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Calculation of Mass Energy Density for a Collapsing Dust Sphere

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ABSTRACT

In this paper, we have considered an internal state of a collapsing body including in the course of the process its compression below the Schwarzschild (Schwarzschild, 1916) sphere, which requires the solution of the Einstein equations for the gravitational field in the material medium. In the centrally symmetric i.e. the velocity at each point must be directed along the radius, it represents the field outside a spherically symmetric mass in otherwise, empty space, later was also recognized as the solution representing both the outside and the inside of a non-rotating black hole. Schwarzschild solution allows the exact calculation of several of the post Newtonian effects of general relativity (GR) including the precession of planetary orbits, the bending of light around the sun, the exact gravitational frequency shift, the Shapiro (Shapiro and Teukolsky, 1991) time delay of light passing near the sun. and the precession of orbiting gyroscopes. In this work, we have considered that the fields equations can be solved in general form if we neglect the pressure of the dust like sphere, i.e. $p = 0$. Although the approximation made is not usually admissible in real situations, the general solution of this problem has considerable methodological interest. Here we have calculated the gravitational collapse of a dust like star in consultation with some coordinates, i.e. the time coordinates $\tau \sim \tau_0$. Ultimately our conclusion towards the total gravitational collapse of the large scale structure of the universe when time τ synchronizes to τ_0 , then a singular situation arise, which is called the beginning of the universe, i.e. big bang, at this moment the density was infinitely high.

Keywords: Schwarzschild solution, material medium, gravitational field, collapsing dust star, mass energy density.

INTRODUCTION

When the star is heavier than a few solar masses, it could undergo an endless gravitational collapse without achieving any equilibrium state. This happens when the star has exhausted its internal nuclear fuel which provides the outwards pressure against the inwards pulling gravitational forces. In the process of such a compression the central temperature of the material rises to ignite a nuclear fuel burning cycle in which the hydrogen, which forms the bulk of the cloud, burns to make helium, the gravitational contraction is halted and the star enters a quasi-static periodic when it supports itself against gravity by means of the thermal and radiation pressure. Such a phase may continue for billions of years, depending on the original mass of the star. If $M < M_{\odot}$ (where $M_{\odot}$ denotes the mass of the sun ~ 2×10^{33} gm), this period is longer than 10^{10} years, but if $M > 10M_{\odot}$, it has to be less than 2×10^{7} years, that is, very massive stars burn out their nuclear fuel much faster. For a review, see for example, Blandford, (1987), Blandford and Throne, (1979). The point of infinite density singularity is the final state for such an evolution is either an equilibrium star or a state of continual, endless gravitational collapse. The key factor for such stability analysis is the equation of state for the cool matter of the star in its ground state, that is, when all possible nuclear reactions have taken place and no further energy can be derived from such burning. The support in such a case must come either from the electron degeneracy pressure, when the star becomes a white dwarf, or from neutron degeneracy pressure giving a neutron star. Chandrasekhar, (1931, 1934, 1983) approximated the equation of state, in this case by an ideal electron Fermi gas and showed that this is a maximum mass limit for the mass of a spherical, non rotating star to achieve a white dwarf stable state, which is given by $M_c \sim 1.4\left(\frac{2}{\mu_e}\right)M_{\odot}$ where μ_e is the constant mean nuclear weight per electron and escape such 'compressed stars' they were initially called dark or frozen stars. A more catchy named was coined years later by John Wheeler, (1968) who called them 'black holes', black because they cannot emit light, hole because anything getting too close fall into them, never to return, the name stuck. To solve these types of problems we consider the Einstein's field equations:

$$R_{ab} - \frac{1}{2}g_{ab}R = -8\pi GT_{ab} \tag{1}$$

where T_{ab} is the energy of the matter (including the electromagnetic field) and R is the Ricci scalar.

The equation (1) shows the universal mass energy density relation. The Einstein equations are highly nonlinear. Therefore for gravitational fields the principle of superposition is not valid. The principle is valid only approximately for weak fields which permit a linearization of the Einstein equations. Einstein published his theory in 1915 and due to high nonlinearity and complexity of the equations; Einstein himself had not expected that the exact solution to the problem could be formulated, but within one year after the publication of the theory, K. Schwarzschild found the first physically significant exact solution to Einstein equations in 1916. Schwarzschild exterior solution contained a complete description of the external field of a massive body such as a star where the leading assumptions were:

- The field was static
- The field was spherically symmetric
- The space time was asymptotically flat
- The field was electrically neutral and non rotating

In this work our aim is to investigate the Schwarzschild interior solution which describes the line element for a sphere of incompressible perfect fluid of constant proper density ε such that at the boundary of the sphere, the pressure is equal to zero and the solution agrees with the exterior solution. It is obvious that if a massive centrally symmetric body is gravitationally unstable, this instability will remain for small disturbances of the symmetry so that such a body will collapse. The line element holds in the interior of a massive body which is at rest at the origin. We have also calculated the mass energy density for a collapsing dust star in connection with some incoming coordinates.

PRELIMINARIES INTERIOR SOLUTION OF THE FIELD EQUATIONS

For a dust like medium one can choose a references system which is both synchronous and co-moving. Denoting the time t and the radial co-ordinate r chosen in this way by τ and R respectively (Landau and Lifshitz, 1975), we write the spherically symmetric line element in the form (Chandrasekhar, 1983),

$$ds^2 = d\tau^2 - e^{\lambda(\tau,R)}dR^2 - r^2(\tau,R)(d\theta^2 + sin^2\theta d\phi^2). \qquad (2)$$

The function $r(\tau, R)$ is the 'radius', defined so that $2\pi r$ is the circumference of a circle (with centre at the origin). The form (2) fixes the choice of τ uniquely, but still per units arbitrary transformations of the radial co-ordinate of the form $R = R(R')$.

Now, from equation (2), we have:

$$g_{00} = 1, \quad g_{11} = -e^{\lambda}, \quad g_{22} = -r^2, \quad g_{33} = -r^2\sin^2\theta. \qquad (3)$$

Then, the contravariant forms can be written as:

$$g^{00} = 1, \quad g^{11} = -e^{-\lambda}, \quad g^{22} = -r^{-2}, \quad g^{33} = -(r\sin\theta)^{-2}. \qquad (4)$$

Again, the determinant of the metric tensor g_{ij} is given by: $g = |g_{ij}| = -r^4e^{\lambda}\sin^2\theta$.

Now, the energy-momentum tensor in the material medium is given by:

$$T_{\mu\nu} = (\varepsilon + p)u_{\mu}u_{\nu} - pg_{\mu\nu} \qquad (5)$$

$$T^{00} = \varepsilon = \text{Mass Energy Density} \qquad (6)$$

$$T^{11} = T^{22} = T^{33} = 0. \qquad (7)$$

Thus, in the co-moving system and pressure free medium, the energy-momentum tensors are:

$$T^{\mu\nu} = (\varepsilon, 0, 0, 0). \qquad (8)$$

The non-vanishing values of Γ's can be calculated as follows:

$$\Gamma^1_{11} = \frac{1}{2}\lambda', \quad \Gamma^1_{22} = -e^{-\lambda}rr', \quad \Gamma^1_{33} = -e^{-\lambda}rr'\sin^2\theta \qquad (9a)$$

$$\Gamma^2_{12} = \Gamma^2_{21} = \frac{r'}{r} \quad , \quad \Gamma^3_{13} = \Gamma^3_{31} = \frac{r'}{r} \tag{9b}$$

$$\Gamma^2_{33} = -\sin\theta\cos\theta, \quad \Gamma^3_{23} = \Gamma^3_{32} = \cot\theta \tag{9c}$$

$$\Gamma^1_{01} = \Gamma^1_{10} = \frac{\dot{\lambda}}{2}, \quad \Gamma^2_{02} = \Gamma^2_{20} = \frac{\dot{r}}{r}, \quad \Gamma^3_{03} = \Gamma^3_{30} = \frac{\dot{r}}{r} \tag{9d}$$

$$\Gamma^0_{11} = \frac{1}{2}e^{\lambda}\dot{\lambda}, \quad \Gamma^0_{22} = r\dot{r}\ , \ \Gamma^0_{33} = r\,\dot{r}\sin^2\theta\ . \tag{9e}$$

Now, from Ricci tensor, we have:

$$R_{ab} = -\Gamma^c_{ab,c} + \Gamma^c_{ac,b} - \Gamma^d_{ab}\Gamma^c_{dc} + \Gamma^d_{ac}\Gamma^c_{db}\,. \tag{10}$$

The values of the different components of the Ricci tensors are as follows:

$$R_{00} = \frac{\ddot{\lambda}}{2} + \frac{2\ddot{r}}{r} + \frac{\dot{\lambda}^2}{4} \tag{11}$$

$$R_{01} = \frac{2\dot{r}'}{r} - \frac{\dot{\lambda}r'}{r} \tag{12}$$

$$R_{11} = -\frac{e^{\lambda}\ddot{\lambda}}{2} + \frac{2r''}{r} - \frac{e^{\lambda}\dot{\lambda}^2}{4} - \frac{e^{\lambda}\dot{\lambda}\dot{r}}{r} - \frac{\lambda' r'}{r} \tag{13}$$

$$R_{22} = -\dot{r}^2 - r\ddot{r} - \frac{1}{2}e^{-\lambda}\lambda' r' r + e^{-\lambda}r'^2 + e^{-\lambda}rr'' - \frac{1}{2}r\dot{r}\dot{\lambda} - 1 \tag{14}$$

$$\begin{aligned} R_{33} = &-r\ddot{r}\sin^2\theta + \dot{r}^2\sin^2\theta + e^{-\lambda}rr''\sin^2\theta + e^{-\lambda}r'^2\sin^2\theta \\ &-\frac{1}{2}e^{-\lambda}rr'\lambda'\sin^2\theta - \frac{1}{2}r\dot{r}\dot{\lambda}\sin^2\theta\ - \sin^2\theta. \end{aligned} \tag{15}$$

Now, we have the Ricci scalar (or scalar curvature)

$$\begin{aligned} R &= g^{ab}R_{ab} \\ &= g^{00}R_{00} + g^{11}R_{11} + g^{22}R_{22} + g^{33}R_{33} \end{aligned} \tag{16}$$

which implies,

$$R = \ddot{\lambda} + \frac{\dot{\lambda}^2}{2} + \frac{4\ddot{r}}{r} - \frac{4e^{-\lambda}r''}{r} + \frac{2\dot{\lambda}\dot{r}}{r} + \frac{2e^{-\lambda}\lambda' r'}{r} - \frac{2e^{-\lambda}r'^2}{r^2} + \frac{2\dot{r}^2}{r^2} + \frac{2}{r^2}\ . \tag{17}$$

We are now in a position to solve the Einstein interior field equation,

$$R_{ab} - \frac{1}{2} g_{ab} R = -8\pi G T_{ab} \tag{18}$$

After detail calculations we have the following interior solution of the Einstein field equations:

$$-\frac{e^{-\lambda}}{r^2}\left[2rr'' + r'^2 - rr'\lambda'\right] + \frac{1}{r^2}\left[r\dot{r}\dot{\lambda} + \dot{r}^2 + 1\right] = 8\pi G\varepsilon \tag{19}$$

$$-e^{-\lambda}r'^2 + 2r\ddot{r} + \dot{r}^2 + 1 = 0 \tag{20}$$

$$-\frac{e^{-\lambda}}{r}\left[2r'' - r'\lambda'\right] + \left[\frac{2\ddot{r}}{r} + \ddot{\lambda} + \frac{\dot{\lambda}^2}{2} + \frac{\dot{r}\dot{\lambda}}{r}\right] = 0 \tag{21}$$

$$2\dot{r}' - \dot{\lambda}r' = 0 \tag{22}$$

where the prime denotes the differentiation with respect to R and the dot with respect to τ. These are the set of equation describing the interior of a collapsing dust star of mass energy density ε.

Now, the equation (22) can be solved as follows:

$$e^{-\lambda}r'^2 = 1 + f(R) \tag{23}$$

where $f\,(R)$ is an arbitrary function, subject only to the condition that $1 + f > 0$.

Again, from the equation (20), we get

$$-e^{-\lambda}r'^2 + 2r\ddot{r} + \dot{r}^2 + 1 = 0$$

$$\Rightarrow 2r\ddot{r} + \dot{r}^2 = f(R); \qquad \text{using equation (23).}$$

$$\Rightarrow 2rp\frac{dp}{dr} + p^2 = f(R) \tag{24}$$

$$\left[\because \dot{r} = p = \frac{\partial r}{\partial \tau} \quad \therefore \frac{dp}{dt} = \frac{dp}{d\tau}\frac{d\tau}{dt} = \frac{\ddot{r}}{\dot{r}} = \frac{\ddot{r}}{p} \quad \Rightarrow \ddot{r} = p\frac{dp}{dr}\right]$$

$$\Rightarrow r\frac{d}{dr}(p^2)+p^2=f(R)$$
$$\Rightarrow \frac{d}{dr}(rp^2)=f(R)$$

$\therefore$ integrating, we get

$$rp^2=rf(R)+f_0(R)$$

where $f_0(R)$ is another constant of integration.

Thus, $\dot{r}^2=p^2=f(R)+\dfrac{f_0(R)}{r}$ (24a)

$$\Rightarrow\left(\frac{dr}{d\tau}\right)^2=f(R)+\frac{f_0(R)}{r}$$
$$\Rightarrow\frac{dr}{d\tau}=\pm\sqrt{f(R)+\frac{f_0(R)}{r}}$$
$$\therefore\ d\tau=\pm\frac{dr}{\sqrt{f(R)+\dfrac{f_0(R)}{r}}}$$

$\therefore$ integrating, we get

$$\tau=\pm\int\frac{rdr}{\sqrt{f(R)r^2+f_0(R)r}}+\tau_0(R) \tag{25}$$

where, $\tau_0(R)$ is constant of integration.

Putting $f_0(R)=F(R)$, we get from (25)

$$\tau=\pm\int\frac{rdr}{\sqrt{f\,r^2+Fr}}+\tau_0(R)$$

The function $r(\tau, R)$ obtained from the integration can be written in the parametric form:

$$\left.\begin{aligned} r &= \frac{F(R)}{2f(R)}(\cosh\eta - 1) \\ \tau_0(R) - \tau &= \frac{F(R)}{2f^{3/2}(R)}(\sinh\eta - 1) \end{aligned}\right\} \quad \text{if } f(R) > 0 \tag{26}$$

Again,

$$\left.\begin{aligned} r &= \frac{F(R)}{-2f(R)}(1 - \cos\eta) \\ \tau_0(R) - \tau &= \frac{F(R)}{2\{-f(R)\}^{3/2}}(\eta - \sin\eta) \end{aligned}\right\} \quad \text{if } f(R) < 0 \tag{27}$$

If, $f(R) = 0$, $\quad r = \left\{\frac{9}{4}F(R)\right\}^{1/3}\left[\tau_0(R) - \tau\right]^{2/3}$ (28)

$$r^2 r' = \left(\frac{9F}{4}\right)^{2/3}(\tau_0 - \tau)\left[\left(\frac{9}{4}\right)^{1/3}\frac{1}{3}F^{-2/3}F'(\tau_0 - \tau) + \left(\frac{9F}{4}\right)^{1/3}\frac{2}{3}\tau_0'\right] \tag{29}$$

$$r' = \frac{dr}{dR} = \left(\frac{9}{4}\right)^{1/3}\frac{1}{3}F^{-2/3}F'(\tau_0 - \tau)^{2/3} + \left(\frac{9F}{4}\right)^{1/3}\frac{2}{3}(\tau_0 - \tau)^{-1/3}\tau_0' \tag{30}$$

NEW SOLUTIONS

From equation (23) we can write,

$$e^{\lambda} = \frac{r'^2}{1 + f(R)}$$

$$\therefore \lambda = 2\log r' - \log(1 + f)$$

$$\Rightarrow \lambda' = \frac{2r''}{r'} - \frac{f'}{(1+f)}$$

$$\Rightarrow 2r'' - r'\lambda' = \frac{r'f'}{(1+f)}$$

$$\Rightarrow 2rr'' - rr'\lambda' = \frac{rr'f'}{(1+f)}$$

$$\Rightarrow 2rr'' + r'^2 - rr'\lambda' = r'^2 + \frac{rr'f'}{(1+f)}$$

$$\Rightarrow \left(2rr'' + r'^2 - rr'\lambda'\right)\frac{e^{-\lambda}}{r^2} = \frac{r'^2 e^{-\lambda}}{r^2} + \frac{r'f'e^{-\lambda}}{r(1+f)}$$

$$\Rightarrow -\frac{e^{-\lambda}}{r^2}\left(2rr'' + r'^2 - rr'\lambda'\right) = -\frac{1+f}{r^2} - \frac{r'f'e^{-\lambda}}{r(1+f)} \tag{31}$$

Now, from equation (22),

$$\dot{\lambda} = \frac{2\dot{r}'}{r'}. \tag{32}$$

Again from equation (24*a*) we can write,

$$\dot{r} = \pm\sqrt{f + \frac{F}{r}}$$

$$\therefore \dot{r}' = \pm\frac{1}{2}\left(f + \frac{F}{r}\right)^{-\frac{1}{2}}\left(f' + \frac{F'}{r} - \frac{Fr'}{r^2}\right)$$

Also, $$\dot{r}\dot{\lambda} = \frac{2\dot{r}'\dot{r}}{r'} = \frac{1}{r'}\left(f' + \frac{F'}{r} - \frac{Fr'}{r^2}\right)$$

$$\Rightarrow r\dot{r}\dot{\lambda} + \dot{r}^2 + 1 = \frac{r}{r'}\left(f' + \frac{F'}{r} - \frac{Fr'}{r^2}\right) + f + \frac{F}{r} + 1$$

$$\Rightarrow r\dot{r}\dot{\lambda} + \dot{r}^2 + 1 = \frac{r}{r'}\left(f' + \frac{F'}{r}\right) + f + 1$$

$$\Rightarrow \frac{1}{r^2}\left[r\dot{r}\dot{\lambda} + \dot{r}^2 + 1\right] = \frac{1}{rr'}\left(f' + \frac{F'}{r}\right) + \frac{f+1}{r^2} \tag{33}$$

Now, adding equations (31) and (33) we obtain,

$$-\frac{e^{-\lambda}}{r^2}\left(2rr'' + r'^2 - rr'\lambda'\right) + \frac{1}{r^2}\left(r\dot{r}\dot{\lambda} + \dot{r}^2 + 1\right)$$

$$= -\frac{r'f'e^{-\lambda}}{r(1+f)} - \frac{1+f}{r^2} + \frac{1}{rr'}\left(f' + \frac{F'}{r}\right) + \frac{f+1}{r^2}$$

$$= -\frac{f'}{rr'} + \frac{1}{rr'}\left(f' + \frac{F'}{r}\right), \quad \left[\text{using } \frac{e^{-\lambda}}{1+f} = \frac{1}{r'^2}\right]$$

$$= \frac{F'}{r^2 r'}$$

Now, from equation (19) we get,

$$8\pi G\varepsilon = \frac{F'}{r^2 r'}. \tag{34}$$

Applying the values of equations (28) and (30) in equation (34) we get,

$$8\pi G\varepsilon = \frac{F'}{\left(\frac{9F}{4}\right)^{\frac{2}{3}}(\tau_0-\tau)^{\frac{4}{3}}\left\{\left(\frac{1}{12}\right)^{\frac{1}{3}}\left(F^{-\frac{2}{3}}F'\right)(\tau_0-\tau)^{\frac{2}{3}} + \left(\frac{2F}{3}\right)^{\frac{1}{3}}(\tau_0-\tau)^{-\frac{1}{3}}\tau_0'\right\}}$$

which gives the following result (Biswas, 2007*a*),

$$8\pi G\varepsilon = \frac{F'}{\left(\frac{9F}{4}\right)^{\frac{2}{3}}(\tau_0-\tau)\left\{\left(\frac{1}{12}\right)^{\frac{1}{3}}\left(F^{-\frac{2}{3}}F'\right)(\tau_0-\tau) + \left(\frac{2F}{3}\right)^{\frac{1}{3}}\tau_0'\right\}}. \tag{35}$$

CONCLUDING REMARKS

From the equations (26), (27) and (28) we get both contraction and expansion of the sphere (depending on the range of values taken by the parameter η, where the parameter η runs through values from 0 to 2π). The contraction and expansion both are equally admissible for the field equations. The important problem of behavior of an unstable massive body corresponds to contraction; the massive body turns to gravitational collapse. The solutions (26), (27) and (28) are written so that contraction occurs when τ, while increasing tends to τ_0. To the moment $\tau = \tau_0(R)$ these corresponds the arrival at the center of the matter with a given radial coordinate R_0 the time τ (for a given *R*) decreases monotonically. Now the important fact is that in this calculation the solution collapse occurs for any mass of the sphere is a natural consequence of neglecting pressure i.e. $p = 0$. Clearly, as $\tau \to \tau_0$ or $\tau_0 \to \tau$ the density $\varepsilon \to \infty$, from the physical point of view the assumption that the matter is dust like is never admissible, and we should use the ultra relativistic equation of state $p = \frac{1}{3}\varepsilon_0$. It appears however, that the general character of the solution collapse is independent of the equation of state of the matter (Lifshitz and Khatatnikov, 1961).

Here we find that the point of infinite density singularity is a concept of classical (that is non quantum) general relativity predicts its own downfall, just as classical mechanics, predicted its downfall by suggesting that blackbodies should radiate infinite energy or that the atoms should collapse to infinite density. And as with classical mechanics, we hope to eliminate these in acceptable singularities by making classical general relativity into quantum theory-that is creating many things from nothingness by developing the quantum gravity. In the central depths of a black hole an enormous mass is crushed to a mine scale size. At the moment of the big bang the whole of the universe erupted from a microscopic nugget whose size makes a grain of sand look colossal. These are realms that are tiny and yet incredibly massive, therefore requiring that both quantum mechanics and general relativity simultaneously brought to bear. For reasons that will become increasingly close as we proceed, the equations of general relativity and quantum mechanics, when combined, begin to shake, rattle and gush with steam like a red-lined automobile. Even if we are willing to keep the deep interior of a black hole and the beginning of the universe shrouded in mystery we can not help feeling that the hostility between quantum mechanics and general relativity cries out for a deeper level of understanding.

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