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On Huntsberger Type Shrinkage Estimator for the Mean of Normal Distribution

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ABSTRACT

The present article deals with an improved estimator of normal mean which is obtained by considering single stage procedure with appropriate shrinkage weight function. Indeed two shrunken testimators of Huntsberger-type are proposed for the mean μ of normal distribution when a prior estimate μ_0 of the mean μ is available. The goodness of this procedure as a means of maximizing the relative efficiency is explained in this paper. The expressions for the bias mean squared error and relative efficiency of the proposed testimators are derived. The performances of the proposed testimators are compared with classical and existing testimators based on the criteria of biased ratio and relative efficiency to search for a 'better' estimator.

Keywords: Normal distribution; Huntsberger-type shrinkage testimator; Preliminary test; Bias ratio; Relative efficiency.

INTRODUCTION

The model

The normal distribution plays an important role in both the application and inferential statistics. In modeling applications, the normal curve is an excellent approximation to the frequency distributions of observations taken on a variety of variables and as a limiting form of various other distributions (Davison, (2003)). Many psychological measurements and physical phenomena can be approximated well by the normal distribution. In addition, there are many applications of the normal distribution in engineering. One application deals with analysis of items which exhibit failure due to wear, such as mechanical devices. Other applications are, the analysis of the variation of component dimensions in manufacturing, modeling global irradiation data, and the intensity of laser light, and so on. Indeed the wide application and occurrence of the normal distribution in life testing and reliability problems are a wonder. In the context of reliability problems and life testing, a number of failure time data have been examined (Bain and Engelharadt, (1991)) and it was shown that the normal distribution give quite a good fit for the most cases.

Incorporating a guess value, and PSE

In many problems, the experimenter has some prior information regarding the value of μ either due to past experiences or to his familiarity with the behavior of the population. However, in certain situations the prior information is available only in the form of an initial guess value (natural origin) μ_0 of μ. In such a situation it is natural to start with an estimator $\overline{X}$ of μ and modify it by moving it closer to μ_0, so that the resulting estimator, though perhaps biased, has smaller mean squared error than that of $\overline{X}$ in some interval around μ_0. This method of constructing an estimator of μ that incorporates the prior information μ_0 leads to what is known as a shrunken estimator. Consider the Huntsberger, (1955) type shrinkage estimator

$$\tilde{\mu}_\varphi = \{\psi(\overline{X})(\overline{X} - \mu_0) + \mu_0\}, \tag{1}$$

where $\psi(\overline{X})(0 \le \psi(\overline{X}) \le 1)$, represents a weighting function specifying the degree of belief in μ_0. A number of authors (Goodman, (1953), Thompson, (1968a), Arnold, (1969) and Mehta and Srinivasan, (1971)) have tried to develop new s shrinkage estimators of the form (1) for special populations by choosing different weight functions. The relevance of such types of shrinkage estimators lies in the fact that, though perhaps they are biased, have smaller MSE than $\overline{X}$ in some interval around μ_0. Shrinkage estimators of the form (1) have the disadvantage that it necessarily uses the prior value in the construction of final estimators. However, it is not necessary that the prior value is close to true value. To employ this idea in estimation of the mean μ, a preliminary test is first conducted to check the closeness of μ_0 to μ before using it in a shrinkage estimator. Define

$$\psi(\overline{X}) = \begin{cases} \varphi(\overline{X}), & if \;\; H_0 : \mu_0 = \mu, \\ 1, & if \;\; H_1 : \mu_0 \ne \mu, \end{cases} \tag{2}$$

where $0 \le \varphi(\overline{X}) \le 1$. Thus, the preliminary shrinkage estimator (PSE) of μ corresponding to $\psi(\overline{X})$ is given by,

$$\hat{\mu} = \{[\varphi(\overline{X})(\overline{X} - \mu_0) + \mu_0]I_R + [\overline{X}]I_{\overline{R}}\}, \tag{3}$$

where I_R and $I_{\bar{R}}$ are respectively the indicator functions of the acceptance region R and the rejection region $\bar{R}$. A number of other authors (Thompson, (1968a, 1968b), Davis and Arnold, (1970), Hirano, (1977), Kambo *et al.*, (1990), Singh *et al.*, (2004), Singh and Saxena, (2005), Lemmer, (2006), Al-Hemyari *et al.*, (2009b), Al-Hemyari, (2009a, 2010)) have tried to develop new shrinkage estimators of the form (3) for special populations by choosing different weight functions and *R*. In this paper two preliminary shrinkage estimators of Huntsberger-type for the mean μ of normal distribution $N(\mu,\sigma^2)$ when σ^2 is known or unknown are proposed, and the expressions for the bias, mean squared error, and relative efficiency are derived and studied numerically. The discussion regarding the comparisons with the earlier known results are made and the usefulness of these testimators under different situations is provided as conclusions from various numerical tables obtained from simulation results.

TESTIMATOR $\tilde{\mu}_1$ AND ITS PROPERTIES WITH KNOWN σ^2

Let X be normally distributed with unknown μ and known variance σ^2. Assume that a prior estimate μ_o about μ is available from the past. The first proposed testimator is,

$$\tilde{\mu}_1 = \{[\ \bar{X} - ae^{-n\, b(\bar{X}\ -\mu_0)^2/\sigma^2}\ (\bar{X} - \mu_0)]\, I_{R_1} + [\bar{X}]\ I_{\bar{R}_1}\ \}. \tag{4}$$

For this testimator we considered R_1 as the commonly used acceptance region of the hypothesis $H_0:\theta=\theta_0$ against the alternative $H_1:\theta\neq\theta_0$. If α is the level of significance of the test, then the preliminary test region R_1 is given by

$$R_1 = \{\bar{X} : T(\bar{X}) \in [L_{1-\alpha/2},\ U_{\alpha/2}]\}, \tag{5}$$

where $l_{1-\alpha/2}$ and $U_{\alpha/2}$ are the lower and upper 100(α/2) percentile points of the statistic $T(\bar{X})$ used for testing the above hypothesis. If the standard normal statistic $T(\bar{X})$ is used, the region R_1 is given by:

$$R_1 = [\mu_o - z_{\alpha/2}\frac{\sigma}{\sqrt{n}},\ \mu_o + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}],\ \ \varphi(\bar{X}) = [1 - a\, e^{[-n\, b(\bar{X}\ -\mu_0)^2/\sigma^2\}}], \tag{6}$$

$b \geq 0,\ 0 \leq a \leq 1$, and $z_{\alpha/2}$ is the upper $100(\alpha/2)$ percentile point of the standard normal distribution. The bias expression of $\tilde{\mu}_1$ is defined by,

$$\begin{aligned} B(\tilde{\mu}_1 \mid \mu) &= E(\tilde{\mu}_1) - \mu \\ &= \int_{R_1} ((1 - ae^{-nb(\overline{X}-\mu_0)^2/\sigma^2})(\overline{X} - \mu_0) + \mu_0) f(\overline{X}/\mu, \sigma^2) d\overline{X} \\ &+ \int_{\overline{R}} \overline{X} f(\overline{X}/\mu, \sigma^2) d\overline{X} - \mu, \end{aligned} \tag{7}$$

where $\overline{R}$ is the rejection region and $f(\overline{X}\mu, \sigma^2)$ is the p.d.f. of $\overline{X}$. Simple calculations lead to

$$B(\tilde{\mu}_1 \mid \mu, R_1) = (\frac{\sigma}{\sqrt{n}}) \left\{ -\frac{ae^{-b\lambda_1^2/(2b+1)}}{(2b+1)^{3/2}} [\sqrt{2b+1} J_1(a_2, b_2) + \lambda_1 J_o(a_2, b_2)] \right\}. \tag{8}$$

The mean squared error expression of $\tilde{\mu}_1$ is defined by,

$$\begin{aligned} MSE(\tilde{\mu}_1 \mid \mu, R_1) &= E(\tilde{\mu}_1 - \mu)^2 = \int_{R_1} ((1 - ae^{-nb(\overline{X}-\mu_0)^2/\sigma^2})(\overline{X} - \mu_0) + \mu_0 - \mu)^2 \\ & f(\overline{X}/\mu, \sigma^2) d\overline{X} + \int_{\overline{R}} (\overline{X} - \mu)^2 f(\overline{X}/\mu, \sigma^2) d\overline{X}. \end{aligned} \tag{9}$$

After some algebraic manipulations, it can be easily shown that

$$\begin{aligned} MSE\ (\tilde{\mu}_1 \mid \mu, R_1) &= \frac{\sigma^2}{n} \Big\{ 1 - 2a(2b+1)^{-5/2} e^{-b\lambda_1^2/(2b+1)} [\ (2b+1) J_2(a_2, b_2) \\ &+ \lambda_1 (1-2b)\sqrt{2b+1}\ \ J_1(a_2, b_2) - 2b\lambda_1^2\ J_o(a_2, b_2)\ ] + a^2 (4b+1)^{-5/2} e^{-2b\lambda_1^2/(4b+1)} \\ &\text{x} \Big[(4b+1) J_2(a_3, b_3) + 2\lambda_1 \sqrt{4b+1}\ \ J_1(a_3, b_3) + \lambda_1^2 J_0(a_3, b_3) \Big] \Big\}, \end{aligned} \tag{10}$$

where

$a_1 = \lambda_1 - z_{\alpha/2},\ b_1 = \lambda_1 + z_{\alpha/2},\ a_2 = (\lambda_1 - z_{\alpha/2})/\sqrt{2b+1},\ b_2 = (\lambda_2 + z_{\alpha/2})/\sqrt{2b+1},$

$a_3 = (\lambda_1 - z_{\alpha/2})/\sqrt{4b-1},\ b_3 = (\lambda_1 + z_{\alpha/2})/\sqrt{4b-1},\ \lambda_1 = \sqrt{n}(\mu - \mu_0)/\sigma,$

and

$$J_i(a_j,b_j)=\int_{a_j}^{b_j}\frac{1}{\sqrt{2\pi}}y^i e^{-y^2/2}dy, \qquad i=0,\ 1,\ 2,\ \ j=1,\ 2.$$

For numerical computations we may use the relations:

$$J_1(a,b)=\left(\exp(-a^2/2)-\exp(-b^2/2)\big/\sqrt{2\pi}\right), \tag{11}$$

and

$$J_2(a,b)=J_0(a,b)+\left(\exp(-a^2/2)-\exp(-b^2/2)\big/\sqrt{2\pi}\right). \tag{12}$$

Remark 1: Choice of a
It seems reasonable to select 'a' that minimizes the $MSE(\tilde{\mu}_1 \mid \mu)$. Setting $((\partial/\partial a)MSE(\tilde{\mu}_1 \mid \mu_0))$ to zero, we get,

$$a=\overline{a}_1=\left((4b+1)/(2b+1)\right)^{5/2}e^{-b\lambda_1^2/(2b+1)(4b+1)}\{[(2b+1)J_2(a_2,b_2)+\lambda_1(1-2b)$$
$$\sqrt{2b+1}J_1(a_2,b_2)-2b\lambda_1{}^2J_0(a_2,b_2)]/[(4b+1)J_2(a_3,b_3)+2\lambda_1\sqrt{4b+1}J_1(a_3,b_3)$$
$$+\lambda_1{}^2J_0(a_3,b_3)]\} \tag{13}$$

Since

$$\frac{\partial^2}{\partial a^2}MSE\ (\tilde{\mu}_1 \mid \mu)=$$
$$\frac{\sigma^2}{n}(4b+1)^{-5/2}e^{-2b\lambda_1^2/(4b+1)}\left[(4b+1)J_2(a_3,b_3)+2\lambda_1\sqrt{4b+1}\ J_1(a_3,b_3)+\lambda_1{}^2J_0(a_3,b_3)\ \right]\geq 0,$$

it follows that the minimizing value of $a\in[0,1]$ is given by:

$$\tilde{a}=\begin{cases}0 & if\quad \overline{a_1}\leq 0,\\ \overline{a_1} & if\quad 0\leq \overline{a_1}\leq 1,\\ 1 & if\quad \overline{a_1}\geq 1.\end{cases} \tag{14}$$

The efficiency of $\tilde{\mu}_1$ relative to $\overline{X}$ is defined by,

$$Eff\,(\tilde{\mu}_1 \mid \mu)=\sigma^2\,/\,MSE(\tilde{\mu}_1 \mid \mu). \tag{15}$$

Remark 2: Some properties.
It is easily seen that $B(\tilde{\mu}_1 \mid \mu)$ is an odd function of λ_1, whereas $E(n \mid \tilde{\mu}_1)$, $MSE(\tilde{\mu}_1 \mid \mu)$ and $Eff(\tilde{\mu}_1 \mid \mu)$ are all even functions of λ_1. Also $Eff(\tilde{\mu}_1 \mid \mu) = 1 \;\; as \;\; \lambda_1 \to \pm\infty$.

Since $\lim_{n \to \infty} B(\tilde{\mu}_1 \mid \mu) = 0,$ and $\lim_{n \to \infty} MSE\ (\tilde{\mu}_1 \mid \mu) = 0$, $\tilde{\mu}_1$ is a asymptotically unbiased and consistent estimator of μ. Also $\tilde{\mu}_1$ dominates $\bar{X}$ in large n and n in the sense that

$$\lim_{n \to \infty} [MSE\ (\tilde{\mu}_1 \mid \mu) - MSE\ (\bar{X})] \leq 0.$$

Remark 3: Special cases.
It may be noted here, when $\varphi(\overline{X}) = c,\ 0 \leq c \leq 1$ is constant the equations (8), (10) & (15) agree with the result of Thompson, (1968a); when the hypothesis $H_o : \mu = \mu_o$ is accepted with probability one the same expression aggress with the result of Mehta and Srinivasan, (1971); also when $a = 0$, the same expression aggress with the result of Hirano, (1977), and when $\varphi(\overline{X}) = c,\ 0 \leq c \leq 1,$ if $\overline{X} \in R$ and $\varphi(\overline{X}) = 1 - c,$ if $\overline{X} \notin R,$ the result agrees with the result of Al-Hemyari *et al.*, (2009b).

TESTIMATOR $\tilde{\mu}_2$ WITH σ^2 UNKNOWN

When σ^2 is unknown, it is estimated by $s^2 = \sum_{i=1}^{n} (X_i - \bar{X})^2 / (n-1)$. Again taking region R_2 as the pretest region of size α for testing $H_o : \mu = \mu_o$ against $H_1 : \mu \neq \mu_o$ in the testimator $\tilde{\mu}_1$ defined in (3) and denoting the resulting estimator as $\tilde{\mu}_2$. The testimator $\tilde{\mu}_2$ employs the interval R_2 given by,

$$R_2 = \left[\mu_o - t_{\alpha/2, n-1} s / \sqrt{n}, \mu_o + t_{\alpha/2, n-1} s / \sqrt{n} \right], \tag{16}$$

where $t_{\alpha/2, n_1 - 1}$ is the upper $100(\alpha/2)$ percentile point of the *t* distribution with $n-1$ degrees of freedom. The expressions for bias and MSE and are given respectively by:

$$B(\tilde{\mu}_2 \mid \mu) = \frac{\sigma}{\sqrt{n}} \int_0^\infty \left\{ -\frac{ae^{-b\lambda^2_1/(2b+1)}}{(2b+1)^{3/2}} [\sqrt{2b+1} J_1(a_2^*, b_2^*) + \lambda_1 J_o(a_2^*, b_2^*)] \right\} f(s^2 \mid \sigma^2) ds^2 \quad (17)$$

$MSE\ (\tilde{\mu}_2 \mid \mu) =$

$$\frac{\sigma^2}{n_1} \int_0^\infty \{ 1 - 2a(2b+1)^{-5/2} e^{-b\lambda_1^2/(2b+1)^2} [(2b+1) J_2(a_2^*, b_2^*) + \lambda_1 (1-2b)\sqrt{2b+1}\ J_1(a_2^*, b_2^*)$$

$$-2b\lambda_1^2 J_0(a_2^*, b_2^*)] + a^2 (4b+1)^{-5/2} e^{-2b\lambda_1^2/(4b+1)} [(4b+1) J_2(a_3^*, b_3^*) + 2\lambda_1 \sqrt{4b+1} J_1(a_3^*, b_3^*)$$

$$+\lambda_1^2 J_o(a_3^*, b_3^*)]\} f(s^2 \mid \sigma^2) ds^2,$$

where

$a_3^* = (\lambda_1 - t_{\alpha/2,n-1} \frac{s}{\sigma}) / \sqrt{4b+1},\ b_3^* = (\lambda_1 + t_{\alpha/2,n} \frac{s}{\sigma}) / \sqrt{4b+1},\ a_2^* = (\lambda_1 - t_{\alpha/2,n-1} x$

$x \frac{s}{\sigma}) / \sqrt{2b+1},\ b_2^* = (\lambda_1 + t_{\alpha/2,n-1} \frac{s_1}{\sigma}) / \sqrt{2b+1},\ a_1^* = \lambda_1 - t_{\alpha/2,n-1} \frac{s}{\sigma},\ b_1^* = \lambda_1 + t_{\alpha/2,n-1},$

and $f(s^2 \mid \sigma^2)$ is the p. d .f. of s^2. If $\mu = \mu_o$, the above expressions reduce to:

$$B(\tilde{\mu}_2 \mid \mu_o) = 0, \quad (19)$$

$$MSE(\tilde{\mu} \mid \mu_o, R_1) = \frac{\sigma^2}{n} \left\{ 1 + (1-\alpha)[\frac{2a}{(2b+1)^{3/2}} - \frac{a^2}{(4b+1)^{3/2}}] - 4a\ \ t_{\alpha/2,n_1-1} \Gamma(n_1/2) / \right.$$

$$[n\sqrt{\pi(n-1)}\ \ \Gamma((n-1)/2) x\ \ x(1 + \frac{t^2_{\alpha/2,n_1-1}(4b+1)}{n-1})^{n_1/2}] + 2a^2 t_{\alpha/2,n_1-1} \Gamma\left(\frac{n}{2}\right) /$$

$$\left. [\ \sqrt{\pi(n-1)}\ \ \Gamma\left(\frac{n-1}{2}\right)(4b+1)\left(1 + \frac{t^2_{\alpha/2,n\ -1}(4b+1)}{n-1}\right)^{n_1/2} \right\}. \quad (20)$$

Remark 4: Choice of a

Proceeding in the manner as in last section, we get the minimizing value of 'a' as follows,

$$a = \bar{a}_2 = [\ \frac{(1-\alpha)}{(2b+1)^{-3/2}} - 2t_{\alpha/2,n_1-1} \Gamma\left(\frac{n}{2}\right) \Big/ [\ \sqrt{\pi(n-1)}\, \Gamma\left(\frac{n_1-1}{2}\right)(2b+1)$$

$$\left(1 + t^2_{\alpha/2,n-1}(2b+1) x\ \ x(n-1)\right)^{-n/2}]] \Big/ [\frac{(1-\alpha)}{(4b+1)^{-3/2}} + 2t_{\alpha/2,n-1} \Gamma\left(\frac{n_1}{2}\right) \Big/$$

$$[(\sqrt{\pi(n-1)}\ \ \Gamma\left(\frac{n-1}{2}\right)(4b+1) x\ \ x\left(1 + t^2_{\alpha/2,n-1}(4b+1) \big/ (n-1)\right)^{n/2}]]$$

Since

$$\frac{\partial^2}{\partial a^2}MSE(\tilde{\mu}_Y \mid \mu_o, R_1) = \frac{\sigma^2}{n}\left\{\frac{2(1-\alpha)}{(4b+1)^{3/2}} + 4t_{\alpha/2,n-1}\Gamma\left(\frac{n}{2}\right)/[\sqrt{\pi(n-1)}\ \ \Gamma\left(\frac{n-1}{2}\right)(4b+1)\right.$$

$$\left.\left(1+\frac{t^2_{\alpha/2,n-1}(4b+1)}{n-1}\right)^{n_1/2}\right\} > 0, \tag{21}$$

it follows that the minimizing value of $a \in [0,1]$ is given by:

$$\tilde{a} = \begin{cases} 0 & if \quad \bar{a}_2 \le 0 \\ \bar{a}_2 & if \quad 0 \le a_2 \le 1 \\ 1 & if \quad a_2 \ge 1 \end{cases} \tag{22}$$

The relative efficiency of $\tilde{\mu}_2$ defined by,

$$Eff(\tilde{\mu}_2 \mid \mu) = \sigma^2 / MSE(\tilde{\mu}_2 \mid \mu_0). \tag{23}$$

SIMULATION AND NUMERICAL RESULTS

To observe the behavior of the proposed testimators, and to give useful comparison between the proposed, classical, and existing estimators, the computations of relative efficiency, bias ratio, were done for the testimators $\tilde{\mu}_1$ and $\tilde{\mu}_2$. Specifically, for testimator $\tilde{\mu}_1$ these computations were done for $\alpha = 0.01,\ 0.02,\ 0.05,\ 0.01,\ 0.015,$ $b = 0.001,\ 0.01,\ 0.02,$ and $\lambda_1 = 0.0(0.1)\ 4,$ whereas for $\tilde{\mu}_2$ this was done for $\alpha = 0.01,\ 0.02,\ 0.05, b = 0.001,\ 0.01,\ n = 4(4)20$ and in order to compare our testimators $\tilde{\mu}_1$ and $\tilde{\mu}_2$ with those of Al-Hemyari *et al.*,(2009), Saxena and Singh, (2006), Kambo *et al.*, (1992), Hirano, (1977), Davis and Arnold, (1970), Arnold, (1969) and Thompson, (1968a). Some of these computations are given in Tables 1 to 3. We make the following observations from tables of computations presented in this paper:

i) The testimator $\tilde{\mu}_1$ is biased (see "Table 2"). The bias ratio is reasonably small if the prior point estimate μ_o doesn't deviate too much from the true value μ .

ii) It is observed that, for $0 \le |\lambda_1| \le 3$, $\tilde{\mu}_1$ has smaller mean squared error than the classical estimator $\overline{X}$. Thus $\tilde{\mu}_1$ may be used to improve the efficiency if λ_1 belong to the region $0 \le |\lambda_1| \le 3$.

iii) For fixed n and α, the relative efficiency of $\tilde{\mu}_1$ is increases with decreasing value of b.

iv) As expected our computations (see "Table1") showed that the relative efficiency of $\tilde{\mu}_1$ decreases with size α of the pretest region, i.e. $\alpha = 0.01$ gives higher relative efficiency than for other values of α.

v) For fixed b, and α, the relative efficiency of $\tilde{\mu}_1$ is maximum when $\lambda_1 \cong 0$, and decreases with increasing value of λ_1.

v) For any fixed α and b, the relative efficiency is a decreasing function of n when $\lambda_1 \cong 0$.

xi) The behavioural pattern of testimator $\tilde{\mu}_2$ is similar to that of $\tilde{\mu}_1$ as for relative efficiency, and bias ratio are concerned (see "Table 3").

TABLE 1: Showing $Eff(\tilde{\mu}_1 \mid \mu)$ when $b = 0.001, 0.01$, $\alpha = 0.01, 0.05, 0.1, 0.135$, and $\lambda = 0.0(0.1)2(1)3$.

b	α	0.0	0.1	0.2	0.3	0.4	0.5	0.6
0.001	0.01	8.643	7.352	6.644	6.135	5.700	5.377	4.627
	0.05	6.554	5.399	4.937	4.239	3.923	3.663	3.100
	0.1	4.893	4.503	3.847	3.893	3.185	2.964	2.700
	0.135	3.776	3.783	2.655	2.444	2.321	2.196	2.092
0.01	0.01	6.530	6.087	5.724	5.182	4.693	4.235	3.813
	0.05	5.527	5.114	4.707	4.163	3.864	3.522	2.932
	0.1	3.973	3.602	3.164	2.953	2.751	2.432	2.266
	0.135	2.915	2.794	2.725	2.635	2.492	2.297	2.150

TABLE 1 (continued): Showing $Eff(\tilde{\mu}_1 \mid \mu)$ when $b = 0.001, 0.01$, $\alpha = 0.01, 0.05, 0.1, 0.135$, and $\lambda = 0.0(0.1)2(1)3$.

b	α	0.7	0.8	0.9	1.0	1.5	2.0	2.5	3.0
0.001	0.01	4.200	3.856	3.426	2.937	1.909	1.371	1.099	1.025
	0.05	2.803	2.631	2.327	2.084	1.714	1.306	1.024	1.005
	0.1	2.724	2.573	1.985	1.765	1.499	1.214	1.019	1.002
	0.135	1.931	1.893	1.802	1.713	1.225	1.153	0.972	1.000
0.01	0.01	3.503	3.164	2.735	2.595	1.812	1.252	1.083	1.023
	0.05	2.710	2.494	2.202	1.975	1.645	1.124	1.022	1.002
	0.1	2.073	1.990	1.854	1.673	1.588	1.063	1.016	1.000
	0.135	1.908	1.796	1.633	1.432	1.213	1.048	1.003	0.957

TABLE 2: Showing $B(\tilde{\mu}_1 \mid \mu)$ when $b = 0.001, 0.01$, $\alpha = 0.01, 0.05, 0.1, 0.135$, and $\lambda = 0.0(0.1)2(1)3$.

b	α	0.0	0.1	0.2	0.3	0.4	0.5	0.6
0.001	0.01	0.000	-0.049	-0.115	-0.166	-0.202	-0.222	-0.228
	0.05	0.000	-0.036	-0.102	-0.160	-0.194	-0.219	-0.224
	0.1	0.000	-0.022	-0.085	-0.143	-0.172	-0.185	-0.195
	0.135	0.000	-0.015	-0.0537	-0.115	-0.148	-0.162	-0.173
0.01	0.01	0.000	-0.040	-0.114	-0.160	-0.201	-0.221	-0.227
	0.05	0.000	-0.035	-0.101	-0.158	-0.192	-0.210	-0.224
	0.1	0.000	-0.022	-0.083	-0.141	-0.171	-0.185	-0.194
	0.135	0.000	-0.014	-0.052	-0.114	-0.140	-0.162	-0.173

TABLE 2 (continued): Showing $B(\tilde{\mu}_1 \mid \mu)$ when
$b = 0.001, 0.01$, $\alpha = 0.01, 0.05, 0.1, 0.135$, and $\lambda = 0.0(0.1)2(1)3$.

b	α	0.7	0.8	0.9	1.0	1.5	2.0	2.5	3.0
0.001	0.01	-0.232	-0.233	-0.245	-0.251	-0.294	-0.326	-0.385	-0.431
	0.05	-0.226	-0.229	-0.235	-0.238	-0.263	-0.308	-0.362	-0.410
	0.1	-0.198	-0.204	-0.217	-0.223	-0.256	-0.294	-0.333	-0.395
	0.135	-0.188	-0.192	-0.197	-0.198	-0.236	-0.277	-0.320	-0.376
0.01	0.01	-0.230	-0.232	-0.243	-0.245	-0.292	-0.324	-0.383	-0.429
	0.05	-0.221	-0.224	-0.253	-0.236	-0.261	-0.303	-0.360	-0.407
	0.1	-0.197	-0.203	-0.216	-0.222	-0.255	-0.293	-0.331	-0.394
	0.135	-0.187	-0.191	-0.195	-0.197	-0.235	-0.276	-0.320	-0.374

TABLE 3: Showing $Eff(\tilde{\mu}_2 \mid \mu)$ when $b = 0.001, 0.01$, $\alpha = 0.01, 0.05, 0.1, 0.135$, and $\lambda = 0.0$.

b	n	α			
		0.01	0.05	0.1	0.135
0.001	4	14.782	10.304	7.562	5.315
	8	17.304	12.709	10.838	7.823
	12	17.913	14.248	12.278	8.935
	16	18.310	15.261	14.464	9.673
	20	18.582	15.855	13.451	11.138
0.01	4	14.643	10.245	7.534	5.225
	8	17.107	12.611	10.769	7.773
	12	17.700	14.278	12.185	8.862
	16	18.085	15.111	13.347	9.520
	20	18.349	15.690	14.314	11.028

CONCLUSION

Testimator $\tilde{\mu}_1$ is better than that of, Saxena and Singh, (2006), Kambo *et al*., (1992), Hirano, (1977), Davis and Arnold, (1970), Arnold, (1969) and Thompson, (1968a) both in terms of higher relative efficiency and boarder range of λ_1 for which efficiency is greater than unity and better than Al-Hemyari *et al*., (2009b) if $\lambda_1 \geq 0.4$. Also, on comparing Table 3 (for unknown σ^2 case) with the corresponding results of Saxena and Singh, (2006), Hirano, (1977), Davis and Arnold, (1970), Arnold, (1969) and Thompson, (1968a), it has been observed that $\tilde{\mu}_2$ is also much better in terms of higher relative efficiency than the existing testimators with unknown σ^2.

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