{\beta}_{21} = -0.001$ $\hat{\beta}_{30} = 0.001$	0.004	0.156	48.99	10.561	0.032
Model 17	$\hat{\beta}_{00} = 0.216$ $\hat{\beta}_{11} = -0.005$ $\hat{\beta}_{12} = -0.001$ $\hat{\beta}_{21} = -0.001$ $\hat{\beta}_{30} = 0.001$	0.004	0.163	48.94	10.470	0.033
Model 18	$\hat{\beta}_{00} = 0.223$ $\hat{\beta}_{10} = -0.011$ $\hat{\beta}_{01} = -0.006$ $\hat{\beta}_{11} = -0.005$ $\hat{\beta}_{20} = -0.001$ $\hat{\beta}_{30} = 0.001$	0.004	0.158	48.04	8.706	0.121

Some diagnostics plots were also obtained for the residuals of Model 6 and in Figure 2 the histogram of the residuals is shown. In Figure 3 the normal probability plot is shown and in Figure 4 the fitted values versus the residuals is shown.

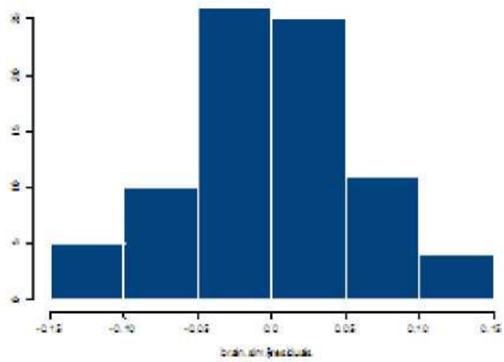

Figure 2: Histograms of the residuals

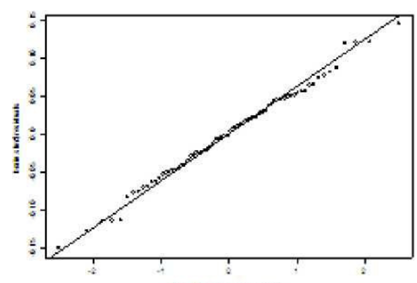

Figure 3: Normal probability plot of the residuals

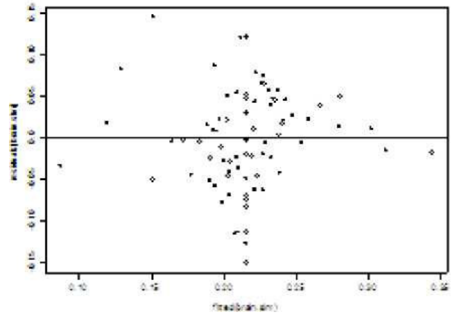

Figure 4: Fitted values vs. Residuals

It is clear from Figures 2 and 3 that the residuals are approximately normally distributed. Plot of the fitted values against the residuals also indicate that Model 6 is an appropriate one.

Other models besides models 6 and 11, that were significant at the 0.05 level were models 5, 12, 13, 16 and 17. The remaining models can be safely discarded.

CONCLUSION

The objective of this research was to fit and illustrate spatial regression modelling that takes accounts of spatial correlation amongst its neighbors. It has been found that the model $Y_i = \beta_{00} + \beta_{11}x_1x_2 + \beta_{21}x_1^2x_2 + \varepsilon_1$ (Model 6) is an appropriate one, in the sense that it has the smallest p value when compared with the null model (Model 1). The coefficients of the covariates were also found to be significant. The parameter ρ was highly significant at 0.001 level explaining the importance of taking the spatial correlation between neighboring points into consideration in the modelling process. The usefulness of this model is that it would help us to estimate the peak frequencies at locations where no observations were recorded and would also lead to an understanding of the phenomenona.

Different neighborhood structures and weights can also be attempted in any further study. Alternatively, further research can be done to fit spatial regression models with either Conditional Autoregressive (CAR) errors or Moving Average (MA) errors and to make comparisons with the proposed model in this research.

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