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Bayesian Estimation of the Parameter and Reliability Function of an Inverse Rayleigh Distribution

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ABSTRACT

In this paper we obtain Bayes' estimators for the unknown parameter of an Inverse Rayleigh distribution (IRD). Bayes estimators are obtained under symmetric (squared error (SE) loss) and asymmetric linear exponential loss functions using a non-informative prior. The performance of the estimators is assessed on the basis of their relative risk under the two loss functions. We also obtain the Bayes estimators of the reliability function using both symmetric as well as asymmetric loss functions and compare its performance based on a Monte Carlo simulation study. Finally, a numerical study is provided to illustrate the results.

Keywords: Bayes' estimator, LINEX loss function, Reliability function, Risk function, Root Mean Square Error (RMSE), squared error loss function.

1. INTRODUCTION

Most of the life time distributions used in reliability studies are characterized by a monotone failure rate. However, one parameter Inverse Rayleigh Distribution (IRD) with probability density function (pdf) and the reliability function is respectively given by:

$$f\left(x;\theta\right)=\frac{2}{\theta x^{3}}e^{-\frac{1}{\theta x^{2}}};\quad x>0,\ \theta>0 \tag{1}$$

and

$$R(t)=\bar{F}(t)=1-e^{-\frac{1}{\theta t^{2}}}\ ;\ t>0 \tag{2}$$

has also been used as a failure time distribution. A variance and higher order moments do not exist for this distribution. Mukherjee and Saran (1984) showed that for a given θ, the distribution is Increasing Failure Rate (IFR) or Decreasing Failure Rate (DFR) according as $t <$ or

$>1.069543/\sqrt{\theta}$. Voda (1972) studied some properties of the MLE of the parameter θ, along with the confidence intervals and tolerance limit. Mukherjee and Maiti (1996) derived percentile estimator of θ and its asymptotic efficiency. Gharraph (1993) derived five measures of the parameter of IRD and also obtained the estimators of the unknown parameter using different methods of estimation. Abdel-Monem (2003) developed some estimation and prediction results for the IRD. El-Helbawy and Abdel-Monem (2005) obtained Bayes estimators of the parameter of the IRD under four loss functions and discussed about the Bayes prediction intervals under one and two – sample plan. Dey (2005) obtained Bayes estimator of θ and reliability $R(t)$ using a vague prior under squared error loss function and also constructed the equal tail $(1-\alpha)$ credible and HPD intervals for θ and $R(t)$. Recently, Soliman *et al.* (2010) discussed about Bayesian and non-Bayesian estimation of the parameter of the IRD along with Bayesian prediction based on a lower record values.

The structure of the article is as follows: Section 2 introduces the prior distribution and loss functions used in this paper. In Section 3, we obtain the Bayes estimators of the Inverse Rayleigh parameter under two loss functions and associated risk functions of two estimators. Section 4 provides the numerical example. Section 5 offers comparisons of the Bayes' estimator of the reliability function under LINEX loss function with the estimator corresponding to the SE loss function using a Monte Carlo Simulation. This paper concludes with a brief discussion in Section 6.

2. PRIOR AND LOSS FUNCTIONS

An appropriate choice of prior(s) is indispensable for Bayesian analysis. However, very often, researchers select prior(s) according to their subjective knowledge and beliefs. Nevertheless, if we have enough information about the parameter(s) we should use informative prior(s), otherwise it is better to consider non-informative prior(s) or vague prior(s). In this paper we consider the following non-informative prior

$$g(\theta)\,\alpha\,\frac{1}{\theta},\quad \theta>0 \tag{3}$$

Taking into account the above prior, we use two different loss functions for the model (1): first is the squared error loss function (SELF) which is symmetric in nature and second is the LINEX loss function which is an asymmetric function. From Bayesian perspective, choice of loss function is an essential part in the estimation and prediction problems. Since, there is no specific analytical procedure to identify the appropriate loss function to be used; in most of the studies on estimation and prediction problems, authors for convenience consider the underlying loss function to be squared error which is symmetric in nature. However, in-discriminate use of squared error loss function is not appropriate particularly in these cases, where the losses are not symmetric. Thus in order to make the statistical inferences more practical and applicable, we often needs to choose an asymmetric loss function. A number of asymmetric loss functions proposed for use, one of the most popular is the LINEX loss function. This loss function was introduced by Varian (1975), and several others including Zellner (1986), Basu and Ebrahimi (1991), Rojo (1987) and Soliman (2000), who have used this loss function in different estimation and prediction problems. In the present work, we consider both symmetric as well as asymmetric loss functions for better comprehension of Bayesian analysis.

(a) The common squared error loss function is given by

$$L(\hat{\theta},\theta)=(\hat{\theta}-\theta)^2 \tag{4}$$

which is symmetric and $\hat{\theta}$ is an estimate of θ.

(b) The LINEX loss function is given by

$$L(\Delta)=e^{a\Delta}-a\Delta-1;\, a\neq 0 \tag{5}$$

where $\Delta=\frac{\hat{\theta}}{\theta}-1,$ where $\hat{\theta}$ is an estimate of θ.

Here a represents the shape parameter of the loss function. The behavior of the LINEX loss function changes with the choice of a. In particular, if a is close to zero, this loss function is almost equivalent to the SE loss function and therefore almost symmetric.

If we define $\Delta_1=\hat{\theta}-\theta$, then $L(\Delta_1)$ is equivalent to the loss function used by Varian (1975) and Zellner (1986).

We use LINEX loss function of the form $L(\Delta)$ for Bayes estimation of the unknown parameter of the IRD and $L(\Delta_1)$ for reliability estimation (following Basu and Ebrahimi (1991)).

3. BAYES ESTIMATION

The likelihood function of the random sample $x=(x_1,x_2,\ldots,x_n)$ is given by

$$L(x\mid\theta)=\frac{2^n}{\theta^n}\prod_{i=1}^{n}\frac{1}{x_i^3}e^{-\frac{S}{\theta}} \tag{6}$$

where $S=\sum_{i=1}^{n}\frac{1}{x_i^2}$.

Note that, $\frac{1}{X}$ follows Raleigh distribution, so $\left(\frac{1}{X}\right)^2$ obeys exponential law. It is known that a sum of independent exponentially distributed random variates, gives a Gamma distributed variable giving the probability density function of S is

$$h(s)=\frac{1}{\Gamma(n)\,\theta^n}s^{n-1}\,e^{-\frac{s}{\theta}},\quad s>0. \tag{7}$$

Bayes Estimator of θ based on Squared Error Loss Function

Combining the prior distribution (3), with the likelihood function (6), the posterior density of θ is

$$\pi(\theta\mid x)=\frac{S^n\,e^{-\frac{S}{\theta}}}{\theta^{n+1}\,\Gamma n},\quad \theta>0. \tag{8}$$

Under squared error loss function (4), the Bayes estimator of θ denoted by $\hat{\theta}_{SB}$ is the posterior mean and is given by

$$\hat{\theta}_{SB}=\left(\frac{S}{n-1}\right) \tag{9}$$

Bayes Estimator of θ based on LINEX Loss Function

Under the LINEX loss function (5), the posterior expectation of the loss function $L(\Delta)$ with respect to $\pi\left(\theta|x\right)$ in (8) is

$$E[L(\Delta)] = \int_0^\infty \{e^{a[(\frac{\hat{\theta}}{\theta})-1]} - a[(\frac{\hat{\theta}}{\theta}) \ -1]-1\}\pi(\theta \mid x)d\theta \tag{10}$$

$$= e^{-a}E[\exp\{a(\frac{\hat{\theta}}{\theta})\}] - aE[(\frac{\hat{\theta}}{\theta})-1]-1 \tag{11}$$

The value of $\hat{\theta}$ that minimizes the posterior expectation of the loss function, $L(\Delta)$, denoted by $\hat{\theta}_{LB}$ is obtained by solving equation

$$\frac{\partial E[L(\Delta)]}{\partial \hat{\theta}} = E[e^{-a}\frac{a}{\theta}\exp(a(\frac{\hat{\theta}}{\theta}))] - aE(\frac{1}{\theta}) = 0 \tag{12}$$

that is, $\hat{\theta}_{LB}$ is the solution for the equation

$$E[\frac{1}{\theta}\exp(a(\frac{\hat{\theta}_{LB}}{\theta}))] = e^{a}E(\frac{1}{\theta}) \tag{13}$$

provided that all expectation exists and are finite. Using (8) and (13), we get the optimal estimate of θ relative to $L(\Delta)$ is

$$\hat{\theta}_{LB} = \frac{S}{a}[1-e^{-\frac{a}{n+1}}] \tag{14}$$

The Risk Efficiency of $\hat{\theta}_{LB}$ with respect to $\hat{\theta}_{SB}$ under LINEX Loss $L(\Delta)$

At this point our objective is to find a risk function of the estimators $\hat{\theta}_{LB}$ and $\hat{\theta}_{SB}$ relative to $L(\Delta)$. These risk functions are denoted by $R_L\left(\hat{\theta}_{LB}\right)$ and $R_L\left(\hat{\theta}_{SB}\right)$, where subscripts L denote the risk relative to $L(\Delta)$ are given by using $h(S)$ in (7) as follows:

$$R_L(\hat{\theta}_{LB}) = E[L(\Delta)] = \int_0^\infty \{e^{a[(\frac{\hat{\theta}_{LB}}{\theta})-1]} - a[(\frac{\hat{\theta}_{LB}}{\theta})-1]-1\}h(s)ds$$

$$= e^{-\frac{a}{n+1}} - n\left(1-e^{-\frac{a}{n+1}}\right) + a - 1 \qquad (15)$$

In the same manner, we get

$$R_L\left(\hat{\theta}_{SB}\right) = E\left[L(\Delta)\right] = \int_0^\infty \left\{e^{a\left[\left(\frac{\hat{\theta}_{SB}}{\theta}\right)-1\right]} - a\left[\left(\frac{\hat{\theta}_{SB}}{\theta}\right)^2 - 1\right] - 1\right\} h(s)ds$$

$$= e^{-a}\left(1-\frac{a}{n-1}\right)^{-n} - \frac{a}{n-1} - 1 \qquad (16)$$

The risk efficiency of $\hat{\theta}_{LB}$ with respect to $\hat{\theta}_{SB}$ under LINEX Loss $L(\Delta)$ can be defined as follows:

$$RE_L(\hat{\theta}_{LB}, \hat{\theta}_{SB}) = \frac{R_L(\hat{\theta}_{SB})}{R_L(\hat{\theta}_{LB})} \qquad (17)$$

The Risk Efficiency of $\hat{\theta}_{LB}$ with respect to $\hat{\theta}_{SB}$ under Squared Error Loss Function

The risk functions of the estimators $\hat{\theta}_{LB}$ and $\hat{\theta}_{SB}$ under squared error loss are denoted by $R_S(\hat{\theta}_{LB})$ and $R_S(\hat{\theta}_{SB})$ are given by

$$R_S(\hat{\theta}_{LB}) = \int_0^\infty (\hat{\theta}_{LB} - \theta)^2 h(s)ds$$

Thus,

$$R_S(\hat{\theta}_{LB}) = \theta^2[\frac{n(n+1)}{a^2}(1-e^{-\frac{a}{n+1}})^2 - \frac{2n}{a}(1-e^{-\frac{a}{n+1}}) + 1] \qquad (18)$$

and

$$R_S(\hat{\theta}_{SB}) = \int_0^{\infty} (\hat{\theta}_{SB} - \theta)^2 h(s) ds$$

Thus,

$$R_S(\hat{\theta}_{SB}) = \theta^2 [\frac{n(n+1)}{(n-1)^2} - \frac{2n}{n-1} + 1] \qquad (19)$$

The risk efficiency of $\hat{\theta}_{LB}$ with respect to $\hat{\theta}_{SB}$ under squared error loss is defined as

$$RE_S(\hat{\theta}_{LB}, \hat{\theta}_{SB}) = \frac{R_S(\hat{\theta}_{SB})}{R_S(\hat{\theta}_{LB})} \qquad (20)$$

4. NUMERICAL EXAMPLE

To compare the proposed estimator $\hat{\theta}_{LB}$ with the estimator $\hat{\theta}_{SB}$, the risk functions are computed to see whether $\hat{\theta}_{LB}$ out performs $\hat{\theta}_{SB}$ under LINEX loss $L(\Delta)$ and how $\hat{\theta}_{LB}$ performs as compared to $\hat{\theta}_{SB}$ when true loss is squared error. A comparison of this type is needed to check whether an estimator is inadmissible under some loss function. If it is so, the estimator would not be used for the losses specified by that loss function. For this purpose the risks of the estimators and risk efficiency have been computed.

We generated N = 500 samples of sizes $n = 10, 20, 30$ from (1) with $\theta = 1$. From Tables 1 to 6, shows that the risk efficiency $RE_L(\hat{\theta}_{LB}, \hat{\theta}_{SB})$ are greater than one for the sample sizes $n = 10, 20, 30$ and for all values of 'a' (a = ± 0.5, ±1, ±2) which indicates that the proposed estimators $\hat{\theta}_{LB}$ is preferable to $\hat{\theta}_{SB}$. In addition to that, the risk efficiencies $RE_L(\hat{\theta}_{LB}, \hat{\theta}_{SB})$ are greater than the risk efficiencies $RE_S(\hat{\theta}_{LB}, \hat{\theta}_{SB})$ for all positive values of 'a' and for all sample sizes $n = 10, 20, 30$ but the risk efficiency $RE_L(\hat{\theta}_{LB}, \hat{\theta}_{SB})$ is less than the risk efficiency $RE_S(\hat{\theta}_{LB}, \hat{\theta}_{SB})$ for all negative values of 'a' and sample sizes $n = 10, 20, 30$.

TABLE 1: The Estimators $\hat{\theta}_{LB}$, $\hat{\theta}_{SB}$, the risk efficiencies $RE_L(\hat{\theta}_{LB},\hat{\theta}_{SB})$ and $RE_S(\hat{\theta}_{LB},\hat{\theta}_{SB})$ under the prior $g(\theta)$ for the values of $a=2$

n	$\hat{\theta}_{SB}$	$\hat{\theta}_{LB}$	$R_L(\hat{\theta}_{LB})$	$R_L(\hat{\theta}_{SB})$	$R_S(\hat{\theta}_{LB})$	$R_S(\hat{\theta}_{SB})$	$RE_L(\hat{\theta}_{LB},\hat{\theta}_{SB})$	$RE_S(\hat{\theta}_{LB},\hat{\theta}_{SB})$
10	1.1042	.8261	.1713	.4483	.0973	.1358	2.6170	1.3928
20	1.0500	.9062	.0923	.1464	.0496	.0582	1.5861	1.1734
30	1.0110	.9159	.0631	.0856	.0332	.0369	1.3566	1.1114

TABLE 2: The Estimators $\hat{\theta}_{LB}$, $\hat{\theta}_{SB}$, the risk efficiencies $RE_L(\hat{\theta}_{LB},\hat{\theta}_{SB})$ and $RE_S(\hat{\theta}_{LB},\hat{\theta}_{SB})$ under the prior $g(\theta)$ for the values of $a=1$

n	$\hat{\theta}_{SB}$	$\hat{\theta}_{LB}$	$R_L(\hat{\theta}_{LB})$	$R_L(\hat{\theta}_{SB})$	$R_S(\hat{\theta}_{LB})$	$R_S(\hat{\theta}_{SB})$	$RE_L(\hat{\theta}_{LB},\hat{\theta}_{SB})$	$RE_S(\hat{\theta}_{LB},\hat{\theta}_{SB})$
10	1.1146	.8718	.0441	.0835	.0927	.1358	1.8934	1.4649
20	1.0434	.9219	.0234	.0321	.0481	.0582	1.3718	1.2099
30	1.0240	.9427	.0160	.0197	.0325	.0369	1.2319	1.1354

TABLE 3: The Estimators $\hat{\theta}_{LB}$, $\hat{\theta}_{SB}$, the risk efficiencies $RE_L(\hat{\theta}_{LB},\hat{\theta}_{SB})$ and $RE_S(\hat{\theta}_{LB},\hat{\theta}_{SB})$ under the prior $g(\theta)$ for the values of $a=0.5$

n	$\hat{\theta}_{SB}$	$\hat{\theta}_{LB}$	$R_L(\hat{\theta}_{LB})$	$R_L(\hat{\theta}_{SB})$	$R_S(\hat{\theta}_{LB})$	$R_S(\hat{\theta}_{SB})$	$RE_L(\hat{\theta}_{LB},\hat{\theta}_{SB})$	$RE_S(\hat{\theta}_{LB},\hat{\theta}_{SB})$
10	1.0947	.8756	.0112	.0186	.0914	.1358	1.6607	1.4858
20	1.0288	.9198	.0059	.0076	.0477	.0582	1.2881	1.2201
30	1.0053	.9329	.0040	.0047	.0323	.0369	1.1750	1.1424

TABLE 4: The Estimators $\hat{\theta}_{LB}$, $\hat{\theta}_{SB}$, the risk efficiencies $RE_L(\hat{\theta}_{LB},\hat{\theta}_{SB})$ and $RE_S(\hat{\theta}_{LB},\hat{\theta}_{SB})$ under the prior $g(\theta)$ for the values of $a=-0.5$

n	$\hat{\theta}_{SB}$	$\hat{\theta}_{LB}$	$R_L(\hat{\theta}_{LB})$	$R_L(\hat{\theta}_{SB})$	$R_S(\hat{\theta}_{LB})$	$R_S(\hat{\theta}_{SB})$	$RE_L(\hat{\theta}_{LB},\hat{\theta}_{SB})$	$RE_S(\hat{\theta}_{LB},\hat{\theta}_{SB})$
10	1.1039	.9241	.0115	.0157	.0914	.1358	1.3652	1.4858
20	1.0399	.9522	.0059	.0069	.0477	.0582	1.1695	1.2201
30	1.0213	.9632	.0041	.0045	.0323	.0369	1.0976	1.1424

TABLE 5: The Estimators $\hat{\theta}_{LB}$, $\hat{\theta}_{SB}$, the risk efficiencies $RE_L(\hat{\theta}_{LB},\hat{\theta}_{SB})$ and $RE_S(\hat{\theta}_{LB},\hat{\theta}_{SB})$ under the prior $g(\theta)$ for the values of $a=-1$

n	$\hat{\theta}_{SB}$	$\hat{\theta}_{LB}$	$R_L(\hat{\theta}_{LB})$	$R_L(\hat{\theta}_{SB})$	$R_S(\hat{\theta}_{LB})$	$R_S(\hat{\theta}_{SB})$	$RE_L(\hat{\theta}_{LB},\hat{\theta}_{SB})$	$RE_S(\hat{\theta}_{LB},\hat{\theta}_{SB})$
10	1.1404	.9767	.0469	.0589	.0929	.1358	1.2559	1.4618
20	1.0690	.9906	.0242	.0271	.0482	.0582	1.1198	1.2075
30	1.0518	.9999	.0163	.0176	.0325	.0369	1.0797	1.1354

TABLE 6: The Estimators $\hat{\theta}_{LB}$, $\hat{\theta}_{SB}$, the risk efficiencies $RE_L(\hat{\theta}_{LB},\hat{\theta}_{SB})$ and $RE_S(\hat{\theta}_{LB},\hat{\theta}_{SB})$ under the prior $g(\theta)$ for the values of $a=-2$

n	$\hat{\theta}_{SB}$	$\hat{\theta}_{LB}$	$R_L(\hat{\theta}_{LB})$	$R_L(\hat{\theta}_{SB})$	$R_S(\hat{\theta}_{LB})$	$R_S(\hat{\theta}_{SB})$	$RE_L(\hat{\theta}_{LB},\hat{\theta}_{SB})$	$RE_S(\hat{\theta}_{LB},\hat{\theta}_{SB})$
10	1.1241	1.009	.1934	.2155	.0994	.1358	1.1143	1.3662
20	1.0455	.9924	.0983	.1036	.0499	.0582	1.0539	1.1663
30	1.0220	.9875	.0659	.0682	.0333	.0368	1.0349	1.1051

5. BAYES ESTIMATOR OF $\overline{F(t)}$

Let $\gamma = \overline{F(t)}$ be the probability that a system will survive a specified mission time t. By substituting $\theta = \frac{1}{-t^2 \log(1-\gamma)}$ in (9), we obtain the posterior pdf of γ as the following

$$\pi_1(\gamma|x) = \frac{(st^2)^n}{\Gamma n}(1-\gamma)^{st^2-1}(-\log(1-\gamma))^{n-1}, \quad 0<\gamma<1 \qquad (21)$$

Using the convex loss function, $L(\Delta_1)$, where $\Delta_1 = \hat{\gamma}-\gamma$. The loss function is quite asymmetric when $a=1$ with overestimation rather than underestimation. When $a<0$, $L(\Delta_1)$ rises almost exponentially when $\Delta_1 < 0$, and almost linearly when $\Delta_1 > 0$. For small values of $|a|$ the loss function $L(\Delta_1)$ is approximately squared error loss and therefore almost symmetric.

The posterior expectation of the LINEX loss function $L(\Delta_1)$ is

$$E_{\gamma_{LB}}[L(\Delta_1)] = \int_0^1 [e^{a(\hat{\gamma}-\gamma)} - a(\hat{\gamma}-\gamma)-1] \ \pi_1(\gamma|x)d\gamma \qquad (22)$$

The Bayes estimate of γ relative to $L(\Delta_1)$, denoted by $\hat{\gamma}_{LB}$ is given by

$$\hat{\gamma}_{LB} = -\frac{1}{a}\log\left[\sum_{j=0}^{a}\frac{(-a)^j}{j!}\sum_{j=0}^{a}\frac{a^j}{j!}(1+\frac{j}{st^2})^{-n}\right] \qquad (23)$$

Under squared error loss function, $(L(\hat{\gamma}, \gamma) = (\hat{\gamma} - \gamma)^2)$, the Bayes estimator of γ, denoted by $\hat{\gamma}_{BS}$ is the posterior mean and is given by

$$\hat{\gamma}_{SB} = 1 - \frac{1}{(1 + \frac{1}{st^2})^n} \tag{24}$$

Therefore, the risk functions relative to loss function $L(\Delta_1)$ do not exist.

Using the same set of data in Section 4 and taking $t = 1$, we compute $\hat{\gamma}_{LB}$ and $\hat{\gamma}_{BS}$, the results with the corresponding values of 'a' and 'n' are given in Table 7. Entries within parentheses represent the corresponding RMSE.

From Table 7, the Bayes estimator of reliability function for positive value of 'a' relative to $L(\Delta_1)$ are lower while the negative values of 'a' are greater than the corresponding Bayes estimator under squared error loss. Additional to that, Bayes estimator of reliability functions relative to $L(\Delta_1)$ have greater RMSE than the corresponding Bayes estimator under squared error loss. Most situation involving reliability estimation and estimation of failure time, asymmetric loss functions are more appropriate than squared error loss function.

TABLE 7: Bayes estimation of reliability function based on LINEX Loss $(\hat{\gamma}_{LB})$ and squared error loss $(\hat{\gamma}_{BS})$ for variation in 'a' (for $t = 1$, the true value $R(t)_{t=1} = 0.6321$)

n	a	$\hat{\gamma}_{LB}$	$\hat{\gamma}_{BS}$
10	2	.6220(.0130)	.6332(.0062)
20	2	.6278(.0076)	.6353(.0058)
30	2	.6364(.0057)	.6399(.0051)
10	1	.6245(.0111)	.6305(.0059)
20	1	.6316(.0062)	.6348(.0057)
30	1	.6340(.0054)	.6362(.0050)
10	.5	.6334(.0064)	.6364(.0050)
20	.5	.6401(.0054)	.6417(.0047)
30	.5	.6421(.0051)	.6432(.0047)
10	-.5	.6360(.0059)	.6330(.0054)
20	-.5	.6369(.0058)	.6353(.0051)
30	-.5	.6385(.0053)	.6374(.0047)
10	-1	.6279(.0079)	.6214(.0055)
20	-1	.6289(.0072)	.6257(.0052)

TABLE 7 (continued): Bayes estimation of reliability function based on LINEX Loss $(\hat{\gamma}_{LB})$ and squared error loss $(\hat{\gamma}_{BS})$ for variation in 'a' (for $t = 1$, the true value $R(t)_{t=1} = 0.6321$)

n	a	$\hat{\gamma}_{LB}$	$\hat{\gamma}_{BS}$
30	-1	.6305(.0056)	.6284(.0049)
10	-2	.6379(.0123)	.6263(.0058)
20	-2	.6398(.0100)	.6336(.0054)
30	-2	.6419(.0068)	.6377(.0042)

6. DISCUSSION

In this paper we discussed Bayesian estimation of parameter and reliability function of one-parameter Inverse Rayleigh distribution using two different loss functions: the SE loss function and the LINEX loss function. From the simulation study we observed that Bayes estimators under the LINEX loss function exhibit better performance than the SE loss function. In the case of reliability function, it is worth mentioning that estimation under the LINEX loss function is superior to the SE loss function with respect to the root mean squared error measure.

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