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Half- and Quarter-Sweeps Implementation of Finite-Difference Time-Domain Method

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ABSTRACT

The propagation, diffraction, scattering, penetration and interaction phenomena of electromagnetic waves are governed by the well known Maxwell's equations. The applications of Maxwell's equations can be found in many disciplines in science and engineering particularly in antenna design and analysis. Finite Difference Time Domain (FDTD) is a popular numerical simulation technique for solving problems related to Maxwell’s equations. Recently, there is other formulation that can potentially be used to solve Maxwell’s equations in source free region.. The new formulation, namely the scalar Wave-Equation Finite-Difference Time-Domain (WE-FDTD), is numerically and mathematically equivalent to the conventional FDTD. Unlike the conventional FDTD, the scalar WE-FDTD allows computing any single field component without the necessity of computing other field components. Therefore, significant savings in the computational time and memory storage can be achieved. In this paper, we presented the explicit formulation of the scalar WE-FDTD for free space wave propagation on one dimensional model problem using full-sweep, half-sweep and quarter-sweep approaches which successfully implemented for solving elliptic problems. We analyzed and compared the performance of the scalar WE-FDTD with all approaches to the conventional FDTD method in terms of the computational accuracy and simulation time. The results found that the proposed formulation significantly reduced the computational time of the method but posed less accuracy as compared to the conventional FDTD method.

Keywords: Maxwell's equation, Finite Difference Time Domain (FDTD), scalar wave-equation.

INTRODUCTION

Finite-Difference Time-Domain (FDTD) method nowadays is one of the most widely used numerical time-domain techniques in electromagnetism, as such antenna design, optics, etc. The primary advantage of the FDTD method is that it is a direct solution of Maxwell's equations. The FDTD method, also known as Yee’s algorithm (K.S.Yee (1966)), computes the field components by discretizing the Maxwell’s curl equations both in time and space, and then solving the discretized equations in a time marching sequence by alternatively calculating the electric and magnetic fields in the computational domain. Therefore no system of linear equations must be solved and the equations used in the FDTD method are fully explicit.

Recently, the FDTD method has been extended to other formulation which can potentially be used to solve Maxwell's equations. The formulation is based on scalar-wave equation which is both mathematically and numerically equivalence to conventional Yee algorithm in source free regions (Aoyagi *et al.* (1993)). Unlike the conventional FDTD approach, the new formulation, called the scalar Wave-Equation FDTD (WE-FDTD), allows computing any single field component without the necessity of computing other field components. Therefore, significant savings in the computational time and memory storage can be achieved. In this paper, the explicit formulations based on the scalar WE-FDTD are presented to solve electromagnetic wave propagation problem in source free region. These formulations are based on the concept of full-sweep, half-sweep and quarter-sweep introduced by Abdullah (1991) and Othman *et al.* (2000). Some numerical simulations were carried out using a model of one-dimensional problem to analyze the accuracy of the formulations and its execution time relative to conventional FDTD method.

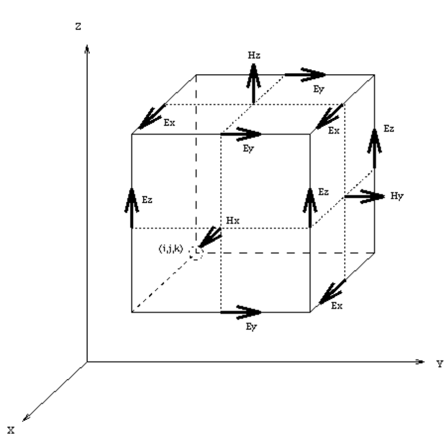

Figure 1: FDTD grid cell showing the staggering of electric and magnetic field components in space.

FORMULATIONS

The Maxwell's curl equations in free space can be written as:

$$\nabla \times H = \varepsilon_0 \frac{\partial E}{\partial t} \tag{1}$$

$$\nabla \times E = -\mu_0 \frac{\partial H}{\partial t} \tag{2}$$

where H is the normalized magnetic field, E is the normalized electric field, μ_0 and ε_0 are the permittivity and the permeability of magnetic and electric field in free space respectively. For one-dimensional free-space environment using E_x and H_y fields in z-direction, the equations become

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\varepsilon_0} \frac{\partial H_y}{\partial z} \tag{3}$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu_0} \frac{\partial E_x}{\partial z} \tag{4}$$

Equations (3) and (4) can be discretized in both space and time using central difference approximation to give the FDTD algorithm written in explicit form (Yee (1966)) as

$$E_x^{n+1}(k) = E_x^n(k) + \frac{\Delta t}{\mu_0 \Delta}\left[H_y^{n+\frac{1}{2}}(k+\frac{1}{2}) - H_y^{n+\frac{1}{2}}(k-\frac{1}{2}) \right] \tag{5}$$

$$H_y^{n+\frac{1}{2}}(k+\frac{1}{2}) = H_y^{n-\frac{1}{2}}(k+\frac{1}{2}) + \frac{\Delta t}{\varepsilon_0 \Delta}\left[E_x^n(k) - E_x^n(k+1) \right] \tag{6}$$

where Δt is the time step size and Δ is the space cell size in z-direction. The relative locations of the E_x and H_y field components in a uniform, Cartesian grid is defined by Yee cell (figure 1). To reduce the computational complexity of the Yee FDTD algorithm described above, equations (5) and (6) can be combined in a source free one-dimensional region as:

$$\frac{\partial^2 E_x}{\partial z^2} = \frac{1}{c_0^2} \frac{\partial^2 E_x}{\partial t^2} \tag{7}$$

where c_0 is the speed of electromagnetic waves in free space. Equation (7) forms the basic of scalar WE-FDTD algorithm by discretizing (7) using the central difference approximation in both time and space, leads to the standard five points explicit formula

$$E_x^{n+1}(k) = (2-2r^2)E_x^n(k) + r^2[E_x^n(k+1) + E_x^n(k-1)] - E_x^{n-1}(k) \quad (8)$$

where $r = \left(\frac{c_0 \Delta t}{\Delta}\right)$ is a courant number that will determine the stability of the algorithm (8) based on the Courant-Frederich Levy (CFL) conditions $r \le 1$. Equation (8) is called the full-sweep scalar WE-FDTD algorithm. Another type of formulation based on equation (7) also can be obtained by discretizing the equation using the same approximation with the grid spacing 2Δ and leads to the half-sweep formulation of scalar WE-FDTD :

$$E_x^{n+1}(k) = (2-\frac{1}{2}r^2)E_x^n(k) + \frac{1}{4}r^2[E_x^n(k+2) + E_x^n(k-2)] - E_x^{n-1}(k) \quad (9)$$

By using the same approximation scheme as full-sweep and half-sweep formulations, but now with the grid spacing 4Δ leads to the quarter-sweep formulation of the scalar WE-FDTD

$$E_x^{n+1}(k) = (2-\frac{1}{8}r^2)E_x^n(k) + \frac{1}{16}r^2[E_x^n(k+4) + E_x^n(k-4)] - E_x^{n-1}(k) \quad (10)$$

Both the half-sweep and the quarter-sweep approaches are inspired from Abdullah (1991) and Othman (2000), which have been successfully implemented for solving large and sparse linear system on the elliptic problems. The implementation of the half-sweep and quarter-sweep algorithms only applicable on the interior grid points. The remaining points, however, can be directly calculated at the required time step. The solution domain for the standard FDTD and the scalar WE-FDTD are shown in figure 2.

Theoretically, the half-sweep formulation (9) and the quarter-sweep formulation (10) give opportunity to solve only half and quarter of the solution domain respectively, therefore can reduce the execution time of the algorithms to nearly half and quarter compared to the full-sweep algorithm (8).

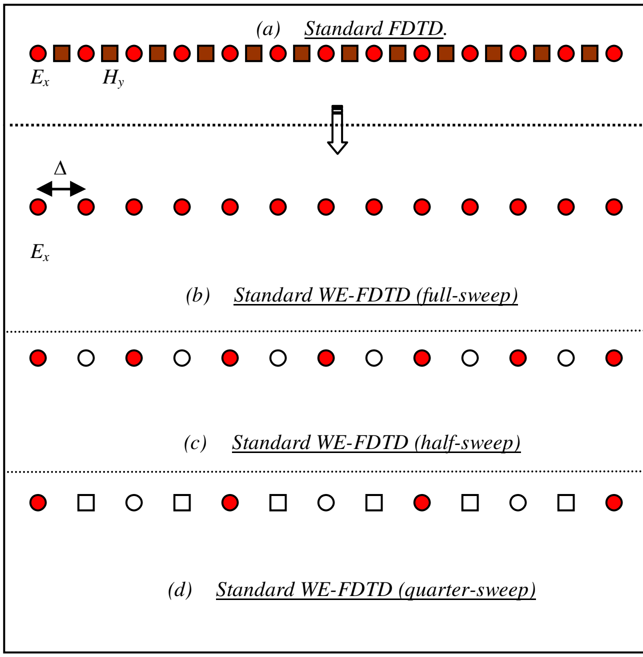

Figure 2: The solution domain for (a) FDTD and (b-d) WE-FDTD.

NUMERICAL EXPERIMENT AND RESULTS

The implementation of scalar WE-FDTD on one-dimensional wave propagation in free space medium is analyzed using Gaussian pulse as the point source. The pulse is excited at the middle of the solution domain of 2 meter, truncated with PEC boundary conditions. The numerical simulations were carried out using different grid size and courant numbers, r = 0.5, 0.75 and 1.0 to identify the optimum value of r that gives the least maximum error. The results of wave propagation from the simulation are shown in figure 3 and figure 4 at different time level. From the experiment, the optimum value of the courant number is obtained at r =1.0 which posses the least maximum error in terms of accuracy of the algorithms (figure 5).

It is shown that the scalar WE-FDTD for all approaches are numerically equivalence and compatible to the conventional FDTD especially at r =1.0 (optimum value). The comparison of simulation time between the FDTD and the scalar WE-FDTD methods for various grid size is shown in figure 5. It was found that, the scalar WE-FDTD with full-sweep, half-sweep, and quarter-sweep approaches significantly reduce the simulation time especially when the grid size becomes larger. The quarter-sweep approach used in the scalar WE-FDTD provide more advantage in terms of processing time but give less accuracy in its formulation.

CONCLUSION

It can be concluded that the scalar WE-FDTD gives an alternative method for solving problems related to electromagnetism. In this paper, the performance of the scalar WE-FDTD in terms of the simulation time can be improved by using half-sweep and quarter-sweep approaches. A major advantage of the scalar WE-FDTD is that it allows computing any single field component without the necessity of computing other field components. To remove the CFL stability conditions in the WE-FDTD method, the unconditionally stable methods can be applied in the algorithm. In order to extend the applicability of the scalar WE-FDTD method, the inspiration from the existing iterative and high speed algorithms are necessary to developed a new scheme for WE-FDTD that can increase the performance of the algorithm.

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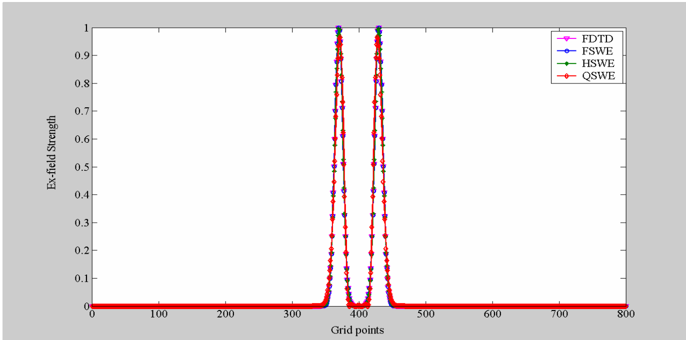
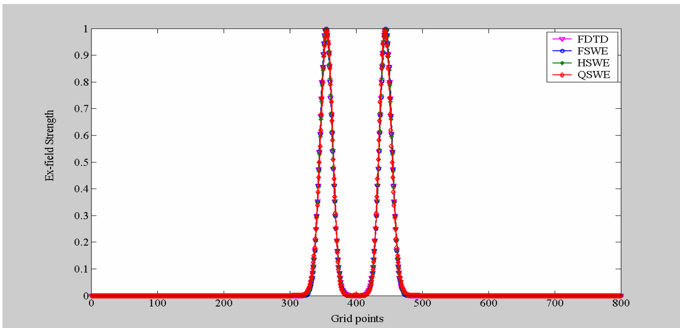


(a)



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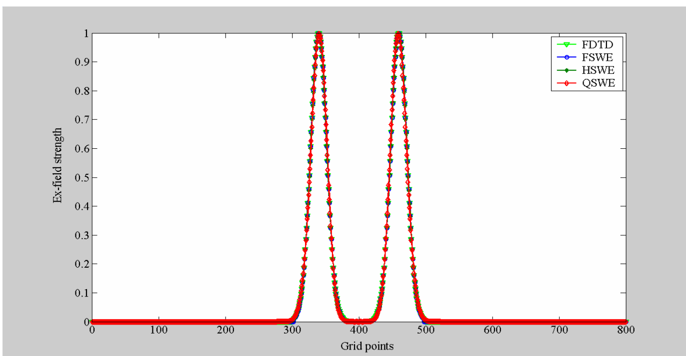
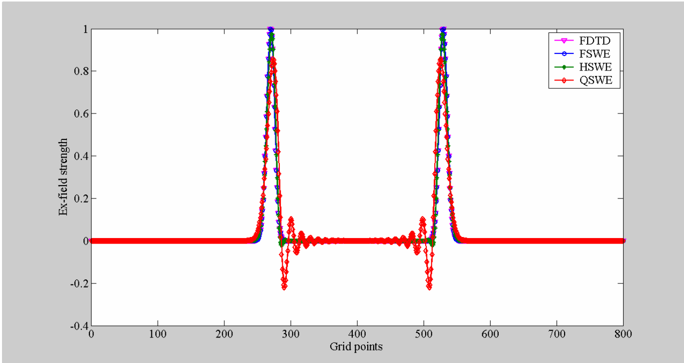


(c)

Figure 3: Wave propagation of the FDTD and the WE-FDTD methods from the center of solution domain after T=100 time steps with courant number (a) r=0.5, (b) r =0.75 (c) r=1.0 respectively.



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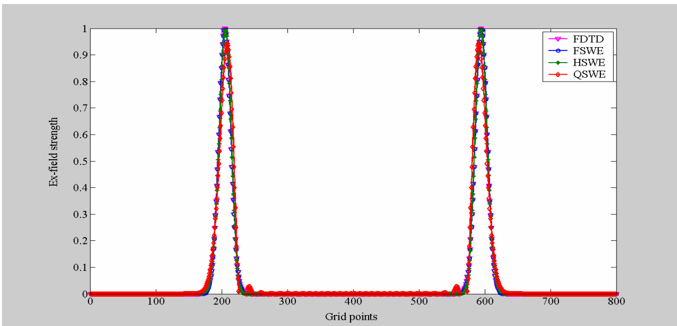

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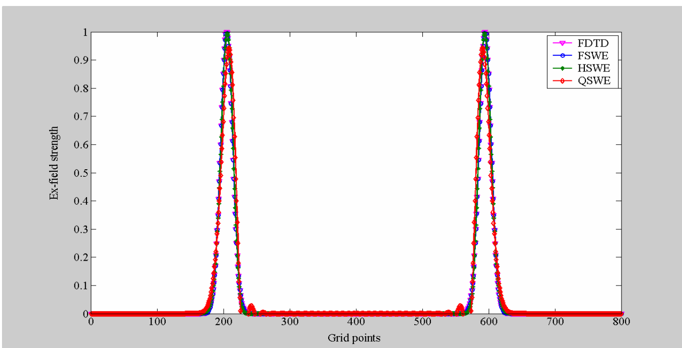

(c)

Figure 4: Wave propagation of the FDTD and the WE-FDTD methods from the center of solution domain after T=300 time steps with courant number (a) r=0.5, (b) r =0.75 (c) r=1.0 respectively.

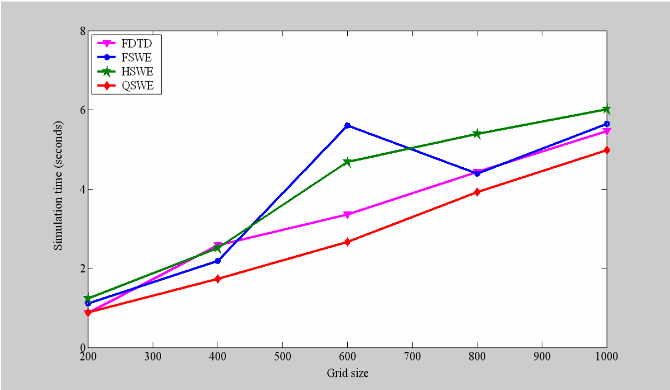

Figure 5: Comparison of the processing time between FDTD and scalar WE-FDTD versus the grid size after 100 time steps.