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Curvature Identities Special Generalized Manifolds Kenmotsu Second Kind

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ABSTRACT

In this paper, we obtain an analytical expression of the structural tensor, brought additional identities curvature of special generalized manifolds Kenmotsu type II and based on them are highlighted in some of the classes of this class of manifolds and obtain a local characterization of the selected classes.

Keywords: Riemann curvature tensor, special generalized manifolds Kenmotsu type II, flat manifolds, curvature additional identity.

1. INTRODUCTION

The notions of almost contact structures and almost contact metric manifolds introduced by Gray, 1959. A careful analysis is subjected to special classes of almost contact metric manifolds. In 1972 Kenmotsu, 1972 introduced the class of almost contact metric structures, characterized by the identity $\nabla_X(\Phi)Y = \langle \Phi X, Y\rangle\xi - \eta(Y)\Phi X; X, Y \in \mathcal{X}(M)$. Kenmotsu structure, for example, arise naturally in the classification Tanno, 1969 connected almost contact metric manifolds whose automorphism group has maximal dimension . They have a number of remarkable properties. For example, the structure Kenmotsu normality and integrability. They are not a contact structure, and hence Sasakian. There are examples of Kenmotsu structures on odd Lobachevsky spaces of curvature (-1). Such structures are obtained by the construction of the warped product $\boldsymbol{R} \times_f \boldsymbol{C}^n$ in the sense of Bishop and O'Neill, 1969 complex Euclidean space and the real line, where $f(t) = ce^t$ (see Kenmotsu, 1972).

Kenmotsu varieties studied by many authors, such as Bagewadi and Venkatesha, 2007; De and Pathak, 2004 and Pitis, 2007 and many others. But we would like to acknowledge the work of Kirichenko, 2001. In this paper it is proved that the class of manifolds Kenmotsu coincides with the class of almost contact metric manifolds derived from cosymplectic manifolds canonical transformation concircular cosymplectic structure. Further Umnova, 2002 studied Kenmotsu manifolds and their generalizations. The author identified class of almost contact metric manifolds, which is a generalization of manifolds Kenmotsu and named class of generalized (in short, *GK*-) Kenmotsu manifolds. In Behzad and Niloufar, 2013, this class of manifolds is called the class of nearly Kenmotsu manifolds. The authors prove that the second-order symmetric closed recurrent tensor recurrence covector which annihilates the characteristic vector ξ, a multiple of the metric tensor g. In addition, the authors examine the Φ-recurrent nearly Kenmotsu manifolds. It is proved that Φ-recurrent nearly Kenmotsu manifold is Einstein and locally Φ-recurrent nearly Kenmotsu manifold is a manifold of constant curvature -1. In Umnova, 2002 identifies two subclasses of generalized manifolds Kenmotsu called special generalized manifolds Kenmotsu (briefly, *SGK*-) I and type II. In Tshikuna-Matamba, 2012 *SGK*-manifolds of type II are called nearly Kenmotsu manifolds. In Umnova, 2002 it is proved that the generalized manifolds of constant curvature Kenmotsu are Kenmotsu manifolds of constant curvature -1. Moreover, it is proved that the class of SGK-manifolds of type II coincides with the class of almost contact metric manifolds, obtained from the closely cosymplectic manifolds canonical transformation closely cosymplectic structure, and given the local structure of these manifolds of constant curvature.

This paper is organized as follows. In Section 2 we present the preliminary information needed in the sequel, we construct the space of the associated *G*-structure. In Section 3 we give the definition of generalized manifolds Kenmotsu and *SGK*-manifolds of type II, we give a complete group of structure equations, the components of Riemann-Christoffel tensor, the Ricci tensor and the scalar curvature in the space of the associated *G*-structure *SGK*-manifolds of type II. In Section 4, we study the properties of the structure tensors *SGK*-manifold of type II. In Section 5, we discuss some identities satisfied by the Riemann curvature tensor *SGK*-manifolds of type

II. On the basis of identities allocated classes *SGK*-manifolds of type II and obtained a local characterization of the classes. The results obtained in sections 4 and 5 are the main results.

2. PRELIMINARIES

Let M – smooth manifold of dimension $2n+1, \mathcal{X}(M)$ – C^{∞}-module of smooth vector fields on *M*. In what follows, all manifolds, tensor fields, etc. objects are assumed to be smooth of class C^{∞}.

Definition 2.1 (Kirichenko, 2013): Almost contact structure on a manifold *M* is a triple (η, ξ, Φ) of tensor fields on the manifold, where η – differential 1-form, called the contact form of the structure, ξ – vector field, called the characteristic, Φ – endomorphism of $\mathcal{X}(M)$ called the structure endomorphism.

In this

$$1)\ \eta(\xi)=1;\ \ 2)\ \eta\circ\Phi=0;\ \ 3)\ \Phi(\xi)=0;\ \ 4)\ \Phi^2=-id+\eta\otimes\xi. \qquad (1)$$

If, moreover, *M* is fixed Riemannian structure $g=<\cdot,\cdot>$, such that

$$\langle\Phi X,\Phi Y\rangle=\langle X,Y\rangle-\eta(X)\eta(Y),\ \ X,Y\in\mathcal{X}(M), \qquad (2)$$

four $(\eta,\xi,\Phi,g=<\cdot,\cdot>)$ is called an almost contact metric (shorter, *AC*-) structure. Manifold with a fixed almost contact (metric) structure is called an almost contact (metric (shorter, *AC*-)) manifold.

Skew-symmetric tensor $\Omega(X,Y)=\langle X,\Phi Y\rangle, X,Y\in\mathcal{X}(M)$ called the fundamental form of *AC*-structure (Kirichenko, 2013).

Let (η,ξ,Φ,g) – almost contact metric structure on the manifold M^{2n+1}. In the module $\mathcal{X}(M)$ internally defined two complementary projectors $m=\eta\otimes\xi$ and $\ell=id-m=-\Phi^2$ (Kirichenko and Rustanov, 2002); thus $\mathcal{X}(M)=\mathcal{L}\oplus\mathcal{M}$, where $\mathcal{L}=Im(\Phi)=ker\eta$ – the so-called contact distribution, $dim\mathcal{L}=2n, \mathcal{M}=Im m=ker(\Phi)=L(\xi)$ – linear hull of structural vector (wherein ℓ and m are the projections onto the submodules $\mathcal{L},\mathcal{M}$, respectively). Clearly, the distribution of the $\mathcal{L}$ and $\mathcal{M}$ are invariant with respect to Φ and mutually orthogonal. It is also obvious that $\widetilde{\Phi}^2=-id, \langle\widetilde{\Phi}X,\widetilde{\Phi}Y\rangle=\langle X,Y\rangle, X,Y\in\mathcal{X}(M)$, where $\widetilde{\Phi}=\Phi|\mathcal{L}$.

Consequently, $\{\widetilde{\Phi}_p, g_p|\mathcal{L}\}$ – Hermitian structure on the space $\mathcal{L}_p$. Complexification $\mathcal{X}(M)^{\mathbf{C}}$ module $\mathcal{X}(M)$ is the direct sum $\mathcal{X}(M)^{\mathbf{C}} = D_\Phi^{\sqrt{-1}} \oplus D_\Phi^{-\sqrt{-1}} \oplus D_\Phi^0$ eigensubspaces of structural endomorphism Φ, corresponding to the eigenvalues $\sqrt{-1}, -\sqrt{-1}$ and 0, respectively. Projectors on the terms of this direct sum will be, respectively, the endomorphisms Gray, 1959 $\pi = \sigma \circ \ell = -\frac{1}{2}\left(\Phi^2 + \sqrt{-1}\Phi\right), \bar{\pi} = \bar{\sigma} \circ \ell = \frac{1}{2}\left(-\Phi^2 + \sqrt{-1}\Phi\right),\ m = id + \Phi^2$ where $\sigma = \frac{1}{2}\left(id - \sqrt{-1}\Phi\right), \bar{\sigma} = \frac{1}{2}\left(id + \sqrt{-1}\Phi\right)$.

The mappings $\sigma_p : \mathcal{L}_p \longrightarrow D_\Phi^{\sqrt{-1}}$ and $\bar{\sigma}_p : \mathcal{L}_p \longrightarrow D_\Phi^{-\sqrt{-1}}$ are isomorphism and anti-isomorphism, respectively, Hermitian spaces. Therefore, each point $p \in M^{2n+1}$ may be connected space frames family $T_p(M)^C$ of the form $(p, \varepsilon_0, \varepsilon_1, \dots, \varepsilon_n, \varepsilon_{\hat{1}}, \dots, \varepsilon_{\hat{n}})$, where $\varepsilon_a = \sqrt{2}\sigma_p(e_a), \varepsilon_{\hat{a}} = \sqrt{2}\bar{\sigma}_p(e_a), \varepsilon_0 = \xi_p$; where $\{e_a\}$ – orthonormal basis of Hermitian space $\mathcal{L}_p$. Such a frame called A-frame (Kirichenko and Rustanov, 2002). It is easy to see that the matrix components of tensors Φ_p and g_p in an A-frame have the form, respectively:

$$\left(\Phi_j^i\right) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sqrt{-1}I_n & 0 \\ 0 & 0 & -\sqrt{-1}I_n \end{pmatrix}, \left(g_{ij}\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & I_n \\ 0 & I_n & 0 \end{pmatrix}, \tag{3}$$

where I_n – identity matrix of order *n*. It is well known (Kirichenko, 2013) that the set of such frames defines a *G*-structure on *M* with structure group $\{1\} \times U(n)$, represented by matrices of the form $\begin{pmatrix} 1 & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & A \end{pmatrix}$, where $A \in U(n)$. This structure is called a *G*-connected (Kirichenko, 2013); Kirichenko and Rustanov, 2002).

3. SPECIALLY GENERALIZED MANIFOLDS KENMOTSU TYPE II

Let $(M^{2n+1}, \Phi, \xi, \eta, g = \langle\cdot,\cdot\rangle)$ – almost contact metric manifold.

Definition 3.1. (Abu-Sleem and Rustanov, 2015;Umnova, 2002): Class of almost contact metric manifolds, characterized by the identity

$$\nabla_X(\Phi)Y + \nabla_Y(\Phi)X = -\eta(Y)\Phi X - \eta(X)\Phi Y; X, Y \in \mathcal{X}(M), \qquad (4)$$

called generalized manifolds Kenmotsu (shorter, *GK*-manifolds).

Definition 3.2. (Abu-Sleem and Rustanov, 2015;Umnova, 2002):*GK*-manifold with a closed contact form, i.e. $d\eta = 0$ are called *SGK*-manifolds of type II.

This class of manifolds in (Tshikuna-Matamba, 2012) is a class of nearly Kenmotsu manifolds. We will refer to these manifolds, as in Umnova, 2002 special generalized manifolds Kenmotsu type II, and briefly *SGK*-manifolds of type II.

In terms of the structure tensors Kirichenko, 2013, Definition 2 can be summarized as follows.

Definition 3.3. (Umnova, 2002) : *GK*-manifolds for which $F^{ab} = F_{ab} = 0$ are called *SGK*-manifolds of type II.

Let *M* – *SGK*-manifolds of type II. Then the first group of structure equations *SGK*-manifolds of type II takes the form (Abu-Sleem and Rustanov, 2015;Umnova, 2002).

$$1)\ d\omega = 0;\ \ 2)\ d\omega^a = -\theta^a_b \wedge \omega^b + C^{abc}\omega_b \wedge \omega_c + \delta^a_b \omega \wedge \omega^b;\ \ 3)\ d\omega_a = \theta^b_a \wedge \omega_b + C_{abc}\omega^b \wedge \omega^c + \delta^b_a \omega \wedge \omega_b, \qquad (5)$$

where

$$C^{abc} = \frac{\sqrt{-1}}{2}\Phi^a_{\hat{b},\hat{c}};\quad C_{abc} = -\frac{\sqrt{-1}}{2}\Phi^{\hat{a}}_{b,c};\quad C^{[abc]} = C^{abc};\quad C_{[abc]} = C_{abc};\quad \overline{C^{abc}} = C_{abc}. \qquad (6)$$

Theorem 2.1 in Abu-Sleem and Rustanov, 2015 takes the form:

Theorem 3.1. Full group of structure equations *SGK*-manifolds of type II in the space of the associated *G*-structure has the form:

$$1)\ d\omega = 0;\ \ 2)\ d\omega^a = -\theta^a_b \wedge \omega^b + C^{abc}\omega_b \wedge \omega_c + \delta^a_b\omega \wedge \omega^b;\ \ 3)\ d\omega_a = \theta^b_a \wedge \omega_b + C_{abc}\omega^b \wedge \omega^c + \delta^b_a\omega \wedge \omega_b;\ \ 4)\ d\theta^a_b = -\theta^a_c \wedge \theta^c_b + \left(A^{ad}_{bc} - 2C^{adh}C_{hbc}\right)\omega^c \wedge \omega_d;\ \ 5)\ dC^{abc} + C^{dbc}\theta^a_d + C^{adc}\theta^b_d + C^{abd}\theta^c_d = C^{abcd}\omega_d - C^{abc}\omega;\ \ 6)\ dC_{abc} - C_{dbc}\theta^d_a - C_{adc}\theta^d_b - C_{abd}\theta^d_c = C_{abcd}\omega^d - C_{abc}\omega. \qquad (7)$$

where

$$A^{ad}_{[bc]} = A^{[ad]}_{bc} = 0, C^{a[bcd]} = 0, C_{a[bcd]} = 0. \quad (8)$$

Taking the exterior derivative of (7(4)), we obtain:

$$dA^{ad}_{bc} + A^{hd}_{bc}\theta^a_h + A^{ah}_{bc}\theta^d_h - A^{ad}_{hc}\theta^h_b - A^{ad}_{bh}\theta^h_c = A^{ad}_{bch}\omega^h + A^{adh}_{bc}\omega_h - 2A^{ad}_{bc}\omega, \quad (9)$$

where

$$1)\ A^{ad}_{b[ch]} = 0;\ \ 2)\ A^{a[dh]}_{bc} = 0;\ \ 3)\ \left(A^{ag}_{b[c} - 2C^{agf}C_{fb[c}\right)C_{|g|dh]} = 0;\ \ 4)\ \left(A^{a[c}_{bg} - 2C^{a[c|f|}C_{fbg}\right)C^{|g|dh]} = 0. \quad (10)$$

Theorems 2.3, 2.4 and 2.5 of Abu-Sleem and Rustanov, 2015 take the form:

Theorem 3.2. Nonzero essential components of Riemann-Christoffel tensor in the space of the associated *G*-structure are of the form:

$$1)\ R^a_{\hat{b}cd} = 2C^{abh}C_{hcd} - 2\delta^a_{[c}\delta^b_{d]};\ \ 2)\ R^a_{\hat{b}\hat{c}\hat{d}} = -2C^{ab[cd]};\ \ 3)\ R^a_{00b} = \delta^a_b;\ \ 4)\ R^a_{bc\hat{d}} = A^{ad}_{bc} - C^{adh}C_{hbc} - \delta^a_c\delta^d_b. \quad (11)$$

Theorem 3.3. Covariant components of the Ricci tensor *SGK*-manifolds II in space of the associated *G*-structure are of the form:

$$1)\ S_{00} = -2n;\ \ 2)\ S_{a\hat{b}} = S_{\hat{b}a} = A^{bc}_{ac} - 3C_{acd}C^{dcb} - \delta^b_a, \quad (12)$$

the other components are zero.

Theorem 3.4. Scalar curvature χ of *SGK*-manifold of type II in the space of the associated G-structure is calculated by the formula

$$\chi = g^{ij}S_{ij} = 2A^{ab}_{ab} - 6C_{acd}C^{dcb} - 2n. \quad (13)$$

4. STRUCTURAL TENSORS *SGK*-MANIFOLDS OF TYPE II

Consider the family of functions $C = \{C^i{}_{jk}\}$; $C^a{}_{\hat{b}\hat{c}} = C^{abc}$; $C^{\hat{a}}{}_{bc} = C_{abc}$; all other components of the family C – zero. We show that this system functions on the space of the associated *G*-structure globally defined tensor of type (2,1) on *M*. In fact, since $\nabla\Phi$ – tensor of type (2,1), by the fundamental theorem of tensor analysis $\nabla\Phi^i_{j,k} = d\Phi^i_{j,k} + \Phi^l_{j,k}\theta^i_l - \Phi^i_{l,k}\theta^l_j - \Phi^i_{j,l}\theta^l_k \equiv 0\ (mod\ \omega^k)$. In particular, $\nabla\Phi^a_{\hat{b},\hat{c}} = d\Phi^a_{\hat{b},\hat{c}} + \Phi^d_{\hat{b},\hat{c}}\theta^a_d - \Phi^a_{\hat{d},\hat{c}}\theta^{\hat{d}}_{\hat{b}} - \Phi^a_{\hat{b},\hat{d}}\theta^{\hat{d}}_{\hat{c}} \equiv 0\ (mod\ \omega^k)$. In view of (6), these relations can be rewritten as $\nabla C^a{}_{\hat{b}\hat{c}} = dC^{abc} + C^{dbc}\theta^a_d + C^{adc}\theta^b_d + C^{abd}\theta^c_d \equiv 0\ (mod\ \omega^k)$. Similarly, $\nabla C^{\hat{a}}{}_{bc} = dC_{abc} - C_{dbc}\theta^d_a - C_{adc}\theta^d_b - C_{abd}\theta^d_c \equiv 0\ (mod\ \omega^k)$.

Consequently, $\nabla C^i{}_{jk} \equiv 0\ (mod\ \omega^k)$. By the fundamental theorem of tensor analysis family of C defines a real tensor field of type (2,1) on M, which we denote by the same symbol. This tensor defines a mapping $C: \mathcal{X}(M) \times \mathcal{X}(M) \longrightarrow \mathcal{X}(M)$, defined by the formula $C(X,Y) = C_{abc}X^bY^c\varepsilon_{\hat{a}} + C^{abc}X_bY_c\varepsilon_a$, where $\{C_{abc}, C^{abc}\}$ – components of the structure tensor *SGK*-manifold of type II. This tensor of type (2,1) is called a composition and determines the module $\mathcal{X}(M)$ the structure of the *Q*-algebra (Kirkchenko, 1983;Kirichenko, 1986). Map *C* has the following properties:

$$1)\ C(\xi,X) = C(X,\xi) = 0;\ \ 2)\ C(X,Y) = -C(Y,X);\ \ 3)\ C(\Phi X,Y) = C(X,\Phi Y) = -\Phi\circ C(X,Y);\ \ 4)\ \eta\circ C(X,Y) = 0;\ \forall X,Y,Z \in \mathcal{X}(M). \quad (14)$$

In fact:
1) Since $\xi^a = \xi_a = 0$, then $C(\xi,X) = C_{abc}\xi^bX^c\varepsilon_{\hat{a}} + C^{abc}\xi_bX_c\varepsilon_a = 0$. Similarly, $C(X,\xi) = C_{abc}X^b\xi^c\varepsilon_{\hat{a}} + C^{abc}X_b\xi_c\varepsilon_a = 0$.

2) In view of (6(3)), (6(4)) we have

$$C(X,Y) = C_{abc}X^bY^c\varepsilon_{\hat{a}} + C^{abc}X_bY_c\varepsilon_a = C_{acb}X^bY^c\varepsilon_{\hat{a}} - C^{acb}X_bY_c\varepsilon_a = -C(Y,X).$$

3) $C(\Phi X,Y) = C_{abc}(\Phi X)^bY^c\varepsilon_{\hat{a}} + C^{abc}(\Phi X)_bY_c\varepsilon_a = \sqrt{-1}C_{abc}X^bY^c\varepsilon_{\hat{a}} - \sqrt{-1}C^{abc}X_bY_c\varepsilon_a = C_{abc}X^b\left(\sqrt{-1}Y\right)^c\varepsilon_{\hat{a}} + C^{abc}X_b\left(-\sqrt{-1}Y\right)_c\varepsilon_a = C_{abc}X^b(\Phi Y)^c\varepsilon_{\hat{a}} + C^{abc}X_b(\Phi Y)_c\varepsilon_a = C(X,\Phi Y)$, and also $\Phi\circ C(X,Y) = \Phi\circ\left(C_{abc}X^bY^c\varepsilon_{\hat{a}} + C^{abc}X_bY_c\varepsilon_a\right) = C_{abc}X^bY^c\Phi(\varepsilon_{\hat{a}}) + C^{abc}X_bY_c\Phi(\varepsilon_a) = -\sqrt{-1}C_{abc}X^bY^c\varepsilon_{\hat{a}} + \sqrt{-1}C^{abc}X_bY_c\varepsilon_a = -\left(C_{abc}\left(\sqrt{-1}X\right)^bY^c\varepsilon_{\hat{a}} + C^{abc}\left(-\sqrt{-1}X\right)_bY_c\varepsilon_a\right) = C(\Phi X,Y)$.

Using the properties of the tensor C, we find an explicit analytic expression for this tensor. By definition, we have: $[C(\varepsilon_{\hat{b}},\varepsilon_{\hat{c}})]^a\varepsilon_a = C^a{}_{\hat{b}\hat{c}}\varepsilon_a = \frac{\sqrt{-1}}{2}\Phi^a_{\hat{b},\hat{c}}\varepsilon_a = \frac{\sqrt{-1}}{2}\left(\nabla_{\varepsilon_{\hat{c}}}(\Phi)\varepsilon_{\hat{b}}\right)^a\varepsilon_a = \frac{1}{2}\left(\Phi\circ\nabla_{\varepsilon_{\hat{c}}}(\Phi)\varepsilon_{\hat{b}}\right)^a\varepsilon_a$, i.e. $[C(\varepsilon_{\hat{b}},\varepsilon_{\hat{c}})]^a\varepsilon_a=\frac{1}{2}\left(\Phi\circ\nabla_{\varepsilon_{\hat{c}}}(\Phi)\varepsilon_{\hat{b}}\right)^a\varepsilon_a$. Since the vectors $\{\varepsilon_a\}$ form a basis of the subspace $\left(D_\Phi^{\sqrt{-1}}\right)_p$, and the vectors $\{\varepsilon_{\hat{a}}\}$ form a basis of the subspace $\left(D_\Phi^{-\sqrt{-1}}\right)_p$ and projectors module $\mathcal{X}(M)^C$ to submodules $D_\Phi^{\sqrt{-1}}, D_\Phi^{-\sqrt{-1}}$ are endomorphisms $\pi=\sigma\circ\ell=-\frac{1}{2}\left(\Phi^2+\sqrt{-1}\Phi\right), \bar{\pi}=\bar{\sigma}\circ\ell=\frac{1}{2}\left(-\Phi^2+\sqrt{-1}\Phi\right)$, then recorded the above equation can be rewritten as $\left(\Phi^2+\sqrt{-1}\Phi\right)\circ C\left(-\Phi^2X+\sqrt{-1}\Phi X,-\Phi^2Y+\sqrt{-1}\Phi Y\right)=\frac{1}{2}\left(\Phi^2+\sqrt{-1}\Phi\right)\circ\left(\Phi\circ\nabla_{-\Phi^2Y+\sqrt{-1}\Phi Y}(\Phi)\left(-\Phi^2X+\sqrt{-1}\Phi X\right)\right);\ \forall X,Y\in\mathcal{X}(M)$. Opening the brackets and simplifying, we obtain

$$C(X,Y)=-\frac{1}{8}\{-\Phi\circ\nabla_{\Phi^2Y}(\Phi)\Phi^2X+\Phi\circ\nabla_{\Phi Y}(\Phi)\Phi X+\Phi^2\circ\nabla_{\Phi^2Y}(\Phi)\Phi X+\Phi^2\circ\nabla_{\Phi Y}(\Phi)\Phi^2X\}. \tag{15}$$

For *SGK*-manifolds of type II have the equality $\Phi^a_{\hat{b},c}=0$, i.e. $\Phi^a_{\hat{b},c}\varepsilon_a=\left(\nabla_{\varepsilon_c}(\Phi)\varepsilon_{\hat{b}}\right)^a\varepsilon_a=0$. Since the vectors $\{\varepsilon_a\}$ form a basis of the subspace $\left(D_\Phi^{\sqrt{-1}}\right)_p$, and the vectors $\{\varepsilon_{\hat{a}}\}$ form a basis of the subspace $\left(D_\Phi^{-\sqrt{-1}}\right)_p$ and projectors module $\mathcal{X}(M)^C$ to submodules $D_\Phi^{\sqrt{-1}}, D_\Phi^{-\sqrt{-1}}$ are endomorphisms $\pi=\sigma\circ\ell=-\frac{1}{2}\left(\Phi^2+\sqrt{-1}\Phi\right), \bar{\pi}=\bar{\sigma}\circ\ell=\frac{1}{2}\left(-\Phi^2+\sqrt{-1}\Phi\right)$, then recorded the above equation can be rewritten as $\left(\Phi^2+\sqrt{-1}\Phi\right)\circ\nabla_{\Phi^2Y+\sqrt{-1}\Phi Y}(\Phi)\left(-\Phi^2X+\sqrt{-1}\Phi X\right)=0;\ \forall X,Y\in\mathcal{X}(M)$.

Opening the brackets and simplifying, we obtain $\Phi^2\circ\nabla_{\Phi^2Y}(\Phi)\Phi^2X+\Phi^2\circ\nabla_{\Phi Y}(\Phi)\Phi X-\Phi\circ\nabla_{\Phi Y}(\Phi)\Phi^2X+\Phi\circ\nabla_{\Phi^2Y}(\Phi)\Phi X=0;\ \forall X,Y\in\mathcal{X}(M)$. We apply the operator Φ on both sides of this equation, then we have

$$\Phi\circ\nabla_{\Phi^2Y}(\Phi)\Phi^2X+\Phi\circ\nabla_{\Phi Y}(\Phi)\Phi X+\Phi^2\circ\nabla_{\Phi Y}(\Phi)\Phi^2X-\Phi^2\circ\nabla_{\Phi^2Y}(\Phi)\Phi X=0;\ \forall X,Y\in\mathcal{X}(M). \tag{16}$$

Given the resulting equation (15), (16) takes the form:

$$C(X,Y)=\frac{1}{4}\{\Phi\circ\nabla_{\Phi^2Y}(\Phi)\Phi^2X-\Phi^2\circ\nabla_{\Phi^2Y}(\Phi)\Phi X\}=-\frac{1}{4}\{\Phi\circ\nabla_{\Phi Y}(\Phi)\Phi X+\Phi^2\circ\nabla_{\Phi Y}(\Phi)\Phi^2X\};\ \forall X,Y\in\mathcal{X}(M). \tag{17}$$

Thus we have proved.

Theorem 4.1. The structure tensor *SGK*-manifold of type II is calculated by the formula $C(X,Y)=\frac{1}{4}\{\Phi\circ\nabla_{\Phi^2Y}(\Phi)\Phi^2X-\Phi^2\circ\nabla_{\Phi^2Y}(\Phi)\Phi X\}=-\frac{1}{4}\{\Phi\circ\nabla_{\Phi Y}(\Phi)\Phi X+\Phi^2\circ\nabla_{\Phi Y}(\Phi)\Phi^2X\};\ \forall X,Y\in\mathcal{X}(M)$.

Since *C* is a tensor of type (2,1), then by the fundamental theorem of tensor analysis, we have:

$$dC^i{}_{jk}+C^l{}_{jk}\theta^i_l-C^i{}_{lk}\theta^l_j-C^i{}_{jl}\theta^l_k=C^i{}_{jk,l}\omega^l, \tag{18}$$

where $\{C^i{}_{jk,l}\}$ – system functions, serving on the space of the bundle of all frames components of a covariant differential structure tensor *C*. Rewriting this equation in the space of the associated *G*-structure, we get:

$$\begin{aligned}&1)\ C^0_{ab,c}=-C_{abc};\ 2)\ C^0_{\hat a\hat b,\hat c}=-C^{abc};\ 3)\ C^a_{0\hat b,\hat c}=C^{abc};\ 4)\ C^c_{ab,\hat d}=\\&-C^{cdh}C_{hab};\ 5)\ C^a_{b\hat c,d}=C^{ach}C_{hbd};\ 6)\ C^a_{\hat b0,\hat c}=-C^{abc};\ 7)\ C^a_{\hat bc,d}=\\&-C^{abh}C_{hcd};\ 8)\ C^a_{\hat b\hat c,\hat d}=C^{abcd};\ 9)\ C^a_{\hat b\hat c,0}=C^{abc};\ 10)\ C^{\hat a}_{0b,c}=\\&C_{abc};\ 11)\ C^{\hat a}_{b0,c}=-C_{abc};\ 12)\ C^{\hat a}_{bc,d}=C_{abcd};\ 13)\ C^{\hat a}_{bc,0}=\\&-C_{abc};\ 14)\ C^{\hat a}_{b\hat c,\hat d}=-C^{cdh}C_{hab};\ 15)\ C^{\hat a}_{\hat bc,\hat d}=C^{bdh}C_{hac};\ 16)\ C^{\hat a}_{\hat b\hat c,d}=\\&-C^{bch}C_{had}.\end{aligned} \tag{19}$$

And the other components are zero.
Thus we have proved the theorem.

Theorem 4.2. The components of the covariant differential structure tensor *C* *SGK*-manifold of type II in the space of the associated *G*-structure are of the form: 1) $C^0_{ab,c}=-C_{abc}$; 2) $C^0_{\hat a\hat b,\hat c}=-C^{abc}$; 3) $C^a_{0\hat b,\hat c}=C^{abc}$; 4) $C^c_{ab,\hat d}=-C^{cdh}C_{hab}$; 5) $C^a_{b\hat c,d}=C^{ach}C_{hbd}$; 6) $C^a_{\hat b0,\hat c}=-C^{abc}$; 7) $C^a_{\hat bc,d}=-C^{abh}C_{hcd}$; 8) $C^a_{\hat b\hat c,\hat d}=C^{abcd}$; 9) $C^a_{\hat b\hat c,0}=C^{abc}$; 10) $C^{\hat a}_{0b,c}=C_{abc}$; 11) $C^{\hat a}_{b0,c}=-C_{abc}$; 12) $C^{\hat a}_{bc,d}=C_{abcd}$; 13) $C^{\hat a}_{bc,0}=-C_{abc}$; 14) $C^{\hat a}_{b\hat c,\hat d}=-C^{cdh}C_{hab}$; 15) $C^{\hat a}_{\hat bc,\hat d}=C^{bdh}C_{hac}$; 16) $C^{\hat a}_{\hat b\hat c,d}=-C^{bch}C_{had}$ and the other components are zero.

Consider the map $\mathcal{C}:\mathcal{X}(M)\times\mathcal{X}(M)\times\mathcal{X}(M)\to\mathcal{X}(M)$, given by the formula

$$\mathcal{C}(X,Y,Z)=C^{abh}C_{hcd}X^cY^dZ_b\varepsilon_a+C_{abh}C^{hcd}X_cY_dZ^a\varepsilon_{\hat a}. \tag{20}$$

Theorem 4.3. For *SGK*-manifold of type II have the following relations:

$$1)\ \mathcal{C}(X_1+X_2,Y,Z)=\mathcal{C}(X_1,Y,Z)+\mathcal{C}(X_2,Y,Z);\ 2)\ \mathcal{C}(\Phi X,Y,Z)=\mathcal{C}(X,\Phi Y,Z)=-\mathcal{C}(X,Y,\Phi Z)=\Phi\circ\mathcal{C}(X,Y,Z);\ 3)\ \mathcal{C}(X,Y,\Phi^2 Z)=-\mathcal{C}(X,Y,Z);\ \ 4)\ \eta\circ\mathcal{C}(X,Y,Z)=0;\ \ 5)\ \mathcal{C}(X,Y,\xi)=\mathcal{C}(X,\xi,Y)=\mathcal{C}(\xi,X,Y)=0;\ \ 6)\ \mathcal{C}(X,Y,Z)=-\mathcal{C}(Y,X,Z);\ \ 7)\ \langle\mathcal{C}(X,Y,Z),W\rangle=-\langle\mathcal{C}(X,Y,W),Z\rangle;\ \ 8)\ \mathcal{C}(X,Y,Z_1+Z_2)=\mathcal{C}(X,Y,Z_1)+\mathcal{C}(X,Y,Z_1);\ \ \forall X,Y,Z,W\in\mathcal{X}(M). \tag{21}$$

Proof. The proof is by direct calculation. For example,

$$\mathcal{C}(X_1+X_2,Y,Z)=C^{abh}C_{hcd}(X_1+X_2)^cY^dZ_b\varepsilon_a+C_{abh}C^{hcd}(X_1+X_2)_cY_dZ^a\varepsilon_{\hat{a}}=C^{abh}C_{hcd}(X_1)^cY^dZ_b\varepsilon_a+C^{abh}C_{hcd}(X_2)^cY^dZ_b\varepsilon_a+C_{abh}C^{hcd}(X_1)_cY_dZ^a\varepsilon_{\hat{a}}+C_{abh}C^{hcd}(X_1)_cY_dZ^a\varepsilon_{\hat{a}}=\mathcal{C}(X_1,Y,Z)+\mathcal{C}(X_2,Y,Z).$$

Similarly we prove the remaining properties.

We consider the endomorphism *c*, given by an *A*-frame matrix

$$(C_b^a)=\left(C^{acd}C_{dcb}\right).$$

This endomorphism Hermitian symmetric, and hence diagonalizable in a suitable A-frame, ie, in this frame

$$C_b^a=C_b\delta_b^a, \tag{22}$$

where $\{C_b\}$ – the eigenvalues of this endomorphism. Moreover, the Hermitian form $\mathfrak{C}(X,Y)=C_b^aX^bY_a$, corresponding to this endomorphism, positive semi-definite, since $\mathfrak{C}(X,X)=C_b^aX^bX_a=C^{acd}C_{dcb}X^bX_a=\sum_{c,d}|C_{cda}X^a|^2\geq 0$. Consequently, $C_a\geq 0, a=1,2,\dots,n$.

Let us consider some properties of the tensor ∇C.

Using the restore identity (Kirichenko, 2013; Kirichenko and Rustanov, 2002) to the relations (19), we obtain the following theorem.

Theorem 4.4. ∇C tensor has the following properties:

$$1)\nabla_{\xi}(C)(\xi,X) = -\nabla_{\xi}(C)(X,\xi) = 0;\ 2)\nabla_{\Phi^2X}(C)(\xi,\Phi^2Y) = -\nabla_{\Phi X}(C)(\xi,\Phi Y) = \nabla_X(C)(\xi,Y) = C(X,Y);\ 3)\nabla_{\xi}(C)(X,Y) - \nabla_{\xi}(C)(\Phi X,\Phi Y) = -2C(X,Y);\ 4)\nabla_{\Phi^2X}(C)(\Phi^2Y,\Phi^2Z) - \nabla_{\Phi^2X}(C)(\Phi Y,\Phi Z) - \nabla_{\Phi X}(C)(\Phi^2Y,\Phi Z) - \nabla_{\Phi X}(C)(\Phi Y,\Phi^2Z) = 4\langle X, C(Y,Z)\rangle\xi;\ \forall X,Y,Z \in \mathcal{X}(M)\,. \tag{23}$$

From the (9) by the fundamental theorem of tensor analysis follows that the family of functions $\{A_{ab}^{cd}\}$ in the space of the associated G-structure, symmetric with respect to the upper and lower indices form a pure tensor on M^{2n+1} called tensor Φ-holomorphic sectional curvature (Abu-Saleem and Rustanov, 2015;Umnova, 2002). This tensor defines a mapping $A: \mathcal{X}(M) \times \mathcal{X}(M) \times \mathcal{X}(M) \longrightarrow \mathcal{X}(M)$, which is given by $A(X,Y,Z) = A_{ab}^{cd}X^aY^bZ_d\varepsilon_c + A_{\hat{a}\hat{b}}^{\hat{c}\hat{d}}X_aY_bZ^d\varepsilon_c$. Direct calculation is easy to check that the tensor Φ-holomorphic sectional curvature has the following properties:

$$1)\ A(\Phi X,Y,Z) = A(X,\Phi Y,Z) = -A(X,Y,\Phi Z);\ 2)\ A(Z,X,\Phi^2Y) = -A(Z,X,Y);\ \ 3)\ \eta\circ A(X,Y,Z) = 0;\ \ 4)\ A(\xi,Y,Z) = A(X,\xi,Z) = A(X,Y,\xi) = 0;\ \ 5)\ A(X,Y,Z) = A(Y,X,Z);\ \ 6)\ \langle A(X,Y,Z),W\rangle = \langle A(X,Y,W),Z\rangle;\ \ \forall X,Y,Z,W \in \mathcal{X}(M). \tag{24}$$

In fact,

$$A(\Phi X,Y,Z) = A_{ab}^{cd}(\Phi X)^aY^bZ_d\varepsilon_c + A_{\hat{a}\hat{b}}^{\hat{c}\hat{d}}(\Phi X)_aY_bZ^d\varepsilon_c = \sqrt{-1}A_{ab}^{cd}X^aY^bZ_d\varepsilon_c - \sqrt{-1}A_{\hat{a}\hat{b}}^{\hat{c}\hat{d}}X_aY_bZ^d\varepsilon_c = A_{ab}^{cd}X^a(\Phi Y)^bZ_d\varepsilon_c + A_{\hat{a}\hat{b}}^{\hat{c}\hat{d}}X_a(\Phi Y)_bZ^d\varepsilon_c = A(X,\Phi Y,Z).$$

Similarly,

$$A(\Phi X,Y,Z) = A_{ab}^{cd}(\Phi X)^aY^bZ_d\varepsilon_c + A_{\hat{a}\hat{b}}^{\hat{c}\hat{d}}(\Phi X)_aY_bZ^d\varepsilon_c = \sqrt{-1}A_{ab}^{cd}X^aY^bZ_d\varepsilon_c - \sqrt{-1}A_{\hat{a}\hat{b}}^{\hat{c}\hat{d}}X_aY_bZ^d\varepsilon_c = -A_{ab}^{cd}X^a\mathrm{Y}^b(\Phi Z)_d\varepsilon_c - A_{\hat{a}\hat{b}}^{\hat{c}\hat{d}}X_a\mathrm{Y}_b(\Phi Z)^d\varepsilon_c = -A(X,Y,\Phi Z).$$

Property 5 follows from the symmetry of the tensor A_{ab}^{cd} on the lower pair of indices. The symmetry of the upper pair of indices implies property 6.

Thus, the theorem is proved.

Theorem 4.5. Tensor Φ-holomorphic sectional curvature *SGK*-manifold of type II has the following properties:

1) $A(\Phi X,Y,Z)=A(X,\Phi Y,Z)=-A(X,Y,\Phi Z)$; 2) $A(Z,X,\Phi^2 Y)=-A(Z,X,Y)$; 3) $\eta\circ A(X,Y,Z)=0$; 4) $A(\xi,Y,Z)=A(X,\xi,Z)=A(X,Y,\xi)=0$; 5) $A(X,Y,Z)=A(Y,X,Z)$; 6) $\langle A(X,Y,Z),W\rangle=\langle A(X,Y,W),Z\rangle$; $\forall X,Y,Z,W\in\mathcal{X}(M)$.

5. CURVATURE IDENTITIES *SGK*-MANIFOLDS OF TYPE II

In Kirichenko, 2013; Kirichenko and Rustanov, 2002 allocated the class of quasi-sasakian manifolds, the Riemann curvature tensor which satisfies identity $R(\xi,\Phi^2X)\Phi^2Y-R(\xi,\Phi X)\Phi Y=0$; $\forall X,Y\in\mathcal{X}(M)$. Following the idea described in these papers, we look at some of the identity satisfied by the Riemann curvature tensor SGK-manifolds of type II.

We apply the restore identity (Kirichenko, 2013; Kirichenko and Rustanov, 2002) to the equations: $R^0_{00b}=\delta^0_b=0$; $R^a_{00b}=\delta^a_b$; $R^{\hat{a}}_{00b}=\delta^{\hat{a}}_b=0$, i.e. to equality $R^i_{00b}=\delta^i_b$. In fixed point $p\in M$ last equality is equivalent to the relation $R(\xi,\varepsilon_b)\xi=\varepsilon_b$. Since $\xi_p=\varepsilon_0$, and the vectors $\{\varepsilon_a\}$ form a basis of the subspace $\left(D_\Phi^{\sqrt{-1}}\right)_p$, given that the projectors module $\mathcal{X}(M)^C$ to submodules $D_\Phi^{\sqrt{-1}}$ and D_Φ^0 are endomorphisms $\pi=\sigma\circ\ell=-\frac{1}{2}\left(\Phi^2+\sqrt{-1}\Phi\right)$, $m=id+\Phi^2$, identity $R(\xi,\varepsilon_b)\xi=\varepsilon_b$ can be rewritten in the form $R\left(\xi,\Phi^2X+\sqrt{-1}\Phi X\right)\xi=\Phi^2X+\sqrt{-1}\Phi X$; $\forall X\in\mathcal{X}(M)$.

Expanding this relation is linear and separating the real and imaginary parts of the resulting equation, we obtain an equivalent identity:

$$R(\xi,\Phi^2X)\xi=\Phi^2X;\ \forall X\in\mathcal{X}(M). \tag{25}$$

We say that the identity (25), the first additional identity curvature *SGK*-manifold of type II.

Since $R(\xi,\xi)\xi=0$ and $\Phi^2=-id+\eta\otimes\xi$, then (25) takes the form:

$$R(\xi,X)\xi=-X+\eta(X)\xi;\ \forall X\in\mathcal{X}(M). \tag{26}$$

Definition 5.1. We call the *AC*-manifold manifold of class R_1, if its curvature tensor satisfies

$$R(\xi,X)\xi=0;\ \ \forall X\in\mathcal{X}(M). \tag{27}$$

Let M – *SGK*-manifold of type II, which is a manifold of class R_1. Then its Riemann curvature tensor satisfies (27). On the space of the associated *G*-structure relation (27) can be written as: $R^i_{00j}X^j = 0$. In view of (11), the last equation can be written as: $R^a_{00b}X^b + R^{\hat{a}}_{00\hat{b}}X^{\hat{b}} = 0$. This equality holds if and only if $R^a_{00b} = R^{\hat{a}}_{00\hat{b}} = 0$. But since $R^a_{00b} = \delta^a_b \neq 0$, then *SGK*-manifold of type II is not a manifold class R_1.

Thus we have proved.

Theorem 5.1. *SGK*-manifold of type II is not a manifold class R_1.

Since $R^0_{0ab} = 0$, $R^c_{0ab} = 0$, $R^{\hat{c}}_{0ab} = 0$, i.e. $R^i_{0ab} = 0$. In fixed point $p \in M$ it is obviously equivalent to the relation $R(\varepsilon_a, \varepsilon_b)\xi = 0$. Since $\xi_p = \varepsilon_0$, and the vectors $\{\varepsilon_a\}$ form a basis of the subspace $\left(D_\Phi^{\sqrt{-1}}\right)_p$, and projectors module $\mathcal{X}(M)^C$ to submodules $D_\Phi^{\sqrt{-1}}$ and D_Φ^0 will endomorphisms $\pi = \sigma \circ \ell = -\frac{1}{2}\left(\Phi^2 + \sqrt{-1}\Phi\right), m = id + \Phi^2$, identity $R(\varepsilon_a, \varepsilon_b)\xi = 0$ can be rewritten in the form $R\left(\Phi^2 X + \sqrt{-1}\Phi X, \Phi^2 Y + \sqrt{-1}\Phi Y\right)\xi = 0;\ \forall X, Y \in \mathcal{X}(M)$.

Expanding this relation is linear and separating the real and imaginary parts of the resulting equation, we obtain an equivalent identity:

$$R(\Phi^2 X, \Phi^2 Y)\xi - R(\Phi X, \Phi Y)\xi = 0;\ \forall X, Y \in \mathcal{X}(M). \tag{28}$$

Taking into account the equality $\Phi^2 = -id + \eta \otimes \xi$ last equation can be written as

$$R(X,Y)\xi - \eta(X)R(\xi,Y)\xi - \eta(Y)R(X,\xi)\xi + \eta(X)\eta(Y)R(\xi,\xi)\xi - R(\Phi X, \Phi Y)\xi = R(X,Y)\xi - \eta(X)R(\xi,Y)\xi + \eta(Y)R(\xi,X)\xi - R(\Phi X, \Phi Y)\xi = 0,$$

which in view of (26) takes the form:

$$R(\Phi X, \Phi Y)\xi - R(X,Y)\xi = -\eta(Y)X + \eta(X)Y;\ \forall X, Y \in \mathcal{X}(M). \tag{29}$$

Similarly, applying the restore identity to the equalities $R^0_{0a\hat{b}} = 0$, $R^c_{0a\hat{b}} = 0$, $R^{\hat{c}}_{0a\hat{b}} = 0$, i.e. $R^i_{0a\hat{b}} = 0$, we obtain the identity:

$$R(\Phi^2 X, \Phi^2 Y)\xi + R(\Phi X, \Phi Y)\xi = 0;\ \forall X, Y \in \mathcal{X}(M). \tag{30}$$

From the (28) and (30), we have:

$$R(\Phi^2 X, \Phi^2 Y)\xi = R(\Phi X, \Phi Y)\xi = 0;\ \forall X, Y \in \mathcal{X}(M). \qquad (31)$$

Taking into account the equality $\Phi^2 = -id + \eta \otimes \xi$ and identity (26), the identity of $R(\Phi^2 X, \Phi^2 Y)\xi = 0$ takes the form:

$$R(X, Y)\xi = -\eta(X)Y + \eta(Y)X;\ \forall X, Y \in \mathcal{X}(M). \qquad (32)$$

Using the restore identity to the relations $R^0_{a0b} = 0$, $R^c_{a0b} = 0$, $R^{\hat{c}}_{a0b} = 0$, i.e. $R^i_{a0b} = 0$, we obtain the identity:

$$R(\xi, \Phi^2 X)\Phi^2 Y - R(\xi, \Phi X)\Phi Y = 0;\ \forall X, Y \in \mathcal{X}(M). \qquad (33)$$

In view of (26) and the relations $R(\xi, \xi)\xi = 0, \Phi^2 = -id + \eta \otimes \xi$ last identity can be rewritten as:

$$R(\xi, X)Y - R(\xi, \Phi X)\Phi Y = -\eta(Y)X + \eta(X)\eta(Y)\xi;\ \forall X, Y \in \mathcal{X}(M). \qquad (34)$$

Now apply recovery procedure identity to the equalities $R^0_{a0\hat{b}} = -\delta^b_a \xi^0, R^c_{a0\hat{b}} = -\delta^b_a \xi^c = 0, R^{\hat{c}}_{a0\hat{b}} = -\delta^b_a \xi^{\hat{c}} = 0$, i.e. $R^i_{a0\hat{b}} = -\delta^b_a \xi^i$. In fixed point $p \in M$ last equality is equivalent to the relation $R(\xi, \varepsilon_{\hat{b}})\varepsilon_a = -\delta^b_a \xi = -\langle \varepsilon_a, \varepsilon_{\hat{b}} \rangle \xi$. Since $\xi_p = \varepsilon_0$, vectors $\{\varepsilon_a\}$ form a basis of the subspace $\left(D^{\sqrt{-1}}_\Phi\right)_p$, and the vectors $\{\varepsilon_{\hat{a}}\}$ form a basis of the subspace $\left(D^{-\sqrt{-1}}_\Phi\right)_p$ and projectors of module $\mathcal{X}(M)^C$ to submodules $D^{\sqrt{-1}}_\Phi, D^{-\sqrt{-1}}_\Phi$ and D^0_Φ will endomorphisms $\pi = \sigma \circ \ell = -\frac{1}{2}\left(\Phi^2 + \sqrt{-1}\Phi\right), \bar{\pi} = \bar{\sigma} \circ \ell = \frac{1}{2}\left(-\Phi^2 + \sqrt{-1}\Phi\right)$, $m = id + \Phi^2$, the identity of $R(\xi, \varepsilon_{\hat{b}})\varepsilon_a = -\langle \varepsilon_a, \varepsilon_{\hat{b}} \rangle \xi$ can be rewritten in the form of $R\left(\xi, -\Phi^2 X + \sqrt{-1}\Phi X\right)\left(\Phi^2 Y + \sqrt{-1}\Phi Y\right) = -\langle -\Phi^2 X + \sqrt{-1}\Phi X, \Phi^2 Y + \sqrt{-1}\Phi Y \rangle \xi;\ \forall X, Y \in \mathcal{X}(M)$. Expanding this relation is linear and separating the real and imaginary parts of the resulting equation, we obtain an equivalent identity:

$$R(\xi, \Phi^2 X)(\Phi^2 Y) + R(\xi, \Phi X)\Phi Y = -2\langle \Phi X, \Phi Y \rangle \xi = -2\langle X, Y \rangle \xi + 2\eta(X)\eta(Y)\xi;\ \forall X, Y \in \mathcal{X}(M). \qquad (35)$$

We say that the identity (35), the second additional identity curvature SGK-manifold of type II.

From (33) and (35) we have:

$$R(\xi, \Phi^2 X)(\Phi^2 Y) = R(\xi, \Phi X)\Phi Y = -\langle \Phi X, \Phi Y\rangle \xi = -\langle X, Y\rangle \xi + \eta(X)\eta(Y)\xi;\ \forall X, Y \in \mathcal{X}(M). \tag{36}$$

Note 1. The identity of $R(\xi, \Phi^2 X)(\Phi^2 Y) = -\langle X, Y\rangle\xi + \eta(X)\eta(Y)\xi$, taking into account (15) and the equation $\Phi^2 = -id + \eta \otimes \xi$, can be written as:

$$R(\xi, X)Y = \eta(Y)\Phi^2 X - \langle \Phi X, \Phi Y\rangle \xi = -\langle X, Y\rangle\xi - \eta(Y)X + 2\eta(X)\eta(Y)\xi;\ \forall X, Y \in \mathcal{X}(M). \tag{37}$$

Definition 5.2. We call the *AC*-manifold manifold of class R_2, if its curvature tensor satisfies the equation:

$$R(\xi, \Phi^2 X)(\Phi^2 Y) + R(\xi, \Phi X)\Phi Y = 0;\ \forall X, Y \in \mathcal{X}(M). \tag{38}$$

Let *M* – *SGK*-manifold of type II, which is a manifold of class R_2. Then its Riemann curvature tensor satisfies the condition (38). In view of (33) the identity (38) can be written as: $R(\xi, \Phi^2 X)(\Phi^2 Y) = R(\xi, \Phi X)\Phi Y = 0;\ \forall X, Y \in \mathcal{X}(M)$. On the space of the associated *G*-structure relation $R(\xi, \Phi X)\Phi Y = 0;\ \forall X, Y \in \mathcal{X}(M)$ can be written as: $R^i_{j0k}(\Phi X)^k(\Phi Y)^j = 0$. In view of (11) and the form of the matrix Φ, the last equation can be written as: $R^0_{a0\hat{b}} X^{\hat{b}} Y^a + R^0_{\hat{a}0b} X^b Y^{\hat{a}} = 0$. This equality holds if and only if $R^0_{a0\hat{b}} = R^0_{\hat{a}0b} = 0$. But since $R^0_{a0\hat{b}} = -\delta^b_a \neq 0$, then the *SGK*-manifold of type II is not a manifold class R_2.

Thus we have proved.

Theorem 5.2. *SGK*-manifold of type II is not a manifold class R_2.

Since $R^0_{abc} = -2C_{0a[bc]} = 0;\ R^d_{abc} = -2C_{\hat{d}a[bc]} = 0;\ R^{\hat{d}}_{abc} = -2C_{da[bc]}$, i.e. $R^i_{abc} = -2C_{ia[bc]}$. In fixed point $p \in M$, taking into account (19), it is obviously equivalent to the relation $R(\varepsilon_b, \varepsilon_c)\varepsilon_a = \nabla_{\varepsilon_b}(C)(\varepsilon_a, \varepsilon_c) - \nabla_{\varepsilon_c}(C)(\varepsilon_a, \varepsilon_b)$. Since the vectors $\{\varepsilon_a\}$ form a basis of the subspace $\left(D_\Phi^{\sqrt{-1}}\right)_p$, and projectors of module $\mathcal{X}(M)^C$ on the submodule $D_\Phi^{\sqrt{-1}}$ is the endomorphism $\pi = \sigma \circ \ell = -\frac{1}{2}\left(\Phi^2 + \sqrt{-1}\Phi\right)$, then the identity of the $R(\varepsilon_b, \varepsilon_c)\varepsilon_a = \nabla_{\varepsilon_b}(C)(\varepsilon_a, \varepsilon_c) - \nabla_{\varepsilon_c}(C)(\varepsilon_a, \varepsilon_b)$ can be rewritten in the form

$$R\left(\Phi^2X+\sqrt{-1}\Phi X,\Phi^2Y+\sqrt{-1}\Phi Y\right)\left(\Phi^2Z+\sqrt{-1}\Phi Z\right)=$$
$$\nabla_{\Phi^2X+\sqrt{-1}\Phi X}(C)\left(\Phi^2Z+\sqrt{-1}\Phi Z,\Phi^2Y+\sqrt{-1}\Phi Y\right)-$$
$$\nabla_{\Phi^2Y+\sqrt{-1}\Phi Y}(C)\left(\Phi^2Z+\sqrt{-1}\Phi Z,\Phi^2X+\sqrt{-1}\Phi X\right);\ \forall X,Y,Z\in\mathcal{X}(M).$$

Expanding this relation is linear and separating the real and imaginary parts of the resulting equation, we obtain an equivalent identity:

$$R(\Phi^2X,\Phi^2Y)\Phi^2Z-R(\Phi^2X,\Phi Y)\Phi Z-R(\Phi X,\Phi^2Y)\Phi Z-$$
$$R(\Phi X,\Phi Y)\Phi^2Z=\nabla_{\Phi^2X}(C)(\Phi^2Z,\Phi^2Y)-\nabla_{\Phi^2X}(C)(\Phi Z,\Phi Y)-$$
$$\nabla_{\Phi X}(C)(\Phi^2Z,\Phi Y)-\nabla_{\Phi X}(C)(\Phi Z,\Phi^2Y)-\nabla_{\Phi^2Y}(C)(\Phi^2Z,\Phi^2X)+$$
$$\nabla_{\Phi^2Y}(C)(\Phi Z,\Phi X)+\nabla_{\Phi Y}(C)(\Phi^2Z,\Phi X)+$$
$$\nabla_{\Phi Y}(C)(\Phi Z,\Phi^2X);\ \forall X,Y,Z\in\mathcal{X}(M). \quad (39)$$

We say that the identity (39) to the third additional identity curvature *SGK*-manifold of type II.

Definition 5.3. We call the *AC*-manifold manifold of class R_3, if its curvature tensor satisfies the equation:

$$R(\Phi^2X,\Phi^2Y)\Phi^2Z-R(\Phi^2X,\Phi Y)\Phi Z-R(\Phi X,\Phi^2Y)\Phi Z-$$
$$R(\Phi X,\Phi Y)\Phi^2Z=0;\ \forall X,Y,Z\in\mathcal{X}(M). \quad (40)$$

From the definition 5.3 and (39).

Theorem 5.3. *SGK*-manifold is a manifold of type II class R_3 if and only if

$$\nabla_{\Phi^2X}(C)(\Phi^2Z,\Phi^2Y)-\nabla_{\Phi^2X}(C)(\Phi Z,\Phi Y)-\nabla_{\Phi X}(C)(\Phi^2Z,\Phi Y)-$$
$$\nabla_{\Phi X}(C)(\Phi Z,\Phi^2Y)-\nabla_{\Phi^2Y}(C)(\Phi^2Z,\Phi^2X)+\nabla_{\Phi^2Y}(C)(\Phi Z,\Phi X)+$$
$$\nabla_{\Phi Y}(C)(\Phi^2Z,\Phi X)+\nabla_{\Phi Y}(C)(\Phi Z,\Phi^2X)=0;\ \forall X,Y,Z\in\mathcal{X}(M).$$

Let M – *SGK*-manifold of type II, which is a manifold of class R_3. Then its Riemann curvature tensor satisfies the condition (40), which in the space of the associated *G*-structure can be written as:

$$R^i_{jkl}(\Phi^2X)^k(\Phi^2Y)^l(\Phi^2Z)^j-R^i_{jkl}(\Phi^2X)^k(\Phi Y)^l(\Phi Z)^j-$$
$$R^i_{jkl}(\Phi X)^k(\Phi^2Y)^l(\Phi Z)^j-R^i_{jkl}(\Phi X)^k(\Phi Y)^l(\Phi^2Z)^j=0.$$

In view of (11) and the form of the matrix Φ, the last equation can be written as: $R^{\hat{d}}_{abc}X^bY^cZ^a+R^{d}_{\hat{a}\hat{b}\hat{c}}X_bY_cZ_a=0$. This equality holds if and only if $R^{\hat{d}}_{abc} = R^{d}_{\hat{a}\hat{b}\hat{c}} = 0$. According to (2.8), $C_{ab[cd]} = 0$. From this equality and (8) we have $C_{abcd} = 0$.

Thus we have proved.

Theorem 5.4. *SGK*-manifold of type II is a manifold class R_3 if and only if on the space of the associated *G*-structure $C_{abcd} = C^{abcd} = 0$.

Since,

$$R^{0}_{ab\hat{c}} = A^{0c}_{ab} - C^{0ch}C_{hab} - \delta^{0}_{b}\delta^{c}_{a} = 0, R^{d}_{ab\hat{c}} = A^{dc}_{ab} - C^{dch}C_{hab} - \delta^{c}_{b}\delta^{d}_{a}, R^{\hat{d}}_{ab\hat{c}} = A^{\hat{d}c}_{ab} - C^{\hat{d}ch}C_{hab} - \delta^{\hat{d}}_{b}\delta^{c}_{a} = 0,$$

i.e. $R^{i}_{ab\hat{c}} = A^{ic}_{ab} - C^{ich}C_{hab} - \delta^{c}_{b}\delta^{i}_{a}$.

In fixed point $p \in M$ it is obviously equivalent to the relation $R(\varepsilon_b, \varepsilon_{\hat{c}})\varepsilon_a = A(\varepsilon_a, \varepsilon_b, \varepsilon_{\hat{c}}) + \nabla_{\varepsilon_{\hat{c}}}(C)(\varepsilon_a, \varepsilon_b) - \varepsilon_b\langle\varepsilon_a, \varepsilon_{\hat{c}}\rangle$. Since the vectors $\{\varepsilon_a\}$ form a basis of the subspace $\left(D^{\sqrt{-1}}_{\Phi}\right)_p$, and the vectors $\{\varepsilon_{\hat{a}}\}$ form a basis of the subspace $\left(D^{-\sqrt{-1}}_{\Phi}\right)_p$ and projectors of the module $\mathcal{X}(M)^C$ to submodules $D^{\sqrt{-1}}_{\Phi}, D^{-\sqrt{-1}}_{\Phi}$ are endomorphisms $\pi = \sigma \circ \ell = -\frac{1}{2}\left(\Phi^2 + \sqrt{-1}\Phi\right)$, $\bar{\pi} = \bar{\sigma} \circ \ell = \frac{1}{2}\left(-\Phi^2 + \sqrt{-1}\Phi\right)$ identity $R(\varepsilon_b, \varepsilon_{\hat{c}})\varepsilon_a = A(\varepsilon_a, \varepsilon_b, \varepsilon_{\hat{c}}) + \nabla_{\varepsilon_{\hat{c}}}(C)(\varepsilon_a, \varepsilon_b) - \varepsilon_b\langle\varepsilon_a, \varepsilon_{\hat{c}}\rangle$ can be rewritten in the form

$$R\left(\Phi^2X + \sqrt{-1}\Phi X, -\Phi^2Y + \sqrt{-1}\Phi Y\right)\left(\Phi^2Z + \sqrt{-1}\Phi Z\right) = A\left(\Phi^2Z + \sqrt{-1}\Phi Z, \Phi^2X + \sqrt{-1}\Phi X, -\Phi^2Y + \sqrt{-1}\Phi Y\right) + \nabla_{-\Phi^2Y+\sqrt{-1}\Phi Y}(C)\left(\Phi^2Z + \sqrt{-1}\Phi Z, \Phi^2X + \sqrt{-1}\Phi X\right) - \left(\Phi^2X + \sqrt{-1}\Phi X\right)\langle -\Phi^2Y + \sqrt{-1}\Phi Y, \Phi^2Z + \sqrt{-1}\Phi Z\rangle;\ \forall X, Y, Z \in \mathcal{X}(M).$$

Expanding this relation by linearity, and separating the real and imaginary parts of the resulting equation and using (24), we obtain an equivalent identity:

$$R(\Phi^2X, \Phi^2Y)\Phi^2Z + R(\Phi^2X, \Phi Y)\Phi Z - R(\Phi X, \Phi^2Y)\Phi Z + R(\Phi X, \Phi Y)\Phi^2Z = -4A(Z, X, Y) + \nabla_{\Phi^2Y}(C)(\Phi^2Z, \Phi^2X) - \nabla_{\Phi^2Y}(C)(\Phi Z, \Phi X) + \nabla_{\Phi Y}(C)(\Phi^2Z, \Phi X) + \nabla_{\Phi Y}(C)(\Phi Z, \Phi^2X) - 2\Phi^2X\langle\Phi Y, \Phi Z\rangle - 2\Phi X\langle Y, \Phi Z\rangle;\ \forall X, Y, Z \in \mathcal{X}(M). \tag{41}$$

We say that the identity (41), fourth additional identity curvature *SGK*-manifold of type II.

Definition 5.4. We call the *AC*-manifold manifold of class R_4, if its curvature tensor satisfies the equation:

$$R(\Phi^2X,\Phi^2Y)\Phi^2Z+R(\Phi^2X,\Phi Y)\Phi Z-R(\Phi X,\Phi^2Y)\Phi Z+R(\Phi X,\Phi Y)\Phi^2Z=0;\ \ \forall X,Y,Z\in\mathcal{X}(M). \quad (42)$$

Let M – *SGK*-manifold of type II, which is a manifold of class R_4. Then its Riemann curvature tensor satisfies the condition (42), which in the space of the associated *G*-structure can be written as:

$$R^i_{jkl}(\Phi^2X)^k(\Phi^2Y)^l(\Phi^2Z)^j+R^i_{jkl}(\Phi^2X)^k(\Phi Y)^l(\Phi Z)^j-R^i_{jkl}(\Phi X)^k(\Phi^2Y)^l(\Phi Z)^j+R^i_{jkl}(\Phi X)^k(\Phi Y)^l(\Phi^2Z)^j=0.$$

In view of (11) and the form of the matrix Φ, the last equation can be written as: $R^d_{ab\hat{c}}X^bY^cZ^a+R^{\hat{d}}_{\hat{a}\hat{b}c}X_bY^cZ_a=0$. This equality holds if and only if $R^d_{ab\hat{c}}=R^{\hat{d}}_{\hat{a}\hat{b}c}=0$. According to (11), $A^{dc}_{ab}-C^{dch}C_{hab}-\delta^c_b\delta^d_a=0$. Simmetriruya last equality first in the indices *a* and *b*, then the indices *c* and *d*, we obtain $A^{(dc)}_{(ab)}=\delta^c_b\delta^d_a=\frac{1}{2}\tilde{\delta}^{cd}_{ab}$. Due to the symmetry of the tensor A^{cd}_{ab} by the upper and lower pair of indices, the resulting identity can be rewritten as: $A^{cd}_{ab}=\frac{1}{2}\tilde{\delta}^{cd}_{ab}$. Then the equality $A^{dc}_{ab}-C^{dch}C_{hab}-\delta^c_b\delta^d_a=0$ can be written as $C^{dch}C_{hab}=\delta^{cd}_{ab}$.

Thus proved the following.

Theorem 5.5. Let M – *SGK*-manifold of type II. Then the following conditions are equivalent:

1) M is a manifold of class R_4;
2) on the space of the associated *G*-structure $C^{dch}C_{hab}=\delta^{cd}_{ab}$;
3) on the space of the associated *G*-structure $A^{cd}_{ab}=\frac{1}{2}\tilde{\delta}^{cd}_{ab}$,

and as an intermediate result can be formulated in the following theorem.

Theorem 5.6. Tensor Φ-holomorphic sectional curvature *SGK*-manifold of type II class R_4 has the form:

$$A(Z,X,Y) = -\frac{1}{2}\{\Phi^2 X\langle \Phi Y, \Phi Z\rangle + \Phi X\langle Y, \Phi Z\rangle\};\ \forall X,Y,Z \in \mathcal{X}(M).$$

Consider the relations $R^0_{b\hat{c}\hat{d}} = 0, R^a_{b\hat{c}\hat{d}} = 0,\ R^{\hat{a}}_{b\hat{c}\hat{d}} = 2C_{abh}C^{hcd} - 2\delta^{[c}_a\delta^{d]}_b$, i.e. $R^i_{b\hat{c}\hat{d}} = 2C_{\hat{\imath}bh}C^{hcd} - 2\delta^{[c}_{\hat{\imath}}\delta^{d]}_b$. The last equality can be written as $R(\varepsilon_{\hat{c}}, \varepsilon_{\hat{d}})\varepsilon_b = -2\nabla_{\varepsilon_{\hat{d}}}(C)(\varepsilon_b, \varepsilon_{\hat{d}}) + \varepsilon_{\hat{d}}\langle\varepsilon_b, \varepsilon_{\hat{c}}\rangle - \varepsilon_{\hat{c}}\langle\varepsilon_b, \varepsilon_{\hat{d}}\rangle$. Since the vectors $\{\varepsilon_a\}$ form a basis of the subspace $\left(D^{\sqrt{-1}}_\Phi\right)_p$, and the vectors $\{\varepsilon_{\hat{a}}\}$ form a basis of the subspace $\left(D^{-\sqrt{-1}}_\Phi\right)_p$ and projectors of module $\mathcal{X}(M)^C$ to submodules $D^{\sqrt{-1}}_\Phi, D^{-\sqrt{-1}}_\Phi$ are endomorphisms $\pi = \sigma \circ \ell = -\frac{1}{2}\left(\Phi^2 + \sqrt{-1}\Phi\right), \bar{\pi} = \bar{\sigma} \circ \ell = \frac{1}{2}\left(-\Phi^2 + \sqrt{-1}\Phi\right)$, identity $R(\varepsilon_{\hat{c}}, \varepsilon_{\hat{d}})\varepsilon_b = -2\nabla_{\varepsilon_{\hat{d}}}(C)(\varepsilon_b, \varepsilon_{\hat{d}}) + \varepsilon_{\hat{d}}\langle\varepsilon_b, \varepsilon_{\hat{c}}\rangle - \varepsilon_{\hat{c}}\langle\varepsilon_b, \varepsilon_{\hat{d}}\rangle$ can be rewritten in the form

$$R\left(-\Phi^2X + \sqrt{-1}\Phi X, -\Phi^2Y + \sqrt{-1}\Phi Y\right)\left(\Phi^2Z + \sqrt{-1}\Phi Z\right) = -2\nabla_{-\Phi^2Y+\sqrt{-1}\Phi Y}(C)\left(-\Phi^2X + \sqrt{-1}\Phi X, \Phi^2Z + \sqrt{-1}\Phi Z\right) + \left(-\Phi^2Y + \sqrt{-1}\Phi Y\right)\langle -\Phi^2X + \sqrt{-1}\Phi X, \Phi^2Z + \sqrt{-1}\Phi Z\rangle - \left(-\Phi^2X + \sqrt{-1}\Phi X\right)\langle -\Phi^2Y + \sqrt{-1}\Phi Y, \Phi^2Z + \sqrt{-1}\Phi Z\rangle;\ \forall X,Y,Z \in \mathcal{X}(M).$$

Expanding this relation by linearity, and separating the real and imaginary parts of the resulting equation, we obtain an equivalent identity:

$$R(\Phi^2X, \Phi^2Y)\Phi^2Z + R(\Phi^2X, \Phi Y)\Phi Z + R(\Phi X, \Phi^2Y)\Phi Z - R(\Phi X, \Phi Y)\Phi^2Z = -2\nabla_{\Phi^2Y}(C)(\Phi^2X, \Phi^2Z) - 2\nabla_{\Phi^2Y}(C)(\Phi X, \Phi Z) - 2\nabla_{\Phi Y}(C)(\Phi^2X, \Phi Z) + 2\nabla_{\Phi Y}(C)(\Phi X, \Phi^2Z) + \Phi^2Y\langle\Phi^2X, \Phi^2Z\rangle + \Phi^2Y\langle\Phi X, \Phi Z\rangle + \Phi Y\langle\Phi^2X, \Phi Z\rangle - \Phi Y\langle\Phi X, \Phi^2Z\rangle - \Phi^2X\langle\Phi^2Y, \Phi^2Z\rangle - \Phi X\langle\Phi^2Y, \Phi Z\rangle - \Phi^2X\langle\Phi Y, \Phi Z\rangle + \Phi X\langle\Phi Y, \Phi^2Z\rangle;\ \forall X,Y,Z \in \mathcal{X}(M). \quad (43)$$

We say that the identity (41), the fifth additional identity curvature *SGK*-manifold of type II.

Definition 5.5. We call the *AC*-manifold manifold of class R_5, if its curvature tensor satisfies the equation:

$$R(\Phi^2X, \Phi^2Y)\Phi^2Z + R(\Phi^2X, \Phi Y)\Phi Z + R(\Phi X, \Phi^2Y)\Phi Z - R(\Phi X, \Phi Y)\Phi^2Z = 0;\ \forall X,Y,Z \in \mathcal{X}(M). \quad (44)$$

Let M – SGK-manifold of type II, which is a manifold of class R_5. Then its Riemann curvature tensor satisfies the condition (44), which in the space of the associated G-structure can be written as:

$$R^i_{jkl}(\Phi^2X)^k(\Phi^2Y)^l(\Phi^2Z)^j + R^i_{jkl}(\Phi^2X)^k(\Phi Y)^l(\Phi Z)^j +$$
$$R^i_{jkl}(\Phi X)^k(\Phi^2Y)^l(\Phi Z)^j - R^i_{jkl}(\Phi X)^k(\Phi Y)^l(\Phi^2Z)^j = 0.$$

In view of (11) and the form of the matrix Φ, the last equation can be written as: $R^d_{\hat{a}bc}X^bY^cZ_a + R^{\hat{d}}_{a\hat{b}\hat{c}}X_bY_cZ^a = 0$. This equality holds if and only if $R^d_{\hat{a}bc} = R^{\hat{d}}_{a\hat{b}\hat{c}} = 0$. According to (11), $C^{adh}C_{hbc} = \delta^a_{[b}\delta^d_{c]}$.

Thus, we have proved.

Theorem 5.7. *SGK*-manifold is a manifold of type II class R_5 if and only if the space of the associated G-structure $C^{adh}C_{hbc} = \delta^a_{[b}\delta^d_{c]}$.

Let's turn the equality $C^{adh}C_{hbc} = \delta^a_{[b}\delta^d_{c]}$ first in the indices a and b, then the indices c and d, we obtain $\sum_{a,b,c}|C_{abc}|^2 = \frac{1}{2}n(n-1)$. From the resulting equation implies that for $n = 1$, we have $C_{abc} = 0$, i.e. manifold is Kenmotsu.

Thus we have proved the following.

Theorem 5.8. *SGK*-manifold of type II class R_5 of 3-dimensional manifold is Kenmotsu.

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