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Effect of Radiation and Magnetohydrodynamic Free Convection Boundary Layer Flow on a Solid Sphere with Convective Boundary Conditions in a Micropolar Fluid

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ABSTRACT

In this paper, the effect of radiation on magnetohydrodynamic free convection boundary layer flow on a solid sphere with convective boundary conditions in a micropolar fluid, is considered. The basic nonlinear system of partial differential equations of boundary layer are first transformed into a non-dimensional form and are then solved numerically using an implicit finite difference scheme known as the Keller-box method. Numerical solutions are obtained for the local Nusselt number and the local skin friction coefficient, as well as the velocity and temperature profiles. The features of the flow and heat transfer characteristics for various values of the Prandtl number Pr, the material or micropolar parameter K, the magnetic parameter M, the radiation parameter N_R, the conjugate parameter γ and the coordinate running along the surface of the sphere, x are analyzed and discussed.

Keywords: Convective Boundary Conditions, Free Convection, Magnetohydrodynamic (MHD), Micropolar Fluid, Radiation Effects, Solid Sphere.

1. Introduction

The effect of radiation on boundary layer flow and heat transfer problems can be quite significant at high operating temperature such as gas turbines, nuclear power plant, and thermal energy store (Bataller (2008a)). Since the process in engineering areas occurs at high temperature, the study on the effect of radiation becomes quite significant for the design of the equipment. Molla et al. (2011), Akhter and Alim (2008) and Miraj et al. 2010 studied the radiation effect on free convection flow from an isothermal sphere in viscous fluid with constant wall temperature, surface heat flux and in presence of heat generation, respectively.

The application of the magnetohydrodynamic (MHD) plays an important role in agriculture, engineering and petroleum industries (Ganesan and Palani (2004)). Alam et al. (2007) and Molla et al. (2005) studied the viscous dissipation and MHD effects on natural convection flow over a sphere in a viscous fluid in the presence of heat generation.

The essence of the theory of micropolar fluid flow lies in the extension of the constitutive equation for Newtonian fluid, so that more complex fluids such as particle suspensions, liquid crystal, animal blood, lubrication, and turbulent shear flows can be described by this theory. The theory of micropolar fluid was first proposed by Eringen (1966) and has been further considered by many researchers. Nazar et al. (2002a, 2002b) considered the free convection boundary layer flows on a sphere in a micropolar fluid with constant wall temperature and constant heat flux, respectively. This paper has been extended by Cheng (2008) to micropolar fluid with constant wall temperature and concentration, while Salleh et al. (2012) extended it to a micropolar fluid with Newtonian heating. We notice, however, that the previous papers studied free convection boundary layer flows on a sphere without effects of radiation and magnetohydrodynamic. It should be mentioned that the mathematical background of the micropolar fluid flow theory is presented in the books by Eringen (2001) and Łukaszewicz (1984) and in the review papers by Ariman et al. (1973, 1974).

On the other hand, convective boundary conditions, namely when heat is supplied through a bounding surface of finite thickness and finite capacity, is the type of boundary condition that has been given much attention recently. The interface temperature is not known a priori for problems of convective boundary conditions, but depends on the intrinsic properties of the systems. This heating process is called conjugate or convective boundary conditions (Merkin (1994)).

Aziz (2009) studied a similarity solution for the forced convection flow and thermal boundary layer over a flat plate with a convective surface boundary condition. The forced convection flow of a uniform stream over a flat surface with a convective surface boundary condition has been studied also by Merkin and Pop (2011). Yao et al. (2011) presented the heat transfer of a viscous fluid flow over a stretching/shrinking sheet with a convective boundary condition. Recently, the numerical solution for stagnation point flow over a stretching surface with convective boundary conditions using the shooting method has been studied by Mohamed et al. (2013).

Therefore, the objective of the present paper is to study numerically the effect of radiation on magnetohydrodynamic free convection boundary layer flow problem past a solid sphere with convective boundary conditions in a micropolar fluid. The governing boundary layer equations are first transformed into a system of non-dimensional equations via the non-dimensional variables, and then into non-similar equations before they are solved numerically by the Keller-box method, as described in the book by Cebeci and Bradshaw (1988).

2. Mathematical Analyses

Consider a heated sphere of radius a, which is immersed in a viscous and incompressible micropolar fluid of ambient temperature T_∞. The surface of the sphere is subjected to a convective boundary conditions, as shown in Figure 1. The gravity vector g acts downward in the opposite direction, where the coordinates $\bar{x}$ and $\bar{y}$ are chosen such that $\bar{x}$ measures the distance along the surface of the sphere from the lower stagnation point and $\bar{y}$ measures the distance normal to the surface of the sphere.

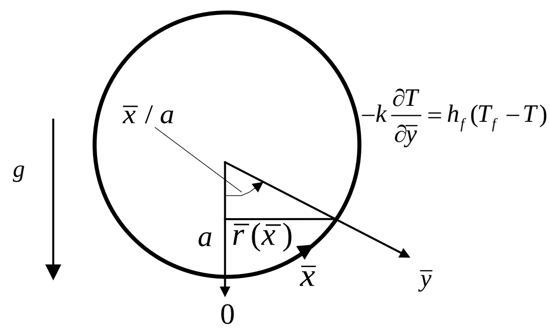

Figure 1: Physical model and coordinate system

We assume that the equations are subjected to convective boundary conditions of the form proposed by Aziz (2009). Under the Boussinesq and boundary layer approximations, the basic equations are

$$\frac{\partial}{\partial \bar{x}}(\bar{r}\,\bar{u}) + \frac{\partial}{\partial \bar{y}}(\bar{r}\,\bar{v}) = 0 \tag{1}$$

$$\rho\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = (\mu+\kappa)\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \rho g\beta(T-T_\infty)\sin\left(\frac{\bar{x}}{a}\right) + \kappa\frac{\partial \bar{H}}{\partial \bar{y}} - \frac{\sigma\beta^2}{\rho}\bar{u} \tag{2}$$

$$\rho j\left(\bar{u}\frac{\partial \bar{H}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{H}}{\partial \bar{y}}\right) = -\kappa\left(2\bar{H} + \frac{\partial \bar{u}}{\partial \bar{y}}\right) + \varphi\frac{\partial^2 \bar{H}}{\partial \bar{y}^2} \tag{3}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha\frac{\partial^2 T}{\partial \bar{y}^2} - \frac{1}{\rho c_\rho}\frac{\partial q_r}{\partial \bar{y}} \tag{4}$$

These equations are subjected to the boundary conditions of (Salleh et al. (2012); Aziz (2009))

$$\bar{u} = \bar{v} = 0,\ -k\frac{\partial T}{\partial \bar{y}} = h_f\,(T_f - T)\ ,\ \bar{H} = -n\frac{\partial \bar{u}}{\partial \bar{y}}\ \text{as}\ \bar{y} = 0,$$
$$\bar{u} \to 0, T \to T_\infty,\ H \to 0,\ \text{as}\ \bar{y} \to \infty, \tag{5}$$

where $\bar{u}$ and $\bar{v}$ are the velocity components along the $\bar{x}$ and $\bar{y}$ directions, respectively, $\bar{H}$ is the angular velocity of micropolar fluid, q_w is the surface heat flux, q_r is the radiative heat flux, κ is the vortex viscosity, T is the local temperature, T_f is the temperature of the hot fluid, g is the gravity acceleration, k is the thermal conductivity, σ is the electric conductivity, α is the thermal diffusivity, β is the thermal expansion coefficient, ν is the kinematic viscosity, μ is the dynamic viscosity, ρ is the fluid density, c_ρ is the specific heat, j is the microinertia density and h_f is the heat transfer coefficient for the convective boundary conditions. It is worth mentioning that in boundary conditions (5), n is constant and $0 \le n \le 1$. The value $n = 0$, which leads to $\bar{H} = 0$ at the wall, represents concentrated particle flows in which the particle density is sufficiently great that microelements close to the wall are unable to rotate or is called “strong” concentration of microelements (Jena, (1980) and Mathur, (1980)).

The case corresponding to $n = 1/2$ results in the vanishing of antisymmetric part of the stress tensor and represents "weak" concentration of microelements Mathur, (1980). In this case, the particle rotation is equal to fluid vorticity at the boundary for fine particle suspension. When $n = 1$, we have flows which are representative of turbulent boundary layer (Mathur (1980)). The case of $n = 1/2$ is considered in this paper.

Let $\bar{r}(\bar{x}) = a\sin(\bar{x}/a)$ be the radial distance from the symmetrical axis to the surface of the sphere and we assume (see Rees and Bassom (1996) or Rees and Pop (1998)) that the spin gradient viscosity φ are given by

$$\varphi = (\mu + \kappa/2)\, j \tag{6}$$

We introduce now the following non-dimensional variables (Salleh et al. (2012); Aziz (2009)):

$$x = \frac{\bar{x}}{a},\ y = Gr^{1/4}\left(\frac{\bar{y}}{a}\right),\ r = \frac{\bar{r}}{a},$$
$$u = \left(\frac{a}{\nu}\right)Gr^{-1/2}\bar{u},\ v = \left(\frac{a}{\nu}\right)Gr^{-1/4}\bar{v},\ H = \left(\frac{a^2}{\nu}\right)Gr^{-3/4}\bar{H},$$
$$\theta = \frac{T - T_\infty}{T_f - T_\infty} \tag{7}$$

where $Gr = g\beta(T_f - T_\infty)a^3/\nu^2$ is the Grashof number for convective boundary conditions. Using the Rosseland approximation for radiation, the radiative heat flux is simplified as (see Bataller (2008b))

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial \bar{y}}, \tag{8}$$

where σ^* and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. We assume that the temperature differences within the flow through the micropolar fluid such as that the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, we get

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{9}$$

Substituting (6)–(9) into Eqs. (1)–(4), we obtain the following non-dimensional equations of the problem under consideration:

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial y}(rv)=0, \tag{10}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=(1+K)\frac{\partial^2 u}{\partial y^2}+\theta\sin x-Mu+K\frac{\partial H}{\partial y} \tag{11}$$

$$u\frac{\partial H}{\partial x}+v\frac{\partial H}{\partial y}=-K\left(2H+\frac{\partial u}{\partial y}\right)+\left(1+\frac{K}{2}\right)\frac{\partial^2 H}{\partial y^2}, \tag{12}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\left(1+\frac{4}{3}N_R\right)\frac{\partial^2 \theta}{\partial y^2}, \tag{13}$$

where $K=\kappa/\mu$ is the material or micropolar parameter, $\Pr=\nu/\alpha$ is the Prandtl number, $M=\sigma\beta^2 a^2/\nu\rho Gr^{1\backslash 2}$ is the magnetic parameter and $N_R=\alpha k^{*}\rho c_{\rho}/4\sigma^{*}T_{\infty}^{3}$ is the radiation parameter. The boundary conditions (5) become

$$u=v=0,\ \frac{\partial \theta}{\partial y}=-\gamma(1-\theta),\ H=-\frac{1}{2}\frac{\partial u}{\partial y}\ \text{ at } y=0,$$
$$u\to 0,\ \theta\to 0,\ H\to 0 \text{ as } y\to\infty, \tag{14}$$

where $\gamma=ah_f Gr^{-1/4}/k$ is the conjugate parameter for convective boundary condition. It is noticed that, if $\gamma\to\infty$ then we have $\theta(0)=1$, which is the constant wall temperature and this case has been studied by Nazar et al. (2002a) .

To solve (10) to (13), subjected to the boundary conditions (14), we assume the following variables:

$$\psi=xr(x)f(x,y),\quad \theta=\theta(x,y),\ H=xh(x,y), \tag{15}$$

where ψ is the stream function defined as

$$u=\frac{1}{r}\frac{\partial \psi}{\partial y}\ \text{ and } v=-\frac{1}{r}\frac{\partial \psi}{\partial x}, \tag{16}$$

which satisfies the continuity equation (10). Thus, (11) to (13) become

$$(1+K)\frac{\partial^3 f}{\partial y^3}+\left(1+x\cot x\right) f\,\frac{\partial^2 f}{\partial y^2}-\left(\frac{\partial f}{\partial y}\right)^2+\frac{\sin x}{x}\theta-M\frac{\partial f}{\partial y}+K\frac{\partial h}{\partial y}= x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}-\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right), \tag{17}$$

$$\left(1+\frac{K}{2}\right)\frac{\partial^2 h}{\partial y^2}+\left(1+x\cot x\right) f\,\frac{\partial h}{\partial y}-\frac{\partial f}{\partial y}h-K\left(2h+\frac{\partial^2 f}{\partial y^2}\right)= x\left(\frac{\partial f}{\partial y}\frac{\partial h}{\partial x}-\frac{\partial f}{\partial x}\frac{\partial h}{\partial y}\right), \tag{18}$$

$$\frac{1}{\Pr}\left(1+\frac{4}{3}N_R\right)\frac{\partial^2\theta}{\partial y^2}+\left(1+x\cot x\right)f\,\frac{\partial\theta}{\partial y}=x\left(\frac{\partial f}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial f}{\partial x}\frac{\partial\theta}{\partial y}\right), \tag{19}$$

subject to the boundary conditions

$$f=\frac{\partial f}{\partial y}=0,\quad \frac{\partial\theta}{\partial y}=-\gamma(1-\theta),\quad h=-\frac{1}{2}\frac{\partial^2 f}{\partial y^2}\ \text{ at } y=0,$$
$$\frac{\partial f}{\partial y}\to 0,\quad \theta\to 0,\quad h\to 0\ \text{ as }\ y\to\infty. \tag{20}$$

It can be seen that at the lower stagnation point of the sphere, $x\approx 0$, equations (17) to (19) reduce to the following nonlinear system of ordinary differential equations:

$$(1+K)f'''+2ff''-f'^2+\theta-Mf'+Kh'=0, \tag{21}$$

$$\left(1+\frac{K}{2}\right)h''+2fh'-f'h-K(2h+f'')=0, \tag{22}$$

$$\frac{1}{\Pr}\left(1+\frac{4}{3}N_R\right)\theta''+2f\,\theta'=0 \tag{23}$$

The boundary conditions (20) become

$$f(0)=f'(0)=0,\quad \theta'(0)=-\gamma(1-\theta(0)),\quad h(0)=-\frac{1}{2}f''(0),$$

$$f\,' \to 0,\ \theta \to 0,\ h \to 0 \text{ as } y \to \infty, \tag{24}$$

where primes denote differentiation with respect to *y*.

The physical quantities of interest in this problem are the local skin friction coefficient C_f and the Nusselt number N_u, and they can be written as

$$C_f = \frac{Gr^{-3/4}a^2}{\mu\nu}\tau_w,\ N_u = \frac{aGr^{-1/4}}{k\left(T_f - T_\infty\right)}q_w, \tag{25}$$

where

$$\tau_w = \left(\mu + \frac{\kappa}{2}\right)\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)_{\bar{y}=0},\ q_w = -k\left(\frac{\partial T}{\partial \bar{y}}\right)_{\bar{y}=0} + q_r \tag{26}$$

Using the non-dimensional variables (7) to (9) and the boundary conditions (14) the local skin friction coefficient C_f and the local Nusselt number N_u are

$$C_f = \left(1 + \frac{K}{2}\right)x\,\frac{\partial^2 f}{\partial y^2}(x,0),\ N_u = \gamma\left(1 + \frac{4}{3}N_R\right)\left(1 - \theta(x,0)\right) \tag{27}$$

3. Results and Discussion

The nonlinear partial differential equations (17) to (19) subject to the boundary conditions (20) were solved numerically using an efficient, implicit finite-difference method known as the Keller-box scheme for convective boundary conditions with several parameters considered, namely the micropolar parameter *K*, the magnetic parameter *M*, the radiation parameter N_R, the Prandtl number Pr, the conjugate parameter γ and the coordinate running along the surface of the sphere, *x*.

The numerical solutions start at the lower stagnation point of the sphere, $x \approx 0$, with initial profiles as given by equations (21) to (23) and proceed round the sphere up to $x = 90^o$.

The heat transfer coefficient $-(\partial\theta/\partial y)(x,0)$ at the lower stagnation point of the sphere, $x \approx 0$, for various values of *K* when Pr = 0.7, 7, without effect of radiation and magnetohydrodynamic (i.e. *M* = 0, $N_R = 0$) and $\gamma \to \infty$ are shown in Table 1. In order to verify the accuracy of the present method, the present results are compared with those reported by Huang and Chen (1987) and Nazar *et al.* (2002a). It is found that the agreement between the previously published results with the present ones is excellent. We can conclude that this method works efficiently for the present problem and we are also confident that the results presented here are accurate.

TABLE 1: Values of the heat transfer coefficient $-(\partial\theta/\partial y)(x,0)$ at the lower stagnation point of the sphere, $x \approx 0,$ for various values of *K* when Pr = 0.7, 7, without the effect of radiation and magnetohydrodynamic (i.e. *M* = 0, $N_R = 0$) and $\gamma \to \infty$

Pr	0.7			7		
K	Huang and Chen (1987)	Nazar *et al.*(2002a)	Present	Huang and Chen (1987)	Nazar *et al.*(2002a)	Present
0	0.4574	0.4576	0.457582	0.9581	0.9595	0.959498
0.5	-	0.4336	0.433616	-	0.8905	0.890523
1	-	0.4166	0.416577	-	0.8443	0.844347
1.5	-	0.4035	0.403509	-	0.8096	0.809569
2	-	0.3930	0.393023	-	0.7805	0.780481

Table 2 shows the values of the wall temperature $\theta(x,0),$ the heat transfer coefficient $-(\partial\theta/\partial y)(x,0)$ and the skin friction coefficient $(\partial^2 f/\partial y^2)(x,0),$ at the lower stagnation point of the sphere, $x \approx 0,$ for various values of N_R when Pr = 0.7, *K* = 2, $\gamma = 0.1$ and *M* = 0, 5. It is observed that, when the magnetic parameter *M* is fixed, an increase in the radiation parameter N_R, causes the values of $\theta(x,0),$ $-(\partial\theta/\partial y)(x,0)$ and $(\partial^2 f/\partial y^2)(x,0),$ to increase. Also when N_R is fixed, and *M* increases, the value of $\theta(x,0),$ increases but the values of $(\partial^2 f/\partial y^2)(x,0),$ and $-(\partial\theta/\partial y)(x,0)$ decrease.

TABLE 2: Values of the wall temperature $\theta(x,0),$ the heat transfer coefficient $-(\partial\theta/\partial y)(x,0)$ and the skin friction coefficient $(\partial^2 f/\partial y^2)(x,0)$ at the lower stagnationpoint of the sphere, $x \approx 0,$ for various values of N_R when Pr = 7, *K* = 2, *M* = 0, 5, and $\gamma = 0.1$

	M = 0			*M* = 5		
N_R	$\theta(x,0)$	$-(\partial\theta/\partial y)$	$(\partial^2 f/\partial y^2)$	$\theta(x,0)$	$-(\partial\theta/\partial y)$	$(\partial^2 f/\partial y^2)$
0	0.159324	0.084067	0.077444	0.215447	0.078455	0.055249
1	0.190971	0.188773	0.098184	0.264358	0.171650	0.067778
2	0.209968	0.289678	0.111751	0.293693	0.258979	0.075365

3	0.224033	0.387983	0.122225	0.314982	0.342509	0.080891
4	0.235386	0.484256	0.130896	0.331738	0.423232	0.085250
5	0.244992	0.578840	0.138354	0.345481	0.501798	0.088828

Figures 2, 3 and 4 show the temperature $\theta(0,y)$, velocity $(\partial f/\partial y)(0,y)$ and angular velocity profiles $h(0,y)$ when Pr = 7, K = 1, M = 5, $N_R = 0,1,3,5$ and $\gamma = 0.1$, respectively. It is found that as N_R increases, the temperature, velocity and angular velocity profiles increase.

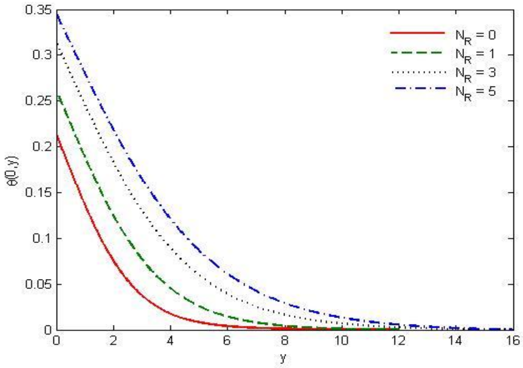

Figure 2: Temperature profiles $\theta(0,y)$ when Pr = 7, M= 5, K = 1, $N_R = 0,1,3,5$ and $\gamma = 0.1$

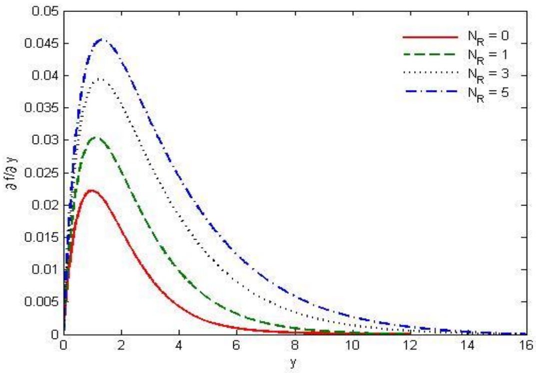

Figure 3: Velocity profiles $(\partial f/\partial y)(0,y)$ when Pr = 7, K = 1, M= 5, $N_R = 0,1,3,5$ and $\gamma = 0.1$

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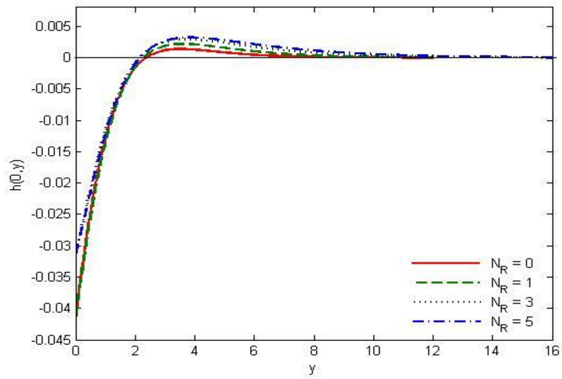

Figure 4: Angular velocity profiles $h(0, y)$ when Pr = 7, K = 1, M = 5, $N_R = 0,1,3,5$ and $\gamma = 0.1$

The temperature $\theta(0, y)$, velocity $(\partial f/\partial y)(0, y)$ and angular velocity profiles $h(0, y)$ are presented in Figures 5, 6 and 7, when Pr = 7, K = 1, $N_R = 1$, M = 0, 5, 10 and $\gamma = 0.1$, respectively. These figures show that when the value of M increases, it is found that the temperature profiles also increase, but the velocity and angular velocity profiles decrease.

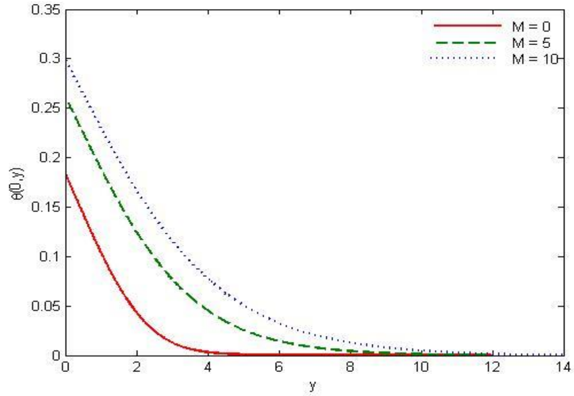

Figure 5: Temperature profiles $\theta(0, y)$ when Pr = 7, K = 1, $N_R = 1$, M = 0, 5, 10 and $\gamma = 0.1$

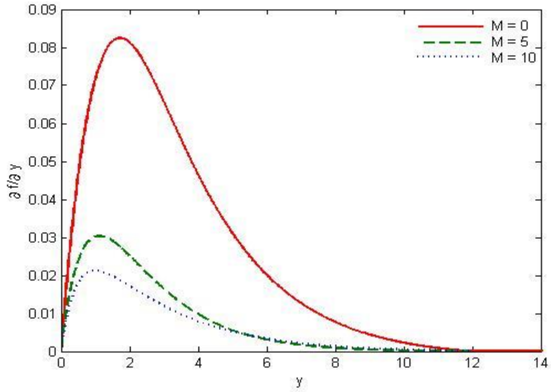

Figure 6: Velocity profiles $(\partial f/\partial y)(0, y)$ when Pr = 7, K = 1, $N_R = 1$, M = 0, 5, 10 and $\gamma = 0.1$

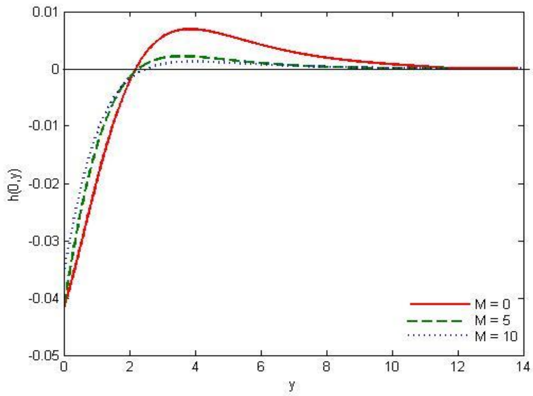

Figure 7: Angular velocity profiles $h(0, y)$ when Pr = 7, K = 1, $N_R = 1$, M = 0, 5, 10 and $\gamma = 0.1$

Variations of the local Nusselt number N_u and the local skin friction coefficient C_f with various values of x when Pr = 0.7, K = 1 $N_R = 1$, M =0, 5, 10 and $\gamma = 0.1$ are plotted in Figures 8 and 9, respectively. It is found that as M increases, both values of the local Nusselt number and the local skin friction coefficient decrease.

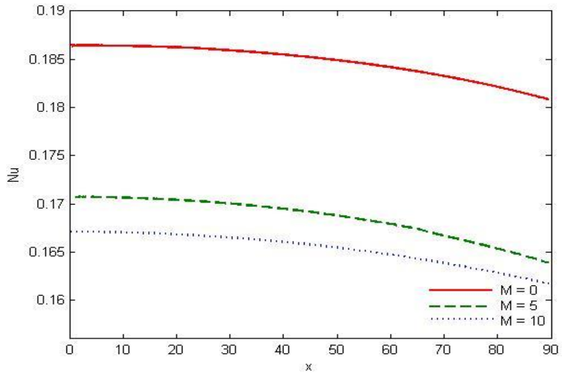

Figure 8: Variation of the local Nusselt number N_u with x when Pr = 0.7, K = 1, $N_R = 1$, M = 0, 5, 10 and $\gamma = 0.1$

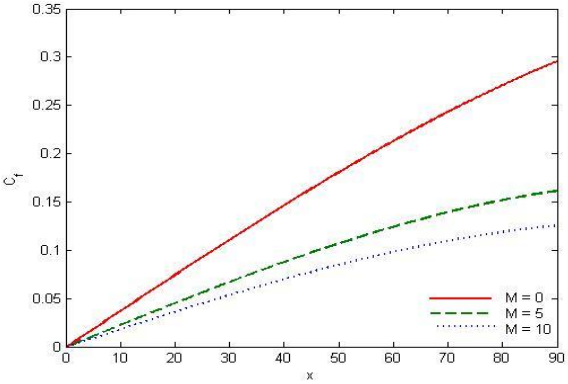

Figure 9: Variation of the local skin friction coefficient, C_f with x when Pr = 0.7, K = 1, $N_R = 1$, M = 0, 5, 10 and $\gamma = 0.1$

Figures 10 and 11 display the local Nusselt number N_u and the local skin friction coefficient C_f with various values of x when Pr = 0.7, K = 1, M = 5, $N_R = 0,1,3,5$ and $\gamma = 0.1$, respectively. It is found that as N_R increases, both values of the local Nusselt number and the local skin friction coefficient increase. We notice from Figures 8 and 10 that the value of N_u is higher at $x = 0^o$ than those at $0^o < x \leq 90^o$, because the sphere temperature is almost equal to fluid temperature at $x = 0^o$, and has a different value when $0^o < x \leq 90^o$.

From Figures 9 and 11, it is found that the value of $C_f = 0$ at $x = 0^o$, because at this point the value of the wall shear stress τ_w is very small, and the maximum value of C_f occurs when $x = 90^o$, because in this case the value of τ_w is very high.

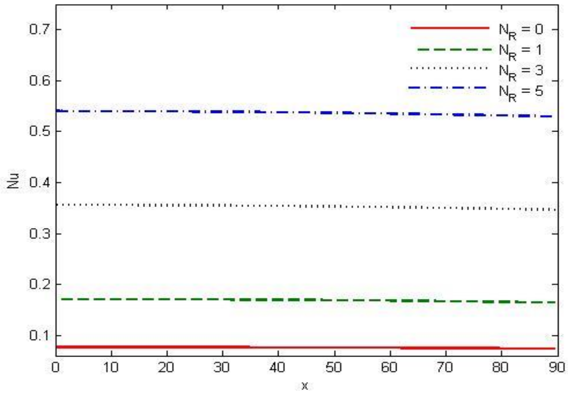

Figure 10: Variation of the local Nusselt number N_u with *x* when Pr = 0.7, *K* = 1, *M*= 5, $N_R = 0,1,3,5$ and $\gamma = 0.1$

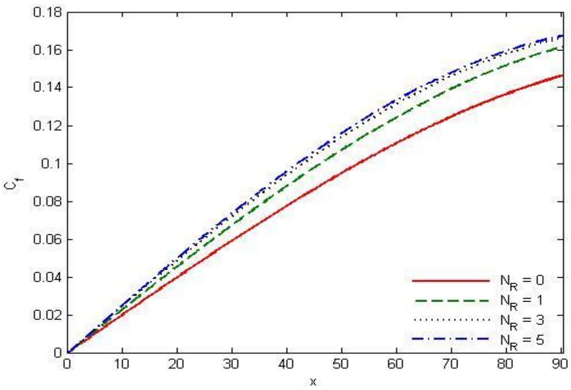

Figure 11: Variation of the local skin friction coefficient, C_f with *x* when Pr = 0.7, *K* = 1, *M* = 5, $N_R = 0,1,3,5$ and $\gamma = 0.1$

4. Conclusions

In this paper, we have numerically studied the effect of radiation on magnetohydrodynamic free convection boundary layer flow on a solid sphere with convective boundary conditions. It shows how the Prandtl number Pr, the magnetic parameter M, the thermal radiation parameter N_R, the micropolar parameter K, the conjugate parameter γ and the coordinate running along the surface of the sphere, x affect the values of the local Nusselt number N_u, the local skin friction coefficient C_f as well as the temperature, velocity and angular velocity profiles.

We can conclude that

- when Pr, γ and M are fixed, as N_R increases, the temperature, velocity and angular velocity profiles, as well as the skin friction coefficient and the heat transfer coefficient increase. When Pr, γ and N_R are fixed, as M increases, the temperature profiles increase, but the velocity and angular velocity profiles decrease.
- when Pr, γ and N_R are fixed, as M increases, both values of the local Nusselt number and the local skin friction coefficient decrease. If Pr, γ and M are fixed, as N_R increases, the local Nusselt number and the local skin friction coefficient increase.

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